ULTRA WIDEBAND SYSTEMS WITH MIMO
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Preface

High data-rate wireless communications, nearing 1 Gb/s transmission rates, are of interest in emerging wireless local area networks (WLANs) and in home audio/video network applications, such as high-speed high-definition television (HDTV) audio/video streams. Currently, WLANs offer peak rates of 54 Mb/s, and a target of 600 Mb/s, such as IEEE 802.11n WLANs, will soon be realized. The IEEE 802.15.3c wireless personal area networks will allow very high data rates over 2 Gb/s for applications such as high-speed internet access, streaming content download (for example, video on demand, HDTV, home theatre), real-time streaming and wireless data bus for cable replacement. Optional data rates in excess of 3 Gb/s will be provided. However, to achieve, say, data rates greater than 50 Mb/s, some technologies such as multiple transmit and multiple receive antennas (MIMO) and orthogonal frequency-division multiplexing should be adopted, as recommended in IEEE 802.11n. To reach the target of 1 Gb/s, more advanced techniques should be used. Ultra wideband (UWB) technology combined with MIMO might provide a solution.

As is well known, a UWB system can make use of a huge frequency band from 3.1 to 10.6 GHz. This provides great potential for increasing data transmission rates according to the Shannon theorem. However, owing to the regulations imposed by the Federal Communications Commission (FCC) in the USA and the European Commission document in Europe, the power spectral density of the transmitted UWB signal is rather limited. This again limits data transmission rates. Incorporating the MIMO technique into UWB provides a viable solution for the bottleneck problem of power limit. For example, if space–time coding is used, the power for a specific transmitted symbol will be strengthened, while the overall transmitted power is still the same as that of a single-transmit-antenna system, thus satisfying the FCC regulation. If a beamforming technique is employed, the power of the signal in a specific direction is increased and may violate the power spectral mask in this direction, while the power in all other directions is still the same as the case without using the beamformer. This will not cause big problems for the indoor applications of UWB.

In this book, we will investigate the benefits of combining UWB and MIMO. We will highlight five aspects of this promising research field: channel capacity, space–time coding, beamforming and localization, time-reversal transmission, and UWB-MIMO relay. The channel capacity describes a limit of the benefits in some sense for a UWB system employing multiple antennas, while the space–time coding provides a realization tool towards reaching the limit. UWB beamforming is of great importance for indoor localization, which has become a hot topic in UWB applications. The time-reversal technique combined with UWB radios provides a nontraditional yet promising way for robust
secure and/or multi-user wireless communications. UWB-MIMO relay provides an important tool for UWB communications in the environments where the direct link between communication partners is blocked or the distance is too large.

The book is organized as follows. Chapter 1 (Introduction) reviews the basics of UWB technology, the principle of MIMO, and the state-of-the-art UWB-MIMO. In Chapter 2 (UWB-MIMO Channel Measurement and Models), several commonly used UWB channel models and a typical spatial correlation model of UWB-MIMO are briefly introduced, which will form the basis of subsequent chapters. The channel sounding equipment developed in our institute and the ray tracing simulation tool will also be presented. In Chapter 3 (UWB Channel Capacity) we investigate how the channel capacity of UWB-MIMO radios depends on the numbers of transmit and receive antennas. In Chapter 4 (UWB-MIMO Space–Time Coding), two Alamouti-like space–time coding schemes and their performances are first reviewed. Then we discuss the space–time coding technique for UWB-MIMO systems with arbitrary numbers of transmit and receive antennas based on the theory of real orthogonal design. Finally, a review of the spatio-frequency multiplexing problem and spatio-time–frequency coding will be presented for multiband UWB systems. In Chapter 5 (UWB Beamforming and Localization) we examine the challenge faced by UWB technology in the problems of beamforming and localization and advantages/opportunities offered also by the UWB technology due to its very narrow pulses and many multipaths. Several methods/ideas are presented. Actually, many kinds of diversified methods by exploiting the advantages can be invented to deal with the challenge in this hot topic. In Chapter 6 (Time-Reversal UWB Systems) we extend the time-reversal technique to UWB-MIMO and investigate several fundamental issues in time-reversal UWB-MIMO systems, such as the motivation, robustness, multiple accessing and the corresponding pre-equalizer design problem. Chapter 7 (UWB Relay Systems) deals with the design and performance analysis of two-hop UWB relay systems which can be equipped with multiple relays and multiple transmit/receive antennas at the source, relay, destination, or all of them.

Chapters 1 and 2 provide a basis for the whole book, while other chapters are relatively independent of each other. The only exception is that Chapter 7 uses the results of Section 4.4.

This book is targeted to graduate students and high-level undergraduate students, researchers in academia, and practising engineers in the field of wireless communications or other related areas. The book can be used as both a reference book for advanced research and a textbook for graduate students. We try our best to make it self-contained, but some preliminary background on probability theory, matrix theory and wireless communications is helpful.
Acknowledgements

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Abbreviations

AF amplify-and-forward
AGC automatic gain control
AoA angle of arrival
AoD angle of departure
AoF amount of fading
AWG arbitrary waveform generator
AWGN additive white Gaussian noise
BER bit error rate
BPAM binary pulse amplitude modulation
cdf cumulative density function
CDMA code division multiple access
CIR channel impulse response
CRB Cramer–Rao bound
CROD companion of a real orthogonal design
CSI channel state information
DCF decouple-and-forward
DCTR differential coded transmitted-reference
DF decode-and-forward
DFT discrete-time Fourier transform
DoA direction of arrival
DPO digital phosphor oscilloscope
DS direct sequence
DTF detect-and-forward
DTR differential transmitted-reference
EIRP equivalent isotropically radiated power
FEQ frequency-domain equalizer
FFT fast Fourier transform
FIR finite impulse response
GI guard interval
GTD geometrical theory of diffraction
HDTV high-definition television
IC inverse channel
IDFT inverse discrete-time Fourier transform
IF intermediate frequency
IFFT inverse fast Fourier transform
IR | impulse radio  
ISI | intersymbol interference  
ISM | industrial, scientific and medical  
LNA | low-noise amplifier  
LOS | line-of-sight  
LPF | low-pass filter  
LS | least squares  
MB | multiband  
MGF | moment-generating function  
MISO | multiple transmit antennas and single receive antenna, or multiple-input single-output  
MIMO | multiple transmit and multiple receive antennas, or multiple-input multiple-output  
MMSE | minimum mean square error  
ML | maximum likelihood  
MLE | maximum likelihood estimation  
MRC | maximum ratio combiner, maximum ratio combining  
MSE | mean square error  
MSI | multi-stream interference  
MUI | multiuser interference  
NLOS | non-line-of-sight  
OFDM | orthogonal frequency-division multiplexing  
OPSA | optimal power spectrum allocation  
PAM | pulse-amplitude modulation  
pdf | probability density function  
PPM | pulse position modulation  
PSD | power spectral density  
QAM | quadrature amplitude modulation  
RF | radio frequency  
RMS | root mean square  
ROD | real orthogonal design  
RSS | received signal strength  
RX | receiver  
SDMA | space-division multiple access  
SIMO | single transmit antenna and multiple receive antennas, or single-input multiple-output  
SINR | power ratio between signal and interference-plus-noise  
SIR | signal-to-interference (power) ratio  
SISO | single transmit and single receive antennas, or single-input single-output  
SM | spatial multiplexing  
SNR | signal-to-noise (power) ratio  
SS | spread spectrum  
STC | space–time coding  
STDL | stochastic tapped delay line  
S-V | Saleh–Valenzuela
TDoA  time difference of arrival
TH   time hopping
ToA  time of arrival
TR   time reversal
TX   transmitter
UPSA uniform power spectrum allocation
UWB  ultra wideband
VBLAST vertical Bell Laboratory layered space–time
VHDR very high data rate
VGA  voltage-gain amplifier
VNA  vector network analyser
WLAN wireless local area network
WPAN wireless personal area network
ZF   zero-forcing
1 Introduction

1.1 Introduction

High data-rate wireless communications, nearing 1 Gb/s transmission rates, are of interest in emerging wireless local area networks (WLANs) and in home audio/video network applications, such as high-speed high-definition television (HDTV) audio/video streams [189]. Currently, WLANs offer peak rates up to 54 Mb/s, and a target of 600 Mb/s is promised to be realized in the near future, for example, in IEEE 802.11n WLANs [72]. The IEEE 802.15.3c wireless personal area networks (WPANs) will allow very high data rates of over 2 Gb/s for applications such as high-speed internet access, streaming content download (for example, video on demand, HDTV, home theatre), real-time streaming, and wireless data bus for cable replacement. Optional data rates in excess of 3 Gb/s will be provided. However, to achieve, say, more than 50 Mb/s data rates, some technologies such as multiple transmit and multiple receive antennas (MIMO) and orthogonal frequency-division multiplexing (OFDM) should be adopted, as recommended in IEEE 802.11n. To reach the target of 1 Gb/s, more advanced techniques should be used. Ultra wideband (UWB) technology combined with MIMO might provide a solution.

As is well known, a UWB system [22, 165, 243, 264] can make use of the huge frequency band from 3.1 to 10.6 GHz in the USA [63] and Asia [49] and at least 6.0 to 8.5 GHz in Europe [105]. This provides great potential for increasing data transmission rates according to the Shannon theorem. However, owing to the regulations imposed by the Federal Communications Commission (FCC) in the USA [63] and the European Commission (EC) document in Europe [105], the permitted power spectral density of a UWB signal is rather limited. This again limits data transmission rates. Incorporating the MIMO technique into UWB provides a viable solution for the bottleneck problem of power limit. For example, if space–time coding (STC) is used, the power for a specific transmitted symbol will be strengthened, while the overall transmitted power is still the same as that of a single-transmit-antenna system, thus satisfying the FCC regulation (see Chapter 4). If a beamforming technique is employed, the power of the signal in a specific direction is increased and may violate the power spectral mask in this direction.

1 In Europe, the frequency band from 1.6 to 10.6 GHz can be used, but a stricter spectral mask is specified than that in the USA. For the details, see [105].
while the power in all other directions is still the same as the case without using the beamformer (see Chapter 5 about the side lobe discussion of UWB beamformers).

One may argue about why UWB should be combined with the MIMO technology, since UWB itself offers rich diversity owing to its abundant multipaths. A simple answer to this question is that it is due to the general greed of ones’ pursuing higher data rates and higher quality of communications; but this is not the whole picture for the problem. According to Edholm’s law of data rate [44], it can be predicted that indoor data rates of several gigabits per second will become a reality in a couple of years. Therefore, although UWB offers enormous bandwidth and, hence, rich diversity in the time domain, even more bandwidth will be required in the near future. Hence, if it can be shown that the channel capacity of UWB systems is proportional to the number of transmit/receive antennas (this is indeed the case; see Chapter 3), then data rates can be significantly increased further by combining UWB and MIMO. This reason is the same as the one that triggered the research era on MIMO about two decades ago [268]. Even if lower data rates are in focus, the trade-off between bandwidth and the number of antennas could facilitate the antenna and amplifier design, which is still a challenge for UWB systems. For example, the bandwidth requirement could be reduced by almost half if two antennas, instead of one antenna, on both transmitter and receiver sides are deployed.\(^2\)

In this book, we investigate the benefits of combining UWB and MIMO. We highlight five aspects of this promising research field: channel capacity, STC, beamforming, UWB-MIMO relay and time-reversal (TR) transmission. The channel capacity describes a limit of the benefits in some sense for a UWB system employing multiple antennas, while STC provides a realization tool towards reaching the limit. UWB beamforming is of great importance for indoor localization, which has become a hot topic in UWB applications. The TR modulation provides a nontraditional, yet promising, way for secure and/or multi-user wireless communications. UWB-MIMO relay provides an important tool for UWB communications in non-line-of-sight (NLOS) environments where the direct link between communication partners is blocked or the distance is too great.

### 1.2 UWB Basics

A UWB transmission system, by definition [63, 210], is a radio system whose 10 dB bandwidth \((f_H - f_L)\) is at least 500 MHz and whose fractional bandwidth \((f_H - f_L)/(f_H + f_L)/2\) is at least 20\%. A UWB radio system can coexist with other kinds of narrow- and wide-band radio systems. Hence, its power spectrum density is strictly limited by relevant regulatory authorities. In the USA, a UWB device can use the frequency band from 3.1 to 10.6 GHz under the spectral mask specified in [63], as illustrated in Figure 1.1 for the indoor applications of the UWB. Notice that different permitted equivalent isotropically radiated power (EIRP) emission levels are applied to different UWB application categories, but in most applications the magic figure \(-41.3 \text{ dBm/MHz}\) across the 3.1 to

\(^2\)Consider the case where we want to achieve a fixed data rate, say \(R_0\), by using a single transmit and single receive antenna (SISO) UWB system (denoted as \(S_1\)) and a \(2 \times 2\) MIMO UWB system (denoted as \(S_2\)). Except for bandwidth, all other system parameters and configurations (for example, modulations and coding–decoding techniques) are the same for \(S_1\) and \(S_2\). Suppose the bandwidth required by system \(S_1\) to achieve \(R_0\) is \(B_1\). Since the channel capacity of \(S_2\) is doubled compared with that of \(S_1\) if they have the same bandwidth, system \(S_2\) will require only a bandwidth of \(B_1/2\) to achieve the same data rate \(R_0\).
There are two approaches to implementing this kind of radio system. The first is the impulse radio (IR)-based approach, where a pulse train, in which each pulse is very short in the time domain (typically on the order of several tens of picoseconds), is used to carry out information data. This pulse train will be directly transmitted through the

**Figure 1.1**  FCC spectral mask for UWB indoor applications.

10.6 GHz frequency band applies. In Europe, a further wide frequency band from 1.6 to 10.6 GHz can be used, but a more strict spectral mask is specified than that in the USA; see Figure 1.2 for an illustration and [105] for the details. In Asia, some proposals for the spectral masks have been proposed, but final concurrence has not yet been reached [49].

**Figure 1.2**  EU spectral mask for UWB indoor applications.
antenna without any carriers. The second is the multiband (MB)-based approach, where the information data is multiplexed into sub-frequency bands in the entire band from 3.1 to 10.6 GHz or a part of it, with each sub-band having 528 MHz bandwidth. In each sub-band, the information data is transmitted by using traditional multi-carrier orthogonal frequency-division multiplexing (OFDM) technology.

The main advantage of the IR-based UWB systems is the simplicity in the transceiver structure. No up- and down-mixers are needed in this kind of system. This advantage is one of the most important reasons which boosted our interest in UWB systems at the very beginning of UWB-related studies. The drawback of the IR-based UWB systems is the large number of multipaths, typically on the order from several tens to a hundred more [265, 267] for a normal office environment. This causes a big challenge for the synchronizer design with reasonably quick acquisition time and the rake receiver implementation with sufficient fingers to capture enough energy.

The main advantage of the MB-based UWB systems is that the key OFDM technique is already mature for deployment in the market, and its several nice properties, such as high spectral efficiency, inherent resilience to radio-frequency (RF) interference, robustness to multipaths, and the ability to efficiently capture multipath energy, have been proven in other commercial technologies, for example, in IEEE 802.11a/g [1, 2]. The drawback of the MB-based UWB systems is the complexity involved in implementing up- and down-mixers and OFDM. The latter requires fast Fourier transform (FFT) and inverse FFT (IFFT) algorithms at the receiver and transmitter sides respectively [192]. This is not preferable in typical UWB applications.

Currently, it is controversial about which technology will dominate the future market, due to the various pros and cons of these two technologies. However, the IR-based UWB technology is finding its niche in some applications such as indoor wireless localization. Therefore, we will focus our main attention on the IR-based UWB systems in this book.

The popularly used waveforms for the monopulse in the IR-based UWB systems are the first and second derivatives of the Gaussian monopulse [264, 267], which are defined respectively by

$$w_1(t) = \varsigma_1 t \exp \left[ -2\pi \left( \frac{t}{\tau_p} \right)^2 \right],$$

$$w_2(t) = \varsigma_2 \left[ 1 - 4\pi \left( \frac{t}{\tau_p} \right)^2 \right] \exp \left[ -2\pi \left( \frac{t}{\tau_p} \right)^2 \right],$$

where $\tau_p$ is a parameter used to adjust the pulse width and $\varsigma_1$ and $\varsigma_2$ are two constants to normalize the peak amplitudes or powers of the pulses $w_1(t)$ and $w_2(t)$ respectively. Generally, the original Gaussian monopulse

$$w_0(t) = \varsigma_0 \exp \left[ -2\pi \left( \frac{t}{\tau_p} \right)^2 \right]$$

is not adopted since its power spectrum contains a DC component. The monopulses $w_1$ and $w_2$ are illustrated in Figure 1.3.
The information data can be embedded in either the amplitude or the position of the UWB impulse train to transmit, producing pulse-amplitude modulation (PAM) and pulse-position modulation (PPM) respectively. For multi-user access to the UWB channel, there are basically two kinds of accessing techniques: time hopping (TH) spread spectrum (SS) and direct sequence (DS) SS accessing. Since the transmit power is rather low, one information bit in the IR-based UWB system is generally spread over multiple monocycles to achieve a processing gain in reception.

For TH-SS accessing, the data modulation can be generally expressed as [209, 266, 274]

$$s_k(t) = \sum_{j=-\infty}^{\infty} a_k(\lfloor j/N_f \rfloor) w(t - j T_f - c_k(j) T_c - \delta d_k(\lfloor j/N_f \rfloor)),$$  \hspace{1cm} (1.1)

where $\lfloor x \rfloor$ denotes the integer floor of $x$, $s_k$ is the transmitted signal for the $k$th user, $w(t)$ is the monopulse of duration $T_w$, $N_f$ is the number of frames for one data symbol, $T_f$ is the frame duration, $T_c$ is the chip duration, $\delta$ is the modulation index, $\{c_k(j)\}$ is the TH coding sequence, which takes values in $[0, N_c - 1]$ and is assumed to be periodic with period $N_f$, and $a_k$ and $d_k$ are the transmitted data symbols.

It is assumed that $T_f = N_c T_c$ with $N_c$ being the number of chips in one frame duration, $T_w \ll T_c$, and $\delta \ll T_c$.

If $a_k = 1$, then Equation (1.1) reduces to TH-SS-PPM. If $d_k = 0$, then Equation (1.1) reduces to TH-SS-PAM. Clearly, the power spectral density (PSD) of the transmitted signal depends on the spectra of both the monopulse and transmitted data sequence. Therefore, it is the combination of the monopulse and transmitted data sequence that shapes the PSD of the transmitted signal and, hence, satisfies the required spectral mask. A complete analysis of the PSD of the transmitted signal is provided in [262].
For DS-SS accessing, the data modulation is expressed as [274, 292]

\[ s_k(t) = \sum_{j=-\infty}^{\infty} a_k(\lfloor j/N_f \rfloor)c_k(j)w(t - jT_f - \delta_d(\lfloor j/N_f \rfloor)). \] (1.2)

Early research on UWB for wireless communications focused on TH-SS PPM due to its implementation advantage of not requiring to change, or inverse for binary modulation, the pulse amplitude [292]. Besides, the PSD of a TH-SS PPM signal does not have strong spectral lines, since the TH information sequence smooths the PSD of the transmitted signal, which is a big advantage of the TH-SS PPM scheme because the strong spectral lines will introduce noticeable interference to other radio systems in the same frequency band [274, 292].

1.3 MIMO Principle

To see the benefit of MIMO, let us investigate the signal-to-noise power ratio (SNR) or channel capacity gain of the MIMO compared with that of SISO for narrowband wireless communication systems with frequency-flat channels. Since many kinds of system performance (such as channel capacity, data rates, bit error rates, etc.) are determined by the SNR, it is justifiable by investigating the SNR gain of the MIMO systems. To make the comparison fair, we keep the constraint that the transmit power of the MIMO is the same as that of the SISO. Let \( N_T \) and \( N_R \) be the numbers of transmit and receive antennas respectively.

First consider the single transmit antenna and multiple receive antennas (SIMO) case. The input–output relationship can be expressed as

\[ Y_i(t) = h_i X(t) + N_i(t), \] (1.3)

where \( X(t) \) and \( Y_i(t) \), \( i = 1, \ldots, N_R \), are the transmit signal and receive signals respectively, \( h_i \), \( i = 1, \ldots, N_R \), are the channel fading from the transmitter to each receiver, and \( N_i(t) \), \( i = 1, \ldots, N_R \), are the receiver noises with zero mean and variance \( \sigma^2_{N_i} \). For the SISO case, the input–output relationship is expressed as

\[ Y(t) = h X(t) + N(t), \]

where the symbols have the same meaning as those in Equation (1.3) and the noise \( N(t) \) is also of zero mean and variance \( \sigma^2_N \). Suppose that all \( h \) and \( h_i \), \( i = 1, \ldots, N_R \), are complex Gaussian with zero mean and variances \( \sigma^2_h \) and \( \sigma^2_{h_i} \) respectively. Suppose that \( X(t) \), \( h_i \), \( h \), \( N_i(t) \) and \( N(t) \) are mutually independent.

If the receiver does not have the channel state information (CSI), we can combine the received signals with an equal gain, i.e.:

\[ Y_{SIMO}(t) = \sum_{i=1}^{N_R} Y_i(t) = \sum_{i=1}^{N_R} h_i X(t) + \sum_{i=1}^{N_R} N_i(t). \]
Then the SNR of the combined signal is
\[
\text{SNR}_{\text{SIMO}} = \frac{\mathbb{E} \left\{ \left[ \sum_{i=1}^{N_R} h_i X(t) \right] \left[ \sum_{i=1}^{N_R} h_i^* X(t) \right]^* \right\}}{\mathbb{E} \left\{ \left[ \sum_{i=1}^{N_R} N_i(t) \right] \left[ \sum_{i=1}^{N_R} N_i^*(t) \right] \right\}} = \frac{\sum_{i=1}^{N_R} \sigma_{h_i}^2 \mathbb{E}[|X(t)|^2]}{N_R \sigma_N^2} = \frac{\sum_{i=1}^{N_R} \sigma_{h_i}^2}{N_R} \text{SNR}_T, \quad (1.4)
\]

where \( \text{SNR}_T \) denotes the SNR at the transmitter side. From Equation (1.4) we can see that there is no gain in the SNR if the receiver does not know the CSI.

On the other hand, if the receiver knows the CSI, then the receiver can combine the received signals using the maximum ratio combiner (MRC) as follows:
\[
Y_{\text{SIMO}}(t) = \sum_{i=1}^{N_R} h_i^* Y_i(t) = \sum_{i=1}^{N_R} |h_i|^2 X(t) + \sum_{i=1}^{N_R} h_i^* N_i(t).
\]

Then the SNR gained is
\[
\text{SNR}_{\text{SIMO}} = \mathbb{E} \left\{ \left[ \sum_{i=1}^{N_R} |h_i|^2 X(t) \right] \left[ \sum_{i=1}^{N_R} |h_i|^2 X(t) \right]^* \right\} = \mathbb{E} \left\{ \left[ \sum_{i=1}^{N_R} |h_i|^2 N_i(t) \right] \left[ \sum_{i=1}^{N_R} |h_i|^2 N_i(t) \right]^* \right\} = \mathbb{E} \left[ \sum_{i=1}^{N_R} |h_i|^4 \right] \mathbb{E}[|X(t)|^2] + \mathbb{E} \left[ \sum_{i=1}^{N_R} |h_i|^2 \left[ \sum_{i_2=1, i_2 \neq i}^{N_R} |h_{i_2}|^2 \right] \right] \mathbb{E}[|X(t)|^2]
\]
\[
= \frac{2 \sum_{i=1}^{N_R} \sigma_{h_i}^4 + \sum_{i_1=1}^{N_R} \sum_{i_2=1, i_2 \neq i_1}^{N_R} \sigma_{h_{i_1}}^2 \sigma_{h_{i_2}}^2 \mathbb{E}[|X(t)|^2]}{\sum_{i=1}^{N_R} \sigma_{h_i}^2} \mathbb{E}[|X(t)|^2]
\]
\[
= \left( \frac{\sum_{i=1}^{N_R} \sigma_{h_i}^4}{\sum_{i=1}^{N_R} \sigma_{h_i}^2} + \sum_{i=1}^{N_R} \sigma_{h_i}^2 \right) \text{SNR}_T. \quad (1.5)
\]

In the third equality we have used the property \( \mathbb{E}[|h_i|^4] = 2\sigma_{h_i}^4 \) of complex Gaussian random variables [101, p. 91], [122]. From Equation (1.5) we can see that the SNR is increased exactly \( N_R \) times (i.e., the SNR of the SIMO is \( N_R \)-times that of the SISO) by using the MRC if the CSI is available at the receiver and all the links in the SIMO have the same fading power as the link in the SISO.

Next, consider the multiple transmit antennas and single receive antenna (MISO) case. The input–output relationship can be expressed as
\[
Y(t) = \sum_{i=1}^{N_T} h_i X_i(t) + N(t), \quad (1.6)
\]
where \(Y(t)\) is the receive signal, \(X_i(t), i = 1, \ldots, N_T\), are the transmit signals and \(h_i, i = 1, \ldots, N_T\), is the channel fading from each transmitter to the receiver. For the statistical properties of the model (1.6), we make similar assumptions as those for model (1.3) except that \(\mathbb{E}[|X_i(t)|^2] = \mathbb{E}[|X(t)|^2]/N_T\), where \(\mathbb{E}[|X(t)|^2]\) is the power of the transmit signal for the SISO case.

If the transmitter does not have the CSI, then the SNR achieved at the receiver will be

\[
\text{SNR}_{\text{MISO}} = \frac{\mathbb{E}\left\{ \left[ \sum_{i=1}^{N_T} h_i X_i(t) \right]\left[ \sum_{i=1}^{N_T} h_i X_i(t) \right]^* \right\}}{\mathbb{E}[|N(t)|^2]} = \frac{\sum_{i=1}^{N_T} \sigma_{h_i}^2}{N_T} \frac{\mathbb{E}[|X(t)|^2]}{\sigma_N^2}
\]

\[
= \frac{\sum_{i=1}^{N_T} \sigma_{h_i}^2}{N_T} \text{SNR}_T.
\]

It can be seen that there is no gain in the received SNR.

On the other hand, if the transmitter knows the CSI, then it can preprocess the transmitted signal for each transmit antenna so that some gain is achieved in the received SNR. Suppose the transmitted signal for each transmit antenna is weighted by its channel fading:

\[
X_i(t) \rightarrow \frac{h_i^*}{\sqrt{\sum_{i=1}^{N_T} \sigma_{h_i}^2}} X_i(t).
\]

Note that \(\mathbb{E}[|X_i(t)|^2] = \mathbb{E}[|X(t)|^2]\) in this situation. Then the overall transmitted power will be the same as the SISO case, and the received signal is

\[
Y(t) = \frac{1}{\sqrt{\sum_{i=1}^{N_T} \sigma_{h_i}^2}} \sum_{i=1}^{N_T} h_i h_i^* X_i(t) + N(t).
\]

Thus the received SNR is

\[
\text{SNR}_{\text{MISO}} = \frac{1}{\sum_{i=1}^{N_T} \sigma_{h_i}^2} \frac{\mathbb{E}\left\{ \left[ \sum_{i=1}^{N_T} |h_i|^2 X_i(t) \right]\left[ \sum_{i=1}^{N_T} |h_i|^2 X_i(t) \right]^* \right\}}{\mathbb{E}[|N(t)|^2]} = \frac{\mathbb{E}\left\{ \sum_{i=1}^{N_T} |h_i|^4 |X_i(t)|^2 \right\}}{\sigma_N^2 \sum_{i=1}^{N_T} \sigma_{h_i}^2} + \frac{\mathbb{E}\left\{ \left[ \sum_{i=1}^{N_T} |h_i|^2 X_i(t) \right]\left[ \sum_{i_2=1,i_2 \neq i_1}^{N_T} |h_{i_2}|^2 X_{i_2}(t) \right]^* \right\}}{\sigma_N^2 \sum_{i=1}^{N_T} \sigma_{h_i}^2}
\]

Let us divide the problem into two cases. The first case is that \(X_{i_1}(t)\) and \(X_{i_2}(t)\) are different streams of symbols for \(i_1 \neq i_2\) and are mutually independent with zero mean. This corresponds to the case of multiplexing. Then

\[
\text{SNR}_{\text{MISO}} = \frac{2 \sum_{i=1}^{N_T} \sigma_{h_i}^4 \mathbb{E}[|X_i(t)|^2]}{\sigma_N^2 \sum_{i=1}^{N_T} \sigma_{h_i}^2} = \frac{2 \sum_{i=1}^{N_T} \sigma_{h_i}^4 \mathbb{E}[|X(t)|^2]}{\sum_{i=1}^{N_T} \sigma_{h_i}^2 \sigma_N^2} = \frac{2 \sum_{i=1}^{N_T} \sigma_{h_i}^4}{\sum_{i=1}^{N_T} \sigma_{h_i}^2 \text{SNR}_T}.
\]

If all the links have the same fading power as the SISO link, then we can see that the received SNR is doubled compared with the SISO case. Notice that the symbol rate of the MISO in this case is \(N_T\)-times that of the SISO.
The second case is that \( X_{i_1}(t) = X_{i_2}(t) \) for all \( i_1 \) and \( i_2 \) and all are of zero mean. This corresponds to the case of diversity combining. In this case, we have

\[
\text{SNR}_{\text{MISO}} = \frac{\mathbb{E} \left[ \sum_{i=1}^{N_T} |h_i|^4 |X_i(t)|^2 \right] + \mathbb{E} \left[ \sum_{i_1=1}^{N_T} |h_{i_1}|^2 X_{i_1}(t) \right] \left[ \sum_{i_2=1, i_2 \neq i}^{N_T} |h_{i_2}|^2 X_{i_2}(t) \right]^*}{\sigma_N^2 \sum_{i=1}^{N_T} \sigma_{h_i}^2}
\]

This leads to the same result as the SIMO case, i.e., the SNR is increased exactly \( N_T \)-fold by using the preprocessing technique if the CSI is available at the transmitter and all the links in the MISO have the same fading power as the link in the SISO.

Now consider the MIMO case. The input–output relationship can be expressed as

\[
Y(t) = HX(t) + N(t),
\]

where \( X(t), Y(t), H \) and \( N(t) \) are the \( N_T \)-dimensional transmit signal, the \( N_R \)-dimensional receive signal, the \( (N_R \times N_T) \)-dimensional channel matrix and the \( N_R \)-dimensional receiver noise respectively. Let us express \( H \) in the singular value decomposition form [107, p. 414]:

\[
H = U \Sigma V^*,
\]

where \( U \) and \( V \) are unitary matrices of \( N_R \times N_R \) and \( N_T \times N_T \) dimensions respectively, \( \Sigma = [\sigma_{ij}] \) is an \( (N_R \times N_T) \)-dimensional diagonal matrix in the sense of \( \sigma_{ij} = 0 \) for all \( i \neq j \). The diagonal entries \( \sigma_{ii} \) are of the property \( \sigma_{11} \geq \sigma_{22} \geq \cdots \geq \sigma_{N_T N_R} \geq 0 \), where \( N_{\text{TR}} = \min\{N_T, N_R\} \). If \( H \) is of full rank (this holds true generally for a random matrix), then we have \( \sigma_{N_T N_R} > 0 \). Suppose that the CSI \( H \) is available at both transmitter and receiver. Let us preprocess and post-process the transmit signal and receive signal respectively in the following way:

\[
\tilde{X}(t) = V^*X(t), \quad \tilde{Y}(t) = U^*Y(t),
\]

where \( \tilde{X}(t) \) and \( \tilde{Y}(t) \) are new transmit and receive signals respectively. Then we have

\[
\tilde{Y}(t) = \Sigma \tilde{X}(t) + U^*N(t).
\] (1.7)
compared with the SISO channel. From the viewpoint of channel capacity, the ergodic channel capacity \([24, 90]\) of the MIMO channel can also be increased \(N_{TR}\)-fold compared with the SISO channel, since this capacity is determined by the determinant of the matrix \(I + HH^*\) \([244]\).

From the above discussion, one can see that the MIMO technology can yield a considerable gain in system performance compared with the SISO system.

### 1.4 State-of-the-Art UWB-MIMO

UWB technology has been widely used in radar and information sensing for more than 30 years. The study of UWB for communications started during the late 1990s. A dramatic change in the study happened in 2002, when the US FCC issued the regulation on the spectral mask of UWB radios. Under the regulation, the extremely wide radio spectrum from 3.1 to 10.6 GHz can be freely used without a licence application if the transmission satisfies the spectral mask condition. This event triggered a great interest in the community of wireless communications from both academia and industry. Since then, almost all concepts, ideas and techniques in narrow- or wide-band wireless communications have been immigrated into UWB communications. However, UWB-MIMO is still in its research infancy. The reason might be twofold. First, MIMO itself is a quite new technology. Its implementation in practical communication equipment is a recent matter. Second, a UWB channel itself possesses rich diversity due to its abundant multipaths. This raises some doubt about whether it is necessary to combine MIMO with UWB. It is our belief that UWB-MIMO will become a powerful candidate for extremely high data-rate communications in the near future. The recent surge of research reports on UWB-MIMO also verifies this belief.

Compared with the volumes of literature in narrowband MIMO research, there are only a few studies of UWB-MIMO. Basically, these studies can be categorized into four fields: UWB-MIMO channel measurement and modelling, channel capacity, STC and beamforming.

With regard to channel measurements and characterizations, several reports have been published; for example, see \([123, 152, 156]\). The full characterization of the spatial correlation of UWB channels is provided in \([152]\), where it is found that in the range of 2.5 times the coherence distance (about 4 cm) the antenna correlation follows a pattern of the first kind zeroth-order Bessel function with distance, while an almost constant correlation coefficient (smaller than 0.4) is observed when the antenna distance is greater than 2.5 times the coherence distance. It is particularly interesting that, as shown in \([65, 114, 152, 155, 156]\), the antenna angular orientation and the signal polarization can be used to decrease the correlation of the spatial channels or to improve the system performance. Another approach describing the correlation property is from the deterministic viewpoint \([247]\). It is defined as the average value of all the cross-correlation functions of different spatial channels normalized by the autocorrelation functions of corresponding spatial channels. A possible unfavourable electromagnetic coupling between UWB antenna elements has been proven to be small, even for marginal antenna separations \([216]\).

Regarding the channel capacity of UWB-MIMO systems, the research results can be found in \([159, 197, 284, 285, 287]\). In \([284, 285]\), it is shown that for \(N_T\) transmit and
$N_R$ receive antennas (for simplicity, it is assumed that $N_T = N_R$ there) the UWB-MIMO ergodic channel capacity increases linearly with $N_R$. However, for the MISO case, it is not always beneficial to employ more transmit antennas. It is shown in [284, 285, 287] that the outage probability decreases with the number of transmit antennas when the communication rate is lower than the critical transmission rate, but it increases when the rate is higher than another value. This critical transmission rate is determined by the fading power and the SNR of the system at the transmitter side. We can roughly say that it is not beneficial to use multiple transmit antennas if the required transmission rate (normalized by the system bandwidth) is higher than the critical transmission rate or equivalently when the available power at the transmitter side is too low. In [159], a fixed region of scattering environments for UWB-MIMO systems is considered. Thus, the number of spatial degrees of freedom of the scattered field, denoted $\eta$, is limited. It is shown in [159] that the system capacity is fundamentally limited by the three numbers $N_T$, $N_R$ and $\eta$. This is not strange, since $\eta$ will place a limit on the rank of the UWB-MIMO channel matrix; hence, it will affect the number of the independently separate channels. In [197], it is shown that if several different antennas (a loop antenna and two orthogonal bow-tie antennas there) are placed in the same place instead of separately in different places with sufficient distance as in the traditional spatial antenna array, the spectral efficiency of such a system approaches that of the traditional array system. This is due to the fact that the rank of the channel matrix involved is well maintained to be equal for both kinds of systems.

Since a UWB system is often required to work in a low power regime by the relevant regulation bodies, it is important to investigate the system capacity at low power or in a low SNR regime. For widebands, it is shown in [80, 171] that very large bandwidths yield poor performance for systems that spread the available power uniformly over time and frequency. In [245] it is shown that the input signals needed to achieve a capacity must be peaky in time or frequency for a wideband fading channel composed of a number of time-varying paths. We can witness this phenomenon for UWB-MIMO systems, as illustrated in [287]. In [287], the uniform power spectrum allocation (UPSA) and optimal power spectrum allocation (OPSA) policies are investigated for transmitted UWB signals, where, for the OPSA, a water-filling algorithm is applied to adjust the power distribution across both the frequency domain and the antenna domain according to the status of the channel multipath fading, and for the UPSA the transmitted power is uniformly distributed across both domains. It is demonstrated that the efficiency of the UPSA relative to the OPSA is low when the SNR is lower than $-20\,\text{dB}$. However, when the SNR is higher than 10 dB, the UPSA policy almost produces the same channel capacity as the OPSA policy. Therefore, an optimal power distribution algorithm, such as the water-filling approach, should be considered if the SNR is rather low, while the water-filling algorithm is just to make the transmitted signals ‘peaky’ in both the frequency and the antenna domains. When the CSI is unknown at the receiver, the system performance is characterized in [203] for MIMO wideband systems.

For the STC, the first result was reported in [273] for IR UWB systems, where it is shown that the receive diversity order is equal to the product of the number of receive antennas and the number of rake fingers. Note that a larger number of antennas promises only a limited diversity gain because of the distinct UWB multipath diversity [260]. In [247], a spatial-multiplexing coherent scheme for a $2 \times 2$ UWB-MIMO system is
experimentally investigated. In [251], the performance of a space–time trellis code for a $2 \times 2$ UWB-MIMO system is evaluated. For general IR-based UWB systems, the STC method was provided in [6, 7, 8, 225]. For OFDM-based UWB systems, [227] presented an STC method which was essentially similar to the STC for wideband OFDM. The report [258] showed an approach to increasing the spatial diversity via antenna selection across data frames. In [39], a space–time selective-rake receiver is proposed considering the presence of narrowband interference and multiple access interference. In [73], a time-interleaving multi-transmit-antenna UWB system is proposed, where monocycle pulses per information symbol are transmitted discontinuously through the time interleaver to get more temporal diversity. Spatial diversity and temporal diversity are compared therein. In [146], a zero-forcing scheme is proposed to remove the interference among the multiple data streams in UWB-MIMO systems. A space–time trellis coding scheme is proposed in [182]. In [238, 240, 260], the multiple access performance of UWB-MIMO systems is investigated. Spatial multiplexing is proposed in [129], where the VBLAST (vertical Bell Laboratory layered space–time) algorithm was applied to UWB systems and a significant multiplexing gain could be proven.

For UWB-MIMO STC, it is found [118] that a fundamental compromise exists among the available SNR, coding interval and the number of transmit antennas. The basic conclusion is that only when the available SNR is sufficiently high (supposing that the total power over all the transmit antennas and the coding interval is fixed), can the coding gain be obtained by deploying the transmit power into more transmit antennas and using a longer coding interval. In other words, if the available SNR is too low, then it is better to use less transmit antennas and shorter space–time codes. Therefore, we can see another kind of ‘peaky’ phenomenon again. It is highly expected to give some quantitative characterization for how high the SNR should be so that the STC can indeed provide rewarding gains. However, this is not available for general UWB-MIMO systems.

Regarding UWB beamforming, systematic studies of the problem were presented in [109, 110, 204], with several fundamental differences being found. In [115], a digital UWB beamforming scheme was proposed. The effect of multipaths on the beamformer output was illustrated in [174], which was simulated by using the ray-tracing technique. In [67, 68, 116], the UWB beamformer was used to find the location of the source. In [154], an adaptive beamformer for MB UWB wireless systems was proposed where it was shown that the signal bandwidth had little impact on the beamwidth or direction; hence, the beam focusing capability will not be sensitive to the signal bandwidth. In [231], the measured transient response of a uniform linear UWB array shows a peaked output without side lobes. In [88], an interesting algorithm for calculating the weighting coefficients of wideband beamformers was developed.

UWB Beamformers have some peculiar properties that are quite different from the narrowband beamformers. For example, the use of unequal weighting filters for the individual antenna branch increases the side-lobe level in UWB beamformers; thus, optimal beamformers, as shown in [204], are those in which the weighting filters for all the antenna branches are identical. A basic difficulty in the UWB beamformer is how to deal with multipaths of the inherited UWB signal propagations. In narrowband array processing technology, this problem can be ignored, but we cannot ignore it in the UWB array since the multipath is one of the most pronounced characteristics of UWB channels. In this research subfield, ranging [116, 117] and sensing are promising applications of
multi-antenna UWB technology. Ultra-short pulses allow spatial resolution even down to subdecimetre range. By means of multi-antenna techniques, additional spatial parameters can be generally extracted (for example, the direction of arrival or direction of departure), leading to an enhanced ranging accuracy. Further applications cover the detection of breast cancer [26] and mine localization [71], as well as the detection of fires by active UWB radiation [261].

Even though a typical power increment (array gain) by multiple antennas is noticed, a general investigation on bandwidth dependence substantiated by quantitative results is still missing. In general, it is evident that a boost of bandwidth is accompanied by a diminished small-scale fading. Hence, a threshold region will exist, but this is not yet quantified. Exceeding this region makes UWB-SIMO less promising, because only the array gain persists.

Overall, the research on UWB-MIMO is still in its infant stage. Further studies, especially on its implementation, are necessary to bring this technology into the market.

1.5 Scope of This Book

We conclude this chapter with a brief overview of the areas discussed in the remainder of this book.

Chapter 2: UWB-MIMO Channel Measurement and Models. Since the monopulses used in UWB radios are very narrow, the received pulses from different paths with a time difference on the order of nanoseconds can be resolved. Owing to this fact, a UWB channel model will be fundamentally different from that of narrowband communications. In this chapter, several commonly used UWB channel models are briefly introduced, which will form the basis of subsequent chapters. The channel sounding equipment developed in our institute and the ray-tracing simulation tool, which is used to simulate the impulse or frequency response of general communication channels, are also presented. The ray-tracing simulation tool is very helpful for investigating the channel model when experimental facilities are limited.

Chapter 3: UWB Channel Capacity. In this chapter we investigate how the channel capacity of UWB-MIMO depends on the numbers of transmit and receive antennas. The results will provide some guidelines for how to use multiple antennas to increase the data rates of UWB communication systems. Three cases are investigated: MISO, SIMO and MIMO. We show in each case that the data rate will increase with the number of antennas in a different way; and in some extreme case for MISO, increasing the number of transmit antennas will be detrimental to the data rate. This is peculiar to UWB systems.

Chapter 4: UWB-MIMO Space–Time Coding. An essential objective of using MIMO is to increase the data rate by its inherent multiplexing gain and/or diversity gain. Different from the narrowband communication channel, the UWB-MIMO channel may exhibit diversities in more dimensions: in the time domain (multipaths) or frequency domain in addition to the spatial domain. A simple yet widely used STC scheme for narrowband wireless communications is the Alamouti code. In the literature, two kinds of STC schemes, which are similar to the Alamouti code, were proposed for IR-based UWB-MIMO systems. In this chapter we first briefly introduce these two coding schemes and investigate their performance. We show that different coding schemes work well in different SNR ranges. After that, we discuss a general design approach for the STC of
IR-based UWB systems with arbitrary numbers of transmit and receive antennas. This design approach is based on real orthogonal design. The concept of the companion of real orthogonal design is proposed for easing the decoding. Finally, a review of the spatio-frequency multiplexing problem and spatio-time–frequency coding is presented for MB UWB systems.

Chapter 5: UWB Beamforming and Localization. As mentioned above, UWB Beamformers have some special properties that are quite different from narrowband beamformers. In this chapter we first investigate how these properties appear in UWB beamformers. Three types of UWB beampattern are defined. The optimal beamformer is presented. Using a UWB beamformer to find the direction of arrival or to estimate the locations of sources is discussed. In principle, the UWB localization problem is similar to the sparse-path radar ranging problem, but the former has both peculiar advantages and great challenges due to very narrow UWB pulses. The main advantage is that its resolution is very high, since the received UWB pulses scattered from different objects can be resolved even if the objects are separated by several centimetres. The main challenge is that mature detection algorithms for the relevant pulse are not yet available, since the received signal consists of so many paths that it is extremely difficult to identify which path is relevant to the object we are concerned with. A UWB array may provide a promising approach to identifying the locations of the objects. By properly processing the two-dimensional signals, only one peak will appear. It is important to investigate the relationship between the peak and the location of the object. In the second part of this chapter, several approaches to the UWB localization problem are examined: beamforming, time of arrival (ToA), and mapping. The methods for dealing with NLOS and multipaths are reviewed and some new relevant ideas discussed.

Chapter 6: Time-Reversal UWB Systems. The TR technique has been applied extensively in acoustic and medical applications and underwater communications. Reports have demonstrated that, in the ultrasonic frequency regime, it is possible to provide error-free communications with five receivers simultaneously. Because of the peculiar property of UWB channel impulse responses, namely abundant multipaths, the TR idea can find wide applications in UWB radios. In this chapter we investigate several fundamental issues in TR-UWB-MIMO systems: why, instead of other kinds of filters, should the TR filter be used at the transmitter and what is the effect of imperfect channels on the system performance? To answer the first question, we analyse the performance of the system with relevant pre-filters at the transmitter when the original channel is of nonminimum phases or corrupted by some estimation errors. To address the second question, we examine how the bit error rate (BER) and received SNR of the TR system change when only a part of the original channel can be obtained and channel estimation errors are suffered. In this chapter we also show that, with an appropriate pre-equalizer, using the TR technique can perform multiuser communications via either MIMO, MISO, or SIMO, combined with the UWB radio. The corresponding pre-equalizer design is presented.

Chapter 7: UWB Relay Systems. Owing to the spectral mask applied to UWB transmission, the transmit power is limited. Thus, the coverage of regularized UWB communication systems is limited to a few metres. To increase the coverage, one possible way is to use multihop relaying. In this chapter we study the system performance of two-hop UWB relay systems in terms of BER, SNR outage probability and the amount of fading. These systems consist of the following: the source, relay and destination can be equipped with
single/multiple antennas, the receivers at the relay and destination can perform coherent or noncoherent detection, and the CSI can be available at the transmitters of the source and relay or at the receivers of the relay and destination. Some guidelines on the design of UWB-MIMO relay systems are illustrated via the analytical and simulation results.

1.6 Notation

Throughout the book we use $I$ to denote an identity matrix, whose dimension is indicated by its subscript if necessary; $P_A(x)$ and $p_A(x)$ respectively represent the cumulative distribution function (cdf) and probability density function (pdf) of a random variable $A$; $E$ (or $E_A$ if necessary) stands for the expectation of a random quantity with respect to the random variable $A$; and $E(\cdot|\cdot)$ denotes the conditional expectation. For a matrix or vector, the use of superscript $T$, $*$ and $\dagger$ denote the transpose, the element-wise conjugate (without transpose) and the Hermitian (conjugate) transpose respectively. The $*$ and $\dagger$ notation also apply to a scalar. The function log is naturally based, if the base is not explicitly stated. We use $\text{diag}$ to denote a diagonal matrix with the diagonal entries being specified by the corresponding arguments.

For other notation, we might use the same symbol to denote different things in different chapters or sections. In such a case we will explicitly explain what the symbol stands for.
Since the monopulses used in IR-UWB radios are impulse-like with a time-domain duration on the order of hundreds of picoseconds, the received pulses from different paths with a time difference on the order of nanoseconds can be resolved. Owing to this fact, a UWB channel model is fundamentally different from that of narrowband communications. Thus, UWB channel modelling has been a lasting research topic since the late 1990s. The research reports on this modelling can be categorized as relating to:

- large-scale path-loss models;
- small-scale amplitude fading models;
- path delay (path arrival time) models.

The basic characteristics of large-scale path-loss models for narrowband channels can be extended to UWB channels. For research reports on this aspect, readers are referred to [17, 130]. A basic conclusion is that the path loss is an increasing function of the distance between the transmitter and receiver, while this function is again related to the frequencies used in UWB systems.

Several models for the small-scale fading of UWB systems have been proposed; for example, see [35, 48, 50, 77, 164]: the Nakagami distribution is shown to fit the amplitude fading in [35]; Chong and co-workers claim that amplitude fading admits a lognormal distribution [48] and a Weibull distribution [50]; in [77, 164], the lognormal distribution is proposed to approximate the amplitude fading, and it is also found that both lognormal and Nakagami distributions can fit the measurement data equally well. These different models are due to the different measurement environments. All the results in [35, 48, 50, 77, 164] show that the excess delays can be modelled as a Poisson process.

Comprehensive surveys of UWB-SISO channel models are provided in [165, 166, 167]. However, the study of UWB-MIMO channel models is not extensive. Until now, only some correlation properties of UWB-MIMO channels have been investigated; see [10, 152, 247]. In [65, 114, 152, 155, 156], the effect of antenna angular orientation and signal polarization on the system performance, which are mainly used to decrease the correlation of spatial channels, is investigated.
In this chapter, several commonly used UWB channel models will be briefly introduced. These will form the basis of subsequent chapters. The channel sounding equipment developed in our institute and the ray-tracing simulation tool are also presented. The ray-tracing simulation tool can provide very good approximation for physical (deterministic and site-specific) channels if scattering environments are described with reasonable accuracy. Therefore, it is helpful for investigating UWB channel models when experimental facilities are limited.

2.1 UWB-SISO Channel Model

2.1.1 Large-Scale Path-Loss Models

If a narrowband electromagnetic wave propagates in free space, then the path loss, defined as the ratio between the transmitted power and received power, is modelled as [90, 201]

\[ L(d, f_c) = \left( \frac{4\pi f_c d}{c} \right)^2, \]  \hspace{1cm} (2.1)

where \( d \) denotes the distance between the transmitter and receiver, \( c \) is the speed of light, \( f_c \) is the centre frequency of the narrowband electromagnetic wave and \( L \) is the path loss for a given distance \( d \) and centre frequency \( f_c \). Considering signal reflection, diffraction and scattering in practical propagation environments, the path-loss model is revised as

\[ L(d, f_c) = \left( \frac{4\pi f_c d}{c} \right)^\gamma, \]  \hspace{1cm} (2.2)

where \( \gamma \) is the path-loss exponent and \( L \) is the average (averaged over all possible ensembles) path loss for a given distance \( d \) and centre frequency \( f_c \). It is clear that \( \gamma = 2 \) for free space. For the propagation environments varying from office buildings to urban area macrocells, \( \gamma \) may change from 1.6 to 6.5; see [90, Table 2.2] and [201, Table 4.2].

In UWB communications, it is difficult to give a meaningful definition for the centre frequency. However, references [165, 166] extend model (2.2) to the UWB case in the following way:

\[ L(d, f) = L(f) L_d(d), \]  \hspace{1cm} (2.3)

where \( L(f) \) and \( L_d(d) \) denote the frequency-dependent path loss and distance-dependent path loss respectively, where \( f \) is the frequency at which the path-loss measurement is taken. It can be shown (see below) that the distance-dependent path loss follows the same model as in narrowband channels. Thus, from Equation (2.2), we can see that \( L_d(d) \) reads as

\[ L_d(d) \propto d^\gamma. \]  \hspace{1cm} (2.4)

\[ Note that f_c and d in Equation (2.2) may be of different exponents. This fact is implied in [90] but not strengthened in the study of narrowband wireless channel models, since attention is paid to the relationship between \( L \) and \( d \) there. Here, for notational convenience, we assume that the two exponents are of the same value.\]
In [87], the path-loss exponent $\gamma$ itself is modelled as a random variable that changes from one building to another. In [130], $L_t(f)$ is modelled as

$$L_t(f) \propto f^{\gamma_1}, \quad (2.5)$$

where the path-loss exponent $\gamma_1$ takes a value in the interval [1.6, 2.8] for the experimental scenarios in [130]. In [17], $L_t(f)$ is modelled as

$$L_t(f) \propto 10^{-\gamma_2 \exp(-\gamma_3 f)}, \quad (2.6)$$

where $\gamma_2$ and $\gamma_3$ are constants and $\gamma_3$ takes a value in the interval [1.0, 1.4].

In a strict sense, the path loss for UWB systems should be defined as the function of distance only. To achieve this goal, we can calculate the path loss by integrating Equation (2.3) across all the frequencies of the transmitted UWB signals. This procedure produces a rigorous yet cumbersome path-loss model. To show this point, we proceed based on the simple model in Equation (2.2).

Denote by $S_{TX}$ the overall transmit power of the UWB signal. Assume $\gamma \geq 2$. The transmit power is uniformly distributed in the frequency range $[f_L, f_H]$, i.e., the power spectral density (PSD) of the transmitted signal is

$$P_{TX}(f) = \begin{cases} 
S_{TX} / (f_H - f_L) & \text{if } f \in [f_L, f_H], \\
0 & \text{otherwise.}
\end{cases} \quad (2.7)$$

Suppose that the scattering media is so distributed that the electromagnetic wave at every frequency assumes the path-loss model (2.2). Then the received power at distance $d$ is

$$S_{RX} = \int_{f_L}^{f_H} P_{TX}(f) \left[ L(d, f) \right]^{-1} \, df
= \frac{S_{TX}}{f_H - f_L} \left( \frac{4\pi d}{c} \right)^{-\gamma} \frac{f_H^{-\gamma+1} - f_L^{-\gamma+1}}{-\gamma + 1}.
$$

Thus, the path loss reads as

$$L(d; f_L, f_H) = \frac{S_{TX}}{S_{RX}} = (-\gamma + 1) \left( \frac{4\pi d}{c} \right)^{\gamma} \frac{f_H - f_L}{f_H^{-\gamma+1} - f_L^{-\gamma+1}}
= (-\gamma + 1) \left( \frac{4\pi d}{c} \right)^{\gamma} (f_H f_L)^{\gamma-1} \left( \sum_{k=0}^{\gamma-2} f_L^{\gamma-2-k} f_H^k \right)^{-1}, \quad (2.8)$$

where the summation is taken for all items of $f_L^{\gamma-2-k} f_H^k$ with the exponent in $f_H$ satisfying $k \leq \gamma - 2$ when $\gamma$ is not an integer. Therefore, the path loss depends on both the low and high frequencies of the transmitted signal if its power is uniformly distributed in its occupied frequency range. If $f_H \gg f_L$, which is generally valid for UWB systems, then Equation (2.8) can be approximated by

$$L(d; f_L, f_H) \approx \frac{\gamma - 1}{2} \left( \frac{4\pi d f_L}{c} \right)^{\gamma} \frac{f_H^2}{f_c f_L}, \quad (2.9)$$
where \( f_c = (f_L + f_H)/2 \) is the centre frequency.\(^2\) Equation (2.9) shows a simple relationship between the path loss and \((d, f_H, f_c, f_L)\).

If the transmit power is not uniformly distributed in the occupied frequency range, then the path loss will generally depend on the PSD of the transmitted UWB signal.

Since applications of the UWB technology are mainly focused on indoor wireless communications and outdoor sensor networks, the shadowing models have been given little attention in the UWB literature.

\[ f_c = \frac{(f_L + f_H)}{2} \]

2.1.2 Small-Scale Fading Models

The simplest UWB channel model characterizing the small-scale fading is the stochastic tapped delay line (STDL) model [35]

\[ h(t) = \sum_{l=1}^{L} \alpha_l \delta[t - (l - 1)\Delta\tau], \]  

(2.10)

where \( h(t) \) is the channel impulse response (CIR), \( \delta \) is the Dirac delta function, \( L \) is the number of multipath components, \( \Delta\tau \) is the sampling interval and \( \alpha_l \) is the amplitude fading in the \( l \)th delay bin. Represent \( \alpha_l \) as

\[ \alpha_l = \nu_l \zeta_l, \]

where \( \nu_l := \text{sign}(\alpha_l) \), the sign of \( \alpha_l \), and \( \zeta_l := |\alpha_l| \), the magnitude of \( \alpha_l \). The statistics of \( \zeta_l \) are typically modelled by the Nakagami distribution [35] or lognormal distribution [77, 164]. For the Nakagami distribution, the pdf of \( \zeta_l \) is described by

\[ p_{\zeta_l}(x) = \begin{cases} 
2[\kappa/(2\Omega_l)]^{\kappa/2}[1/[\Gamma(\kappa/2)]]\chi^{\kappa-1}e^{-\kappa x^2/(2\Omega_l)} & \text{when } x \geq 0, \\
0 & \text{when } x < 0, \quad \kappa \geq 1, 
\end{cases} \]

(2.11)

where \( \Gamma \) denotes the Gamma function, \( \Omega_l = \mathbb{E}(\alpha_l^2) \) and \( \kappa = 2[\mathbb{E}(\alpha_l^2)/\text{Var}(\alpha_l^2)] \). For the lognormal distribution, the pdf of \( \zeta_l \) is described by

\[ p_{\zeta_l}(x) = \frac{20/\ln(10)}{\sqrt{2\pi\sigma_{\zeta_l}^2 x}} \exp \left\{ -\frac{[10\log_{10}(x^2) - \mu_{\zeta_l}]^2}{2\sigma_{\zeta_l}^2} \right\}, \]

(2.12)

where \( \mu_{\zeta_l} \) is the mean of \( \zeta_l \) in decibels and \( \sigma_{\zeta_l} \) is the variance of \( \zeta_l \).

The variable \( \nu_l \) takes the signs +1 and −1 with equal probability to account for signal inversion due to reflections.

Further characterization for models (2.10) and (2.12) is the parameterizations of \( \Omega_l \) and \( \mu_{\zeta_l} \) respectively. Many experimental studies [35, 41, 167] have shown that the power

\[ \mathcal{L}(d; f_L, f_H) \approx \frac{\gamma - 1}{2} \left( \frac{4\pi df_c^2}{c f_H} \right)^\gamma \frac{f_H^{2\gamma}}{f_c(f_H + f_L)}. \]

\(^2\) Another definition of the centre frequency for UWB systems is \( f_c = \sqrt{f_L f_H} \). If \( f_c \) is so defined, then Equation (2.9) should be revised as
of the amplitude fading is exponentially decreasing with the excess delay. Therefore, a simple model for $\Omega_l$ is as follows:

$$\Omega_l = \varrho \Omega_{l-1},$$

(2.13)

where $\varrho < 1$ is a constant. According to the data reported in [35, 41], $\varrho$ takes a value between 0.91 and 0.98. Of course, the value of $\varrho$ is determined by the scenario under investigation. There are few discussions about the model of $\mu_{\zeta_l}$ in the literature. A possible model for $\mu_{\zeta_l}$ is given by [77, p. 14].

Model (2.10) is suitable for dense scattering, such as industrial environments. In this case, each resolvable delay bin contains significant energy, and the concept of ray arrival rates, as used in more complex models, loses its meaning [167].

The STDL model (2.10) is able to capture the severe frequency selectivity of UWB channels, which is the main characteristic, but it does not reflect the clustering characteristic, which is also observed in many UWB channel sounding campaigns [57, 167, 250].

The S-V model [206], tailored to the UWB case, is widely used to characterize the clustering phenomenon. In the S-V model, multipath arrivals are grouped into two different categories: a cluster arrival and a ray arrival within a cluster. The CIR of the S-V model is described by

$$h(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{k,l} \delta(t - T_k - \tau_{k,l}),$$

(2.14)

where $K$ is the total number of clusters, $L$ the total number of rays within each cluster, $\alpha_{k,l}$ is the tap gain (a real number) of the $l$th multipath component in the $k$th cluster, $T_k$ is the excess delay of the $k$th cluster and $\tau_{k,l}$ is the excess delay of the $l$th multipath component in the $k$th cluster relative to the cluster arrival time $T_k$. Both the cluster and ray arrivals are modelled by Poisson processes:

$$p_{T_k}(T_k | T_{k-1}) = \lambda_C \exp[-\lambda_C(T_k - T_{k-1})], \quad k > 1;$$

(2.15)

$$p_{\tau_{k,l}}(\tau_{k,l} | \tau_{k,l-1}) = \lambda_R \exp[-\lambda_R(\tau_{k,l} - \tau_{k,l-1})], \quad k > 1,$$

(2.16)

where $\lambda_C$ and $\lambda_R$ are the cluster arrival rate and ray arrival rate respectively. By definition, $\tau_{k,0} = 0$ for all $k$. The parameter $1/\lambda_C$ is typically in the range of 10–50 ns, while $1/\lambda_R$ shows a wide variation from 0.5 ns in NLOS situations to more than 5 ns in line-of-sight (LOS) situations [165]. It is generally assumed that $T_k$ and $\tau_{k,l}$ are mutually independent. For a given $T_k$ and $\tau_{k,l}$, the power delay profile admits the following form:

$$\mathbb{E}[|\alpha_{k,l}|^2 \mid T_k, \tau_{k,l}] = \Omega_{1,1} \exp\left(-\frac{T_k}{\gamma_C} - \frac{\tau_{k,l}}{\gamma_R}\right),$$

(2.17)

where $\Omega_{1,1}$ is the integrated energy of the first cluster, and $\gamma_C$ and $\gamma_R$ are the cluster decay time constant and ray decay time constant respectively. The cluster decay time constant $\gamma_C$ is typically around 10–30 ns, while different values (between 1 and 60 ns)

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3 In the most general case, the parameters $\lambda_C$, $\lambda_R$, $\gamma_C$, $\gamma_R$ and $L$ for different clusters may have different values; hence, it is legitimate to attach a subscript $k$ to all these five parameters. For concision, we choose not to do so.
have been reported for the ray decay time constant $\gamma_R$ (see [165, 166] and references cited therein).

Note that different clusters can generally overlap. However, owing to the exponential decay of the power with the excess delay, the power of the overlapping part is too small to be detected [206].

Both the Nakagami distribution and lognormal distribution are widely used to describe the pdf of $|\alpha_{k,l}|$, while occasionally the Weibull distribution [50] or even the Rayleigh distribution [57, 121] is used to model the pdf of $|\alpha_{k,l}|$. As in model (2.10), the sign of $\alpha_{k,l}$ takes the values +1 and −1 with equal probability.

Even though model (2.14) provides a more accurate characterization for the UWB channels than model (2.10), the cluster identification for model (2.14) is somewhat ambiguous. Currently, the determination of the number of clusters is often done by ‘visual inspection’ [167].

Model (2.14) has been adopted in the IEEE 802.15.3a\(^4\) and IEEE 802.15.4a standards for high-rate WPAN and low-rate WPAN respectively [3, 4]. Four standard UWB channel models, namely CM1, CM2, CM3 and CM4, were suggested in [3] and widely used in simulation studies. CM1 characterizes a LOS scenario with a short distance between the transmitter and receiver (less than 4 m), CM2 describes an NLOS scenario with a short distance between the transmitter and receiver, CM3 represents an NLOS scenario with a long distance between the transmitter and receiver (in the range from 4–10 m), and CM4 characterizes an NLOS scenario with a further longer distance between the transmitter and receiver (beyond 10 m) or the root mean square (RMS) delay being around 25 ns. All these models are for indoor (either residential or office) environments. The key channel parameters are illustrated in Table 2.1 [77, 164, 165].

In [4, 167], nine standard UWB channel models, CM1~CM9, are suggested, covering indoor residential, indoor office, industrial, outdoor, and open outdoor environments with distinctions between LOS and NLOS properties. Notice that the terms short distance and long distance in the IEEE 802.15.3a and IEEE 802.15.4a standards are different.

| Table 2.1 | Parameters of standard UWB channel models CM1, CM2, CM3 and CM4. |
|-----------|-------------|-------------|-------------|-------------|
| Parameters | CM1         | CM2         | CM3         | CM4         |
| $\lambda_C$ (ns\(^{-1}\)) | 0.0233      | 0.4         | 0.0667      | 0.0667      |
| $\lambda_R$ (ns\(^{-1}\)) | 2.5         | 0.5         | 2.1         | 2.1         |
| $\gamma_C$ (ns) | 7.1         | 5.5         | 14.0        | 24.0        |
| $\gamma_R$ (ns) | 4.3         | 6.7         | 7.9         | 12.0        |
| Note | LOS         | NLOS        | NLOS        | NLOS        |
|      | $\leq 4$ m  | $\leq 4$ m  | 4–10 m      | $>10$ m or |
|      |             |             |             | RMS delay $\sim 25$ ns |

\(^4\)Note that the draft for the standard IEEE 802.15.3a has been withdrawn, partly due to the disagreement between the two different industry alliances, representing two approaches for the UWB technology (IR-based UWB and MB-based UWB) and partly due to significant regulatory hurdles [3].
A further extension of model (2.14) is to model the total number of clusters $K$ as a random variable. In [167], $K$ is modelled as a Poisson-distributed random variable with pdf:

$$p_K(k) = \frac{\bar{K}^k \exp(-\bar{K})}{k!},$$

where $\bar{K}$ is the mean of the total number of clusters.

### 2.2 UWB-MIMO Channel Model

#### 2.2.1 General Models

In a UWB-MIMO system, the channel model is described by a CIR matrix. Suppose there are $N_T$ transmit antennas and $N_R$ receive antennas. Then the CIR matrix of the system is an $N_R \times N_T$ matrix. If the $N_T$ transmit antennas are packed sufficiently close, and so are the $N_R$ receive antennas, then all the transmit–receive pairs will experience similar clustering structures. In this case, the channel can be modelled by

$$H(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} A_{k,l} \delta(t - T_k - \tau_{k,l}),$$

where $H(t)$ is the CIR matrix of the system, $A_{k,l}$ is the amplitude fading matrix, which describes the path fading in amplitude for the $k$th cluster and $l$th ray, and $T_k$ and $\tau_{k,l}$ have the same meaning as those in model (2.14). Models (2.15) and (2.16) can be used to characterize the statistical distributions of $T_k$ and $\tau_{k,l}$ respectively. Denote by $a^{(k,l)}_{i_1,i_2}$ the $(i_1, i_2)$th entry of $A_{k,l}$. Then $a^{(k,l)}_{i_1,i_2}$ is the tap gain from the $i_2$th transmit antenna to the $i_1$th receive antenna for the $k$th cluster and $l$th ray. Therefore, either the Nakagami model (2.11) or lognormal model (2.12) can be used to characterize the statistic distribution of $|a^{(k,l)}_{i_1,i_2}|$. As usual, we can assume that the sign of $a^{(k,l)}_{i_1,i_2}$ takes values $\pm 1$ with equal probability.

To characterize the model for $H$ completely, one needs to know the correlation functions among all $\{A_{k,l}\}$ and among all the entries of each $A_{k,l}$. Since the matrices $\{A_{k,l}\}$ for different $k$ or $l$ characterize the microwave propagations caused by different scatterers, it is reasonable to assume that the matrices $A_{k,l}$ with different $k$ or $l$ are independent of each other. The correlation property among all the entries of each $A_{k,l}$ is similar to that of a simplified model, which we shall discuss in Section 2.2.2.

If the transmit and receive antennas are separated sufficiently far, any of two transmit–receive pairs will not experience similar clustering structures. Then the channel can be modelled by

$$H(t) = \begin{bmatrix}
h_{11}(t) & h_{12}(t) & \cdots & h_{1N_R}(t) \\
h_{21}(t) & h_{22}(t) & \cdots & h_{1N_R}(t) \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_R1}(t) & h_{N_R1}(t) & \cdots & h_{N_RN_T}(t)
\end{bmatrix},$$
where $h_{i_1i_2}(t)$ is the CIR of the channel from the $i_2$th transmit antenna to the $i_1$th receive antenna, which can be typically modelled as

$$h_{i_1i_2}(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{k,l}^{(i_1i_2)} \delta(t - T_k^{(i_1i_2)} - \tau_{k,l}^{(i_1i_2)})$$,

where $\alpha_{k,l}^{(i_1i_2)}$, $T_k^{(i_1i_2)}$ and $\tau_{k,l}^{(i_1i_2)}$ have the same meanings as those in model (2.14) specified to the channel from the $i_2$th transmit antenna to the $i_1$th receive antenna. In this case it is fair to assume that all $\{\alpha_{k,l}^{(i_1i_2)}\}$, $T_k^{(i_1i_2)}$ and $\tau_{k,l}^{(i_1i_2)}$ will assume the same fading parameters respectively, since all the transmit–receive pairs share the same scattering environment.

For example, all $\{\alpha_{k,l}^{(i_1i_2)}\}$ will admit the same fading model (2.11) or (2.12) and (2.17) with the same parameters $\{\Omega_1, \gamma_C, \gamma_R\}$, and all $\{T_k^{(i_1i_2)}\}$ and $\{\tau_{k,l}^{(i_1i_2)}\}$ will admit the same fading models (2.15) and (2.16) respectively with the same parameters $\{\lambda_C, \lambda_R\}$.

If the scatterers are sufficiently rich, then model (2.10) can be extended to describe the UWB-MIMO channel, i.e.:

$$H(t) = \sum_{l=1}^{L} A_l \delta[t - (l - 1)\Delta \tau], \quad (2.18)$$

where $\Delta \tau$ is the sampling interval and $A_l$, $l = 1, \ldots, L$, are the amplitude fading matrices. Each entry of $A_l$ may assume either the Nakagami distribution (2.11) or lognormal distribution (2.12). Since two different matrices $A_{l_1}$ and $A_{l_2}$ ($l_1 \neq l_2$) correspond to the amplitude fading caused by different groups of scatterers, any two matrices $A_{l_1}$ and $A_{l_2}$ ($l_1 \neq l_2$) can be assumed to be mutually independent. The correlation model among the entries of the matrix $A_l$ will be discussed next.

### 2.2.2 Correlation Property

The correlation property among the entries of $A_l$ characterizes the spatial correlation among the antennas. Similar to narrowband MIMO channels, there are two approaches to modelling the spatial correlation [188]. The first approach is to use a matrix of dimension $N_T N_R \times N_T N_R$ to describe the correlation between any two entries of $A_l$. The second approach is to change the channel matrix as

$$\bar{A}_l = R_t^{1/2} A_l R_r^{1/2}, \quad (2.19)$$

where $R_r$ and $R_t$ are the $N_R \times N_R$ receive covariance matrix and $N_T \times N_T$ transmit covariance matrix respectively. This model is based on the assumption that the correlation among the receive antennas is independent of the correlation among the transmit antennas.

Substituting Equation (2.19) into (2.18) yields

$$\bar{H}(t) = R_t^{1/2} H(t) R_r^{1/2},$$
where $H(t)$ is the CIR matrix of a UWB-MIMO channel taking into account the spatial correlation, and $\bar{H}(t)$ denotes the CIR matrix of a UWB-MIMO channel with all the subchannels being independent of each other.

For a uniform linear array of antennas, the matrices $R_r$ and $R_t$ can be typically described by [188]

$$R_r = \begin{bmatrix}
1 & \rho_{cr} & \rho_{cr}^2 & \cdots & \rho_{cr}^{N_R-1} \\
\rho_{ct} & 1 & \rho_{cr} & \cdots & \rho_{cr}^{N_R-2} \\
\rho_{ct}^2 & \rho_{ct} & 1 & \cdots & \rho_{cr}^{N_R-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_{ct}^{N_R-1} & \rho_{ct}^{N_R-2} & \rho_{ct}^{N_R-3} & \cdots & 1
\end{bmatrix}, \quad (2.20)$$

$$R_t = \begin{bmatrix}
1 & \rho_{ct} & \rho_{ct}^2 & \cdots & \rho_{ct}^{N_T-1} \\
\rho_{ct} & 1 & \rho_{ct} & \cdots & \rho_{ct}^{N_T-2} \\
\rho_{ct}^2 & \rho_{ct} & 1 & \cdots & \rho_{ct}^{N_T-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_{ct}^{N_T-1} & \rho_{ct}^{N_T-2} & \rho_{ct}^{N_T-3} & \cdots & 1
\end{bmatrix}, \quad (2.21)$$

where $\rho_{ct}$ and $\rho_{cr}$ are the correlation coefficients between two neighbouring transmit antennas and between two neighbouring receive antennas respectively. The correlation model defined by Equations (2.20) and (2.21) captures the fact that the spatial correlation of the UWB channel decreases with the distance between the antennas.

Another approach to describing the correlation property is to use the frequency-domain expressions of UWB-MIMO channels. Let us denote by $x(t)$ and $y(t)$ the time-domain $N_T$- and $N_R$-dimensional vectors of the transmit and receive signals respectively. Let $\tilde{x}(f)$, $\tilde{y}(f)$ and $\tilde{H}(f)$ be the Fourier transforms of $x(t)$, $y(t)$ and $H(t)$ respectively. Then the UWB-MIMO system can be generally described by

$$\tilde{y}(f) = \tilde{H}(f)\tilde{x}(f) + \tilde{n}(f), \quad (2.22)$$

where $\tilde{n}(f)$ is the receiver noise expressed in the frequency domain. By sampling in the frequency domain, $\tilde{H}(f)$ can be approximated by the sampled values at a series of frequency tones, say $f \in \{f_1, f_2, \ldots, f_m\}$, where $f_k = f_1 + (k - 1)\Delta f$, $k = 1, \ldots, m$, with $\Delta f$ being the frequency interval. If the bandwidth around each frequency tone is sufficiently narrow, i.e., if $\Delta f$ is sufficiently small, then model (2.22) reduces to $m$ parallel narrowband channels:

$$\tilde{y}_{f_1} = \tilde{H}_{f_1}\tilde{x}_{f_1} + \tilde{n}_{f_1},$$

$$\vdots$$

$$\tilde{y}_{f_m} = \tilde{H}_{f_m}\tilde{x}_{f_m} + \tilde{n}_{f_m},$$

where $\tilde{y}_f = \tilde{y}(f)$, $\tilde{x}_f = \tilde{x}(f)$, $\tilde{H}_f = \tilde{H}(f)$ and $\tilde{n}_f = \tilde{n}(f)$. Let us consider a specific frequency tone $f$, where we abuse the notation $f$ to denote a specific frequency rather
than a general frequency. Following the correlation model of narrowband MIMO channels proposed in [188], we can adopt the model [10]

$$\tilde{H}_f = R_t^{1/2} \tilde{H}_w R_r^{1/2}$$

(2.23)

to describe the spatial correlation among the entries of $\tilde{H}_f$, where $\tilde{H}_w(f)$ is a random matrix with independent and identically distributed entries, and the correlation matrices $R_t$ and $R_r$ are of the same structure as shown in Equations (2.20) and (2.21) respectively. Different from narrowband systems, the assumption that each entry of $\tilde{H}_w$ is complex Gaussian with zero mean and unit variance generally does not hold true for UWB systems. Notice that the parameters $\rho_{ct}$ and $\rho_{ct}$ in $R_t$ and $R_r$ of Equation (2.23) can be frequency dependent.

In [10], it is shown that the BER performance of a MIMO system based on the simulated channel using model (2.23) gives a good approximation to the BER performance of the MIMO systems based on the measured channel if appropriate values for $\rho_{ct}$ and $\rho_{ct}$ are chosen.

In [152], the spatial correlation coefficients are measured in an indoor office with dense scatterers by using a vector network analyser (VNA), where the spatial arrays at both transmitter and receiver are synthesized by moving a single antenna along a preplanned route. This concept in synthesizing an array of antennas is valid when the channel is stationary for the period under investigation and is used in several other studies [155, 168, 190, 191]. The parameters used in [152] are $f_1 = 3.1$ GHz, $f_m = 10.6$ GHz and $m = 1601$, which leads to $\Delta f \approx 4.7$ MHz. The complex correlation coefficient $\rho_c$, power correlation coefficient $\rho_p$ and envelope correlation coefficient $\rho_e$ are defined with respect to $\tilde{H}_f$, $|\tilde{H}_f|^2$ and $|\tilde{H}_f|$ respectively for a fixed frequency but at different positions. Discone antennas with efficient radiation characteristics over the UWB band are used. Figure 2.1 shows the measured results.

It can be seen from Figure 2.1 that, in the range of $\pm 5$ cm, the antenna correlation is a monotonously decreasing function of the antenna distance, while almost a constant correlation coefficient (smaller than 0.4) is observed when the antenna distance is larger than $\pm 5$ cm.

Figure 2.1 also shows that the square-law relation [152]

$$\rho_p \approx \rho_c \approx |\rho_c|^2$$

does not hold for the UWB channels, which is valid for Rayleigh fading channels. Therefore, $\tilde{H}_w$ in Equation (2.23) cannot be complex Gaussian.

As shown in Chapter 3, the MIMO system performance, from the viewpoint of channel capacity, will degrade little if the correlation coefficient is less than 0.4 compared with the case of independent spatial channels. Therefore, the effect of the antenna correlation on MIMO system performance (especially channel capacity) can be neglected if any two antennas are separated by more than 10 cm.

One important issue concerns how the spatial correlation depends on the separation in frequency domain. We measured this kind of frequency dependency using the UWB-MIMO testbed developed in our laboratory [118]. For the details of the testbed, readers

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5 This is actually a virtual antenna array approach. A main drawback of this approach is that it neglects the possible electromagnetic coupling effect between antennas [123, 152].
Figure 2.1  Spatial correlation for different polarizations in dense indoor-office environments with LOS. (a) Along cross-range direction with vertical polarization; (b) along range direction with vertical polarization; (c) along cross-range direction with horizontal polarization. The insets are for NLOS channels. The abscissa is the distance between two receive antennas under consideration. (From [152]. Reproduced by permission of © IEEE 2008.)
are referred to Section 2.3. The results are shown in Figure 2.2, where $\rho_{tx}^2$ denotes the correlation coefficient between two channels from two transmit antennas to one receive antenna and $\rho_{rx}^1$ denotes the correlation coefficient between two channels from one transmit antenna to two receive antennas. As shown in Figure 2.2, both the transmit and receive correlation coefficients are highly frequency dependent and mostly below 0.5, with the
maximum value being 0.65, which appears typical in the case where the LOS transmission is dominant. However, this kind of frequency dependence looks rather random with respect to the separation in the frequency domain.

Another approach to describing the spatial channel correlation property is from the deterministic point of view [247]. It is defined as the average value of all the cross-correlation functions of different spatial channels normalized by the autocorrelation functions of the corresponding spatial channels.

**Remark 2.1** It is noteworthy that the channel correlation model (2.19) changes the form of the pdf of the fading of the single channel, i.e., the pdf of the entry of the matrix $\tilde{A}_j$ is no longer of the Nakagami or lognormal form. This is different from the narrowband MIMO channels, where a linear transformation of the channel matrix only changes the parameters of the pdf, not the form.

**Remark 2.2** Model (2.19) is a simplified mathematical model for the channel correlation, whose validity, indirectly verified in [10], needs to be further examined through extensive UWB channel measurement studies.

## 2.3 Channel Measurement

Testbeds give a fundamental insight into the practical aspects of a developed approach and provide an important tool to verify a proposed model and the results of a model-based system design. In order to simplify the access to the real UWB-MIMO channel, a universal offline testbed with a MATLAB interface and VHDR (very high data rate) MIMO MB-OFDM (multiband OFDM) system simulator was set up in the laboratory of our Institute. It enables various experiments and testing of the proposed algorithms under real propagation constraints and scenarios.

### 2.3.1 Measurement Setup

The testbed is aimed at both channel modeling and verification of several applications. The VHDR applications are targeted at high-resolution media content transfer over short distances up to 1 m mainly in LOS indoor environments. The envisioned VHDRs for an extension of the existing WiMedia standard [112] vary between 1 Gbps to 3 Gbps, depending on the modulation scheme and quality of the channel. The intended doubling of the signal bandwidth to 1056 MHz and the adoption of 16-QAM (quadrature amplitude modulation) modulation inevitably place a higher level requirement to the receiver sensitivity. The receive SNR is typically required to be higher than 15 dB for reliable detection. While the system performance of single antenna VHDR has been reported to provide a sufficient small average BER at this SNR level, it is interesting to check the feasibility of MIMO approaches under the same system constraints. In particular, we have tried to examine the achievable spatial multiplexing gains and to experimentally measure the channel capacity and fading correlation under the specific test environments.

The channel modeling as well as the test and verification of the VHDR MIMO MB-OFDM system were conducted within a typical office environment with desks, metal

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6 This section is reproduced with permission from a part of [118]. Reproduced by permission of © IEEE 2009.
cabinets, wooden cupboards, computers, and different kinds of smaller scattering objects. For the purpose of the envisioned VHDR applications, measurements were performed in pure LOS environment. The equipment components (TX and RX) were placed on desks approximately 70 cm over the floor and 1 m apart. The transmit and receive antenna arrays were aligned linearly with 10 cm inter-antenna separation, corresponding to the largest wavelength of the transmitted signals. The temporal stationarity of the channel was ensured by the absence of mobile objects/persons, thus allowing us to assume quasi-static channel conditions during a single MB-OFDM frame.

The main components of the offline UWB-MIMO testbed are two arbitrary waveform generators (AWGs) and one digital phosphor oscilloscope (DPO). Here AWG7102 and DPO-71604 by Tektronix are used. Each AWG supports two channels with up to 10 GSamples/s (GS/s) sampling rate and 3.5 GHz bandwidth or one channel with up to 20 GS/s sampling rate and 5.8 GHz analog bandwidth using interleaving, as well as 32 mega-Sample memory size. Each channel may be used either for I/Q or direct IF/RF synthesis of arbitrary digital waveforms complying with the maximum available bandwidth. This allows for high flexibility of loading and storing various waveforms directly from a system-model simulator without the need of up-conversion circuits and expensive UWB RF front ends. A set of various sequencing commands also enables the transmission of repetitive portions of the waveform through numerous program branches, jumps, and loops.

The DPO provides 4 channels with up to 50 GS/s sampling rate and 16 GHz frequency span. Therefore, it is currently possible to test and verify arbitrary $2 \times 4$ MIMO transmissions with an upper RF frequency of 5.8 GHz and $4 \times 4$ MIMO with an upper RF frequency of 3.5 GHz without additional up-conversion.

The setup of the UWB-MIMO testbed and a sketch of the whole system are illustrated in Figure 2.3. As can be seen, each AWG in the testbed is connected to one UWB antenna. Since the proposed system employs spatial multiplexing, the signals are loaded into each AWG independently and transmitted synchronously over the interleave channels of each AWG. For most experiments we have used small-size omni-directional UWB patch antennas, providing a maximum directive gain of 3 dBi in the frequency range from 3 GHz to 4.8 GHz. In this aspect we have limited the VHDR MIMO test to the first band group of the existing WiMedia standard [112].

On the receiver side, up to 4 UWB antennas are connected to the DPO through a bank of bandpass filters (1.5 GHz bandwidth) and +55 dB low-noise amplifiers for reliable signal acquisition under harsh SNR conditions. Note that the AWGs have been synchronized by a 10 MHz reference clock signal and separately calibrated to minimize internal RF frontend mismatches. The trigger signal of the DPO is applied to its auxiliary output, which is connected to the trigger input of the AWG. This enables synchronous MIMO transmission and reception by triggering the DPO manually with an internal command. Both AWGs and DPO are connected to the processing computer via Ethernet.

### 2.3.2 Measured Results

The typical equipment setting for experiments conducted in this book is shown in Table 2.2. It is for the case of a $2 \times 3$ UWB-MIMO offline testbed. Other cases use similar settings with some minor changes.

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7 See [http://www.tek.com/site/ps/0,76-19779-INTRO_EN,00.html](http://www.tek.com/site/ps/0,76-19779-INTRO_EN,00.html).
For every experiment, a minimum of 1000 VHDR MB-OFDM frames of 1 KB payload size are generated, sequentially transmitted and acquired by the UWB-MIMO testbed, and finally stored for further offline processing in Matlab.

Figure 2.4 illustrates the power spectrum of the four MIMO VHDR MB-OFDM signal frames acquired during a $2 \times 3$ MIMO VHDR transmission. It shows a good out-band suppression of the bandpass filters and gives a qualitative indication of the influence of channel fading on the received signal spectrum.
Table 2.2 Typical equipment setting for experiments conducted in this paper.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels per AWG</td>
<td>1 (interleaved)</td>
</tr>
<tr>
<td>AWG interleaving</td>
<td>on</td>
</tr>
<tr>
<td>AWG sample rate</td>
<td>20 GS/s</td>
</tr>
<tr>
<td>AWG sample length</td>
<td>20,000,000</td>
</tr>
<tr>
<td>AWG size per sample</td>
<td>32 bit floating point</td>
</tr>
<tr>
<td>Channels per DPO</td>
<td>3</td>
</tr>
<tr>
<td>DPO sample rate</td>
<td>25 GS/s</td>
</tr>
<tr>
<td>DPO sample length</td>
<td>25,000,000</td>
</tr>
<tr>
<td>DPO size per sample</td>
<td>16 bit fixed point</td>
</tr>
</tbody>
</table>

Figure 2.4 Power spectrum of the acquired MIMO VHDR MB-OFDM frame signal.

The spatial correlation coefficients between the fading paths of the estimated UWB-MIMO channel are illustrated in Figure 2.2 in the preceding section.

2.4 Ray-Tracing Simulation Tool

2.4.1 Ray Tracing for Narrowband Systems

The models discussed in the preceding sections are in nature statistical channel models, which are appropriate for the evaluation of channel characteristics and system performance in a general sense. Even though they can be used to predict the general tendency of the
system performance, they may fail in the evaluation of the system performance for real environments if a high accuracy for such an evaluation is required. A site-specific channel model can be developed based on the ray-tracing simulation approach. In [212], it is shown that the ray-tracing prediction model is able to predict path loss with an overall standard deviation of less than 5 dB throughout the two buildings investigated therein. The time delay comparison there shows that the amplitudes and time delays of measured power delay profiles can be predicted accurately via ray tracing.

Ray tracing is a physically tractable method of predicting the delay spread and path loss of in-building radio signals. Ray-tracing techniques approximate the propagation of electromagnetic waves by representing the wavefronts as simple particles. Thus, the effects of reflection and diffraction on the wavefront are approximated using simple geometric equations instead of Maxwell’s equations [90].

In ray tracing, three kinds of wave propagations are considered: transmission, reflection and diffraction; see Figure 2.5. Only large scattering objects (in the sense that the sizes of the objects are at least several times the largest wavelength) are taken into account. This is due to two reasons. The first is that the received signal is generally dominated by transmitted rays or the rays reflected by large objects. The second is that the error of the ray-tracing approximation is small when the reflecting/diffracting objects are large relative to the wavelength and fairly smooth [90, 212]. Therefore, scattering rays are neglected. Thus, a finite number of reflecting and diffracting objects will be assumed with their locations and dielectric properties being known.

For a moderate scattering environment, it is not easy manually to identify all the possible rays from the transmitter to the receiver. However, professional computer programs, such as Lucent’s Wireless Systems Engineering (WiSE) software, Wireless Valley’s SitePlanner®, Marconi’s Planet® EV, and Wireless InSite®, are available to generate all the possible rays based on the office/building layout. General guidelines for creating

![Diagram of transmitted, reflected and diffracted rays](image)

**Figure 2.5** An illustration for transmitted, reflected and diffracted rays. Note that the transmitted rays include both the LOS ray and possibly NLOS rays.
a two-dimensional picture of a three-dimensional (3D) world in the field of graphical ray tracing are also well developed [89, 232]. As can be easily imagined, one ray from the transmitter to the receiver may consist of several rays reflected or diffracted through several objects. Typically, only the rays with the number of transmissions, reflections and diffractions being less than a small integer are considered.

The reflected ray is characterized by a reflection coefficient, which is again a function of the dielectric constant of the reflecting object and the ray incidence angle [141, 256].

Diffraction can be accurately characterized using the geometrical theory of diffraction (GTD) [124], but its complexity limits its application in the channel modelling issue. The wedge diffraction approach [124, 127] gives a good approximation for the GTD by assuming that the diffracting object is a wedge rather than a more general shape. In this model, the diffracted ray is characterized by a diffraction coefficient. In [127], the diffraction coefficient is developed from the canonical scattering solution for perfectly conducting wedges. The canonical problem of scattering via a dielectric wedge is not yet solved. In [30], the formula for the diffraction coefficient developed in [127] is modified to apply to the case of a dielectric wedge.

A further simplified and widely used approach is the Fresnel knife-edge diffraction model, where the diffraction coefficient depends only on the geometrical parameters of the triangle consisting of the transmitter, wedge and receiver [90, 141].

For the $i$th received ray at the receiver, the complex field is given by

$$E_i = E_0 \overline{g_t} \overline{g_r} L_i (d_i) \prod_{k_1} \Gamma_R (\theta_{k_1 i}) \prod_{k_2} \Gamma_T (\theta_{k_2 i}) \prod_{k_3} \Gamma_D (\theta_{k_3 i}) \exp \left( -\frac{2\pi d_i}{\lambda} \right)$$

(2.24)

where $E_i$ (V/m) is the field strength of the $i$th ray component, $E_0$ (V/m) is the reference field strength, $g_t$ is the field amplitude radiation pattern of the transmit antenna, $g_r$ is the field amplitude radiation pattern of the receive antenna, $d_i$ (m) is the path length of the $i$th ray, $L_i$ is the path loss of the $i$th ray, $\theta_{k_i}$ is the incidence angle of the $i$th ray at the $k$th component, $\Gamma_R$ is the reflection coefficient, $\Gamma_T$ is the transmission coefficient, $\Gamma_D$ is the diffraction coefficient, $\lambda$ is the wavelength, $k_1$ is the index of all the reflected components contained in the $i$th ray, and $\prod_{k_1} \Gamma_R (\theta_{k_1 i})$ is the product of all the reflection coefficients contained in the $i$th ray. A similar convention applies to $\prod_{k_2} \Gamma_T (\theta_{k_2 i})$ and $\prod_{k_3} \Gamma_D (\theta_{k_3 i})$.

Finally, the field strength of the received signal is obtained by summing up all the rays:

$$E_r = \sum_i E_i,$$

where the summation is taken over all possible rays from the transmitter to the receiver.

The above approach can be easily extended to the MIMO case by using the superposition principle. First, let us place only one transmit antenna. Then, by repeating the procedure outlined above from this transmitter to each receive antenna, we can obtain the received field strength vector for the SIMO case. Second, repeat the procedure of the SIMO case for each transmit antenna and record all the received field strength vectors. Third, summing up all these field strength vectors, we obtain the MIMO field strength vector.
2.4.2 Ray Tracing for UWB Systems

The ray-tracing technique can be directly extended to wideband or UWB systems in the following way. First, let us fix a frequency. For this fixed frequency, calculate the received field strength (or field strength vector in the MIMO case) for a given input signal. Second, repeat the first step by sweeping all the frequencies in the interested frequency band and then we obtain the frequency response function of the channel. Third, we can calculate, if needed, the CIR of the system by using the inverse Fourier transform of the frequency response function obtained.

The ray-tracing technique has been used in the wideband case [291] and in the UWB case [20, 65, 66, 114, 237, 246]. In the following, some comparisons between measured CIRs and ray-tracing-based simulated CIRs of UWB systems will be illustrated.

2.4.2.1 The SISO Case

The measurements were performed in an office environment in IMST GmbH\(^8\) for both LOS and NLOS scenarios. The floor plan of the offices and the locations of antennas are shown in Figure 2.6. Each room is approximately 5 m × 5 m in size and 3 m in height. Both transmit and receive antennas were put 1.5 m high above the floor. The receive antenna is fixed in a position, while the transmit antenna is placed at one of the two positions shown in Figure 2.6 to cover both LOS and NLOS scenarios. For the NLOS scenario, the transmitter and receiver are separated by a metal cabinet of size 1.78 m × 0.42 m × 1.96 m. Both transmit and receive antennas are identical omni-directional UWB conical antennas of 3 dB gain.

![Figure 2.6](Image)

**Figure 2.6** Floor plan of the offices. (From [66]. Reproduced by permission of © IEEE 2008.)

\(^8\) See the company website at http://www.imst.de/de/home.php.
The frequency response of the channel (including the transmit and receive antennas as a part of the whole channel) is measured by a VNA across the frequency range from $f_1 = 1$ GHz to $f_m = 11$ GHz with a sampling interval $\Delta f = 6.25$ MHz. The inverse Fourier transform is used to obtain the channel impulse response.

In the ray-tracing simulation setup, the maximum numbers of reflections, transmissions and diffractions considered for each path are set to be two, two and one respectively. Wireless InSite is used to generate all the possible rays, as shown in Figure 2.7.

To make the comparison between the measurements and the simulations fair, we choose the same bandwidth from 1 to 11 GHz with a frequency sweeping step of 6.25 MHz for the simulations. Similarly, the inverse Fourier transform is used to obtain the CIR.

The transfer function or radiation pattern of the transmit/receive antenna was obtained by using the electromagnetic simulation tool CST MICROWAVE STUDIO®. This can be done either in the time domain by defining far-field probes surrounding the antenna or in the frequency domain by sweeping the operating frequency from 1 to 11 GHz with a step of 250 MHz and calculating the radiation pattern of the complex electric field at each operating frequency. As an example, the calculated radiation pattern of the transmit antenna at the frequency $f = 6$ GHz is shown in Figure 2.8.

The measured and simulated CIRs of the LOS and NLOS cases are illustrated in Figure 2.9 and Figure 2.10 respectively. It can be seen from Fig 2.9 and Fig 2.10 that the LOS component in the measurement and simulation arrives at the same time instant for the LOS scenario, and that the most conspicuous rays in the measurement and simulation arrive at the same time instants for the NLOS scenario. For the LOS scenario, the simulated CIR agrees well with the measured CIR, while for the NLOS scenario, a lot of rays are missed in the simulated CIR. This is due to the fact that many small scatterers are not taken into account in the simulation setup and the maximum numbers of reflections and diffractions are too small.

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9 Computer Simulation Technology Microwave Studio or CST MWS; http://www.cst.com/.
Figure 2.8 The absolute value of the radiated electric field of the vertical UWB conical antenna at $f = 6$ GHz: (a) $\theta$ component; (b) $\phi$ component. (From [66]. Reproduced by permission of © IEEE 2008.)

2.4.2.2 The MIMO Case

The ray-tracing technique, combined with the active element pattern method [125], can be easily used to investigate the mutual coupling among elements of an antenna array.

When the element separation in an antenna array is not large enough, the mutual coupling among array elements will cause a severe change for the expected radiation pattern of the antenna array. Numerical techniques can be used directly to solve the current distributions on the elements subject to the boundary conditions imposed by the array geometry and, hence, predict the effect of the mutual coupling on the performance of relevant systems [236]. However, this approach is computationally intensive. On the other hand, the pattern multiplication approach can be applied to provide a good approximation for the effect, which is accurate enough for many narrowband arrays encountered in practice when the element separation is reasonably large and the element current distributions for different array elements are identical within a complex multiplicative constant (homogeneous media) [125]. Nevertheless, it is difficult to apply this approach to UWB antenna arrays, since the radiation pattern of each element depends on the frequency used; hence, frequency sweeping should be used.

The active element pattern method applies to the case where the scattering media is static and linear [125], no matter whether the media is inhomogeneous or the array elements are close to each other. When the scattering media is linear, the superposition principle applies, which implies that the total current distribution on a fully excited array is the sum of the individual unit-excitation current distribution components, scaled by the complex-valued feed voltages [125]. By this principle, the radiation pattern of a fully excited array can be written as [125]:

$$E(\theta, \phi) = \sum_{k=1}^{N} V_k g_{a,k}(\theta, \phi),$$

(2.25)
where the two-dimensional vector $\mathbf{E}(\theta, \phi)$ is the radiation pattern of the fully excited array, $V_k$ is the complex-valued feed voltage applied to the $k$th element, $N$ is the number of elements in the array and the two-dimensional vector $\mathbf{g}_{\text{ir},k}$ is the field produced by the $k$th unit-excitation current distribution component. Notice that $\mathbf{g}_{\text{ir},k}$ represents the pattern radiated by the entire array when only one element is directly excited and the other elements are parasitically excited by the active element. It is this characteristic that

**Figure 2.9**  (a) Measured and (b) simulated CIR for the LOS scenario. (From [66]. Reproduced by permission of © IEEE 2008.)
Figure 2.10  (a) Measured and (b) simulated CIR for the NLOS scenario. (From [66]. Reproduced by permission of © IEEE 2008.)

makes the active element pattern method different from the traditional isolated element pattern method.

Applying this principle, we can excite each element turn by turn, use any electromagnetic simulation tool such as CST MICROWAVE STUDIO to calculate actively excited and all the parasitically excited field strength, and store it for later use in array pattern
calculation. In the following, the difference between the CIRs taking into account the mutual coupling and neglecting the mutual coupling will be illustrated. The simulation setup is shown in Figure 2.11.

In Figure 2.11a, two UWB bow-tie transmit antennas are located at the position Tx, as shown in Figure 2.11b, with a separation of 1.5 cm, which is smaller than half of the minimal wavelength (\(\sim 3\) cm). Hence, the mutual coupling between the two antennas is conspicuous. The 36 \(\phi\)-oriented probes and 36 \(\theta\)-oriented probes surrounding the transmit antennas with equal angular step size of \(10^\circ\) are used to sense the radiation pattern of the two-antenna array. The location of the receive antenna is shown in Figure 2.11b, where the possible rays from the transmit antennas to the receive antenna are also shown.

**Figure 2.11** A simulation setup showing the effect of the mutual coupling between antennas on the CIR: (a) transmit antenna array; (b) room setup and all possible rays. (From [114]. Reproduced by permission of © IEEE 2007.)
The ray-tracing technique is used to calculate the CIRs. The frequency is sampled from 1 to 11 GHz with a step of 6.25 MHz. The CIRs taking into account the mutual coupling and neglecting the mutual coupling between the two transmit antennas are illustrated in Figure 2.12 and Figure 2.13 respectively. As can be seen, the CIRs taking into account the mutual coupling and neglecting the mutual coupling are quite different from each other.
2.5 Summary

In this chapter we have reviewed several commonly used UWB channel models. The channel sounding equipment developed in our institute and the ray-tracing simulation tool have also been briefly presented. A simple correlation model for UWB-MIMO channels is proposed. Based on the ray-tracing simulation tool, the effect of mutual electromagnetic coupling between antennas on the UWB-MIMO channel models has been discussed.
Note that the channel models discussed in this chapter are mainly based on the IR-based UWB systems. Having been extensively studied for more than a decade, the UWB channel models for the SISO case are quite standardized. Nowadays, the models CM1, CM2, CM3 and CM4 combined with the lognormal or Nakagami amplitude fading distributions have been widely accepted as the standard SISO UWB channel models for the corresponding scenarios. However, the picture for the study of MIMO UWB channel models is quite different. It is fair to say that the study is in its very early stage. Further extensive investigation from computer simulations, laboratory experiments and measurement campaigns is needed.
3

UWB Channel Capacity

3.1 Introduction

In this chapter, we investigate the channel capacity of UWB-MIMO systems. It gives a picture for how much potential the systems combining together the UWB and MIMO technology can provide.

There is a lot of research work on the capacity of narrowband MIMO fading channels, see, for example, [78, 244] for analytical studies and [81, 168] for measurement studies, but few reports are available on the study of the capacity of frequency selective MIMO fading channels. References [27, 28, 155, 168] are some examples for the research in this field. The results on the channel capacity of UWB-MIMO systems can be found in [155, 202, 284, 285, 287]. On the channel capacity of UWB-SISO systems, there are a few studies, see [70, 143, 198, 199, 200, 280, 282]. In [70, 143, 199, 280, 282], the channel capacity of $M$-ary PPM UWB systems is discussed. However, the UWB channel fading property is not taken into account there. In [198, 200], the channel capacity for multiple-access using time hopping and block waveform encoded $M$-ary PPM is analyzed, where free-space propagation conditions are assumed. Hence channel fading needs not be considered either.

Our first concern in this chapter will be focused on the channel capacity of UWB-MIMO systems. This is because two major benefits can be exploited if the proportionality of the channel capacity to the number of transmit/receive antennas is also proved true for UWB fading channels. First, according to Edholm’s law of data rate [44], it can be predicted that indoor data rates of several Gbit/s will become reality in a couple of years. Therefore, even more bandwidth will be required sometime in the future, although UWB offers enormous bandwidth. If it can be shown that the channel capacity of UWB systems is proportional to the number of transmit/receive antennas, data rates can be significantly increased further by combining MIMO and UWB. However, even if lower data rates are in focus, the trade-off between bandwidth and the number of antennas could facilitate the antenna and front-end amplifier design, which is still a challenge for UWB systems. For

1 A part of this chapter is reproduced with permission from [287] and a part of [118]. Reproduced by permission of IEEE © 2008, 2009.
example, the bandwidth requirement could be almost halved if two antennas on each side of the link are deployed.

Our second concern will be focused on the effect of frequency selectiveness on channel capacity. An interesting phenomenon about the capacity of a frequency-selective channel for wideband communication systems observed in [58] is that when the signal-to-noise power ratio (SNR) is high (higher than 20 dB for the system studied in [58]), there is only a very small loss of channel capacity if a white power spectrum for the transmitted signal instead of the optimal power distribution in frequency domain is used, while the benefit of optimal power distribution can be only obtained when the SNR is low. It is important to investigate whether this finding holds for the UWB case since UWB systems may work in the scenario of low SNR.

3.2 System Model

In this chapter, the simplest UWB channel models (2.10) for the SISO case and (2.18) for the MIMO case will be used to investigate the channel capacity for UWB systems. Let us start with the SISO case. Let $X(t)$ and $Y(t)$ denote the transmitted UWB signal and the received signal after the matching filter and sampling, respectively. Then the relationship between $X(t)$ and $Y(t)$ can be described by

$$Y(t) = \sum_{l=1}^{L} \alpha_l X[(t - (l - 1)\Delta\tau] + N(t), \quad (3.1)$$

where $\alpha_l$ is the amplitude fading in the $l$th delay bin, $N(t)$ denotes the receiver noise, and $\Delta\tau$ can be set to be $1/B$, with $B$ being the bandwidth of the transmitted signal $X$. In channel capacity calculation, we can assume that the transmitted signal will occupy the whole bandwidth of the channel frequency band.

In the MIMO case, the input–output relation (channel model) can be generally described, similar to Equation (3.1), by the following equation:

$$Y(t) = \sum_{l=1}^{L} A_l X[t - (l - 1)\Delta\tau] + N(t), \quad (3.2)$$

where $X(t)$ and $Y(t)$ are the $N_T$- and $N_R$-dimensional vectors of the transmit and receive signals, respectively, with $N_T$ and $N_R$ being the numbers of the transmit and receive antennas, $A_l$, $l = 1, \ldots, L$, are the amplitude fading matrices, and $N(t)$ is the receiver noise vector.

The Nakagami distribution model (2.11) together with the power decay model (2.13) for the amplitude fading $\alpha_l$ in Equation (3.1) and each entry of $A_l$ in Equation (3.2) will be used throughout this chapter.

For a random process $X(t)$ (vector-valued), define, respectively, its correlation matrix and power spectral density (PSD) matrix as

$$R_X(\xi) = \mathbb{E}_X[X(t + \xi)X^T(t)],$$

$$S_X(f) = \int_{-\infty}^{+\infty} \mathbb{E}[X(t + \tilde{\tau})X^T(t)] e^{j2\pi f \tilde{\tau}} d\tilde{\tau}.$$
It is implied in model (3.2) that the number of multipaths across all the MIMO channels is the same $L$. This is physically unrealistic. In practice, we can choose $L$ as the number of time bins of the longest channel and set the amplitude of the extraneous taps for the shorter channels to zero. For the channel model (3.2), we further make the following simplifying assumptions, as usual in discussing the channel capacity problem.

**Assumption 3.1** The amplitude fading matrices $A_l$, $l = 1, \ldots, L$, are assumed to be mutually independent, and all the entries of $A_l$, $l = 1, \ldots, L$, are also assumed to be mutually independent. Suppose $A_l = [\alpha_{l, mn}]_{NR \times NT}$ with $\alpha_{l, mn} = \nu_{l, mn} \zeta_{l, mn}$, $l = 1, \ldots, L$, $m = 1, \ldots, NR$, $n = 1, \ldots, NT$. Then $\zeta_{l, mn} = |\alpha_{l, mn}|$ is of the distribution (2.11) with $\Omega_l$ being governed by the model (2.13), and $\nu_{l, mn} = \text{sign}(\alpha_{l, mn})$ takes values $\pm 1$ with equal probability.

**Assumption 3.2** The noise $N$ is zero-mean Gaussian with PSD matrix being $N_0 I_{NR}$.

**Assumption 3.3** The power of the transmitted signal is bounded by $\bar{S}$, i.e.,

$$\mathbb{E}[X^T(t)X(t)] = \int_{-B/2}^{B/2} \text{tr}(S_X(f))df \leq \bar{S},$$

(3.3)

where $\text{tr}$ denotes the trace of a square matrix.

### 3.3 Channel Capacity with Unknown CSI at the Transmitter

In this chapter, we generally presume that the receiver possesses the complete knowledge of the instantaneous channel parameters, i.e., the realizations of $A_l$ and their statistics. However, the channel state information (CSI) may or may not be available at the transmitter, depending on the cases studied. In this section, we assume that the transmitter is not aware of the CSI.

The Shannon capacity of a communication channel is the maximum transmission rate supportable by the channel. For an additive white Gaussian noise (AWGN) channel, whose input–output relationship is characterized by $y(t) = x(t) + n(t)$, the channel capacity is given by Shannon’s well-known formula [56]:

$$C = B_x \log \left[ 1 + \frac{\mathbb{E}(x^2)}{\mathbb{E}(n^2)} \right],$$

(3.4)

where $x$, $y$, and $n$ are the transmitted signal, received signal, and receiver noise, respectively, and $B_x$ is the channel bandwidth. Equation (3.4) provides a fundamental relationship among the system capacity, channel bandwidth, and the average SNR characterized by $\mathbb{E}(x^2)/\mathbb{E}(n^2)$. Shannon’s coding theorem proves that there exists a code for $x$ that achieves data rates arbitrarily close to the capacity $C$ with arbitrarily small probability of bit errors for a sufficiently long coding block.

For the channel of wireless narrowband communications, the input–output relationship is characterized by

$$y(t) = ax(t) + n(t),$$

(3.5)
where $a$ is a random variable characterizing the fading caused by the communications environment. Since the SNR at the receiver, $a^2\mathbb{E}(x^2)/\mathbb{E}(n^2)$, is also a random variable, two notions of capacity, namely ergodic capacity and outage capacity, are introduced [24], depending on the property of the fading $a$.

First, if the fading changes so quickly that a transmitted codeword experiences many (or an infinite number of, in the extreme case) independently fading blocks, the ergodic capacity defined as

$$C_e = B_x \mathbb{E}_a \left\{ \log \left[ 1 + \frac{a^2\mathbb{E}(x^2)}{\mathbb{E}(n^2)} \right] \right\}$$

(3.6)

gives a characterization for the data rates supportable by the channel. The ergodic capacity represents the average data rate supportable by the channel.

Second, if the fading changes so slowly that a transmitted codeword spans only a single fading block (in other words, the fading is almost constant in one codeword), the outage capacity defined as

$$P_{out}(R) = P_{C_{|a}}(R)$$

(3.7)

gives a characterization for the data rates supportable by the channel, where $R$ is the data transmission rate, $C_{|a}$ is the channel capacity for a given realization or sample of the fading. The so defined $P_{out}(R)$ means that if some codewords are transmitted across the channel with the rate $R$, the codewords cannot be correctly decoded with a probability of $P_{out}(R)$, i.e., the channel can only support the rate $R$ with a probability of $1 - P_{out}(R)$.

For a MIMO channel described by

$$y(t) = x(t) + n(t),$$

(3.8)

where $y(t)$, $x(t)$, and $n(t)$ are $N_x$-dimensional vectors and $n(t)$ is the AWGN vector, the channel capacity is obtained by maximizing the mutual information given by [244]

$$I(x; y) = 2B_x[\mathcal{H}(y) - \mathcal{H}(n)] = B_x \log \det[I + R_x R_n^{-1}]$$

(3.9)

under the constraints applied on the transmitted signal $x$, where $\mathcal{H}$ denotes the differential entropy of a continuous variable, and $B_x$ is the bandwidth occupied by $x$. For an $N_x$-dimensional vector $x$ of Gaussian distribution, its differential entropy is given by [56, p. 230]

$$\mathcal{H}(x) = \frac{1}{2} \log[(2\pi e)^{N_x} \det(R_x)],$$

where $R_x$ is the correlation matrix of $x$. Note that the unit of $I$ is nats/s. A typical constraint on $x$ is that the power carried by $x$ is limited (cf. Assumption 3.3).

It is clear that Equation (3.9) is a direct extension of Equation (3.4) in the MIMO case. The matrix $R_x R_n^{-1}$ can be considered as a generalized SNR. Since multiple antennas are deployed in the MIMO case, we can allocate the overall power across the transmit antennas in different ways. For example, for a MIMO system, the channels between different pairs of transmit–receive antennas may experience severely different fading. If the transmitter has this a priori knowledge, more power can be allocated to the better
channel, while less power to the worse channel, so that the overall data transmission rate can be increased. Therefore, some optimization procedures are needed to maximize the mutual information given by Equation (3.9).

Having the above background, it is easy to obtain the channel capacity for model (3.2). The basic idea is to combine the results of capacity (3.6) and (3.9) for models (3.5) and (3.8), respectively.

Let us define
\[ \Phi_1(t) = \sum_{l=1}^{L} A_l X[t - (l - 1) \Delta \tau]. \]

So Equation (3.2) can be rewritten as
\[ Y(t) = \Phi_1(t) + N(t). \]

Define \( \bar{\omega} = (A_1, \ldots, A_L) \). To address the capacity from the viewpoint of outage probability, we consider a given realization of \( \bar{\omega} \). We divide the whole channel into infinitely many channels in the frequency domain. Consider the sub-channel of which the frequency is from \( f \) to \( f + \Delta f \). In this sub-channel, the spectrum of \( \Phi_1(t) \) can be considered as flat with its value being approximated by \( S_{\Phi_1}(f) \). Hence, the covariance matrices of \( \Phi_1(t) \) and \( N(t) \) confined in the sub-channel \( [f, f + \Delta f] \) are given by \( S_{\Phi_1}(f) \Delta f \) and \( N_0 \Delta f I_{NR} \) respectively. Therefore, the mutual information \( \Delta I|_{\bar{\omega}} \) conveyed by this sub-channel, conditioned upon \( \bar{\omega} \), is given by
\begin{equation}
\Delta I|_{\bar{\omega}} = \Delta f \log \left[ \frac{(2\pi e)^{NR}}{\det(N_0 \Delta f I_{NR})} \right] + \frac{1}{N_0} \log \det \left( I_{NR} + S_{\Phi_1}(f) \right). \tag{3.10}
\end{equation}

Notice that in the above argument we have used the fact that the transmitted signal should be Gaussian to achieve the channel capacity for an AWGN channel. Thus, taking the limit \( \Delta f \to df \) in Equation (3.10), we obtain the conditional mutual information, denoted as \( I|_{\bar{\omega}} \), between \( X \) and \( Y \) as follows:
\begin{equation}
I|_{\bar{\omega}} = \int_{-B/2}^{B/2} \log \det \left[ I_{NR} + \frac{1}{N_0} S_{\Phi}(f) \right] \, df. \tag{3.11}
\end{equation}

So the conditional channel capacity \( C|_{\bar{\omega}} \) is given by the maximization of \( I|_{\bar{\omega}} \) subject to the power constraint (3.3), i.e.:
\begin{equation}
C|_{\bar{\omega}} = \max_{0 \leq \Delta f \leq \bar{S}} \int_{-B/2}^{B/2} \log \det \left[ I_{NR} + \frac{1}{N_0} S_{\Phi}(f) \right] \, df. \tag{3.12}
\end{equation}

The above derivation for Equation (3.11) is based only on intuition. Its rigorous proof can be obtained by repeating the same argument as that in [185, Appendix A] using the covariance matrices of the component in frequency slot \( [f, f + \Delta f] \) of the orthogonal expansion of the signals \( \Phi_1(t) \) and \( N(t) \), respectively, in the frequency domain.

\[ S_{\Phi}(f) \Delta f \text{ and } N_0 \Delta f I_{NR} \text{ are the covariance matrices of the component in frequency slot } [f, f + \Delta f] \text{ of the orthogonal expansion of the signals } \Phi_1(t) \text{ and } N(t), \text{ respectively, in the frequency domain.} \]
Karhunen–Loeve expansion, though the distribution of $F$ discussed here is different from that in [185]. Define

$$H(f) = \sum_{l=1}^{L} A_l e^{-j2\pi f(l-1)\Delta\tau}.$$  

Then we have

$$S\Phi(f) = H(f)S_X(f)H^\dagger(f). \quad (3.13)$$

Substituting Equation (3.13) into Equation (3.12) gives

$$C|_F = \max_{\text{subject to (3.3)}} \int_{-B/2}^{B/2} \log \det \left[ I_{NR} + \frac{1}{N_0} H(f)S_X(f)H^\dagger(f) \right] df. \quad (3.14)$$

It is clear from Equation (3.14) that the capacity is achieved when $X(t)$ is such that, among other conditions, the equality for the power constraint in Equation (3.3) holds.

Now consider the optimal power design for the transmitted signal. Since the transmitter does not have any information about either $H(f)$ or the noise $N(t)$, the best way for distributing the power in the transmitter is to equally allocate it among all antennas and to uniformly distribute it on the frequency band $[-B/2, +B/2]$. Thus we have

$$S_X(f) = \begin{cases} \tilde{S}/N_TB & \text{when } f \in \left[-\frac{B}{2}, \frac{B}{2}\right], \\ 0 & \text{otherwise}. \end{cases} \quad (3.15)$$

Substituting Equation (3.15) into Equation (3.14), using the substitution $u = 2\pi f\Delta\tau$ in the integral and noticing the fact that $1/B = \Delta\tau$, we obtain

$$C|_F = \int_{-B/2}^{B/2} \log \det \left[ I_{NR} + \frac{\tilde{S}}{N_0 N_TB} H(f)H^\dagger(f) \right] df = \frac{B}{\pi} \int_0^\pi \log \det \left[ I_{NR} + \frac{\rho}{N_T} \tilde{H}(u)\tilde{H}^\dagger(u) \right] du, \quad (3.16)$$

where $\rho$ stands for the SNR at the transmitter side:

$$\rho = \frac{\tilde{S}}{BN_0},$$

and

$$\tilde{H}(u) = \sum_{l=1}^{L} A_l e^{-j(l-1)u}.$$
With $P_{C|f}(x)$, the outage probability of the channel capacity can be obtained by

$$P_{\text{out}}(R) = P_{C|f}(R) = P_{C|f} \left( \frac{R}{B} \right)$$

for a given data transmission rate $R$.

**Remark 3.1** Note that the power spectrum allocation policy (3.15) is justified from the information theoretic perspective, but in practice this policy can be only approximated due to the power spectrum mask and the particular waveforms used.

### 3.4 Channel Capacity with Known CSI at the Transmitter

In this section, we assume that the full CSI is available at the transmitter. For the purpose of easy exposition, two cases, i.e., the SISO and MIMO, are separately investigated.

#### 3.4.1 The SISO Case

For the UWB-SISO system, the transmitted and received signals, and the channel fading coefficients are denoted as $X(t)$, $Y(t)$, and $\alpha_1, \ldots, \alpha_L$ respectively. The channel model is described by Equation (3.1). The channel frequency response function is denoted as $H(f)$. Let $\mathcal{F} = (\alpha_1, \ldots, \alpha_L)$.

If the information about $\mathcal{F}$, and hence about $H(f)$, is available to the transmitter, the result of [79, Section 8.3] can be used to obtain the optimal $S_X(f)$ to achieve $C|\mathcal{F}$, which is as follows:

$$S_X(f) = \left[ \Theta - \frac{N_0}{|H(f)|^2} \right]_+,$$

where $[x]_+$ equals $x$ when $x \geq 0$ and 0 otherwise, $\Theta$ is a constant which satisfies the Equation

$$\int_{f \in F_\Theta \cap [-B/2, B/2]} \left( \Theta - \frac{N_0}{|H(f)|^2} \right) \, df = \bar{S},$$

and $F_\Theta$ is the range of $f$ for which $N_0/|H(f)|^2 \leq \Theta$. The above solution for the optimal $S_X(f)$ is the widely known water filling algorithm. In the following, we present the detailed expression of the channel capacity.

Suppose that

$$F_\Theta \cap \left[0, \frac{B}{2}\right] = [f_{11}, f_{12}] \cup [f_{21}, f_{22}] \cup \cdots \cup [f_{\nu 1}, f_{\nu 2}].$$

By the symmetry of $|H(f)|^2$, we can see that the power constraint (3.18) reduces to

$$\sum_{k=1}^{\nu} \int_{f_{k1}}^{f_{k2}} \left( \Theta - \frac{N_0}{|H(f)|^2} \right) \, df = \frac{\bar{S}}{2}.$$
Define
\[ H(u) = \left\{ \sum_{l=1}^{L} \alpha_l \cos[(l-1)u] \right\}^2 + \left\{ \sum_{l=1}^{L} \alpha_l \sin[(l-1)u] \right\}^2. \] (3.19)

Applying the substitution \( u = 2\pi f \Delta \tau \) to the left-hand side of Equation (3.18) and noticing the fact that \( \Delta \tau = 1/B \), we have
\[
\sum_{k=1}^{V} \int_{2\pi f_{k1}/B}^{2\pi f_{k2}/B} \left( \frac{\Theta}{N_0} - \frac{1}{H(u)} \right) du = \frac{S}{2} \frac{2\pi}{BN_0} = \pi \rho. 
\]

Define \( u_{k1} = 2\pi f_{k1}/B, u_{k2} = 2\pi f_{k2}/B, k = 1, 2, \ldots v \), and \( \tilde{\Theta} = \Theta/N_0 \). Then we can see that power constraint (3.18) is equivalent to
\[
\sum_{k=1}^{V} \int_{u_{k1}}^{u_{k2}} \left( \tilde{\Theta} - \frac{1}{H(u)} \right) du = \pi \rho. \quad (3.20)
\]

Notice that the solution \( \tilde{\Theta} \) to the above equation depends only on the SNR \( \rho \), independent of the bandwidth of the channel. It should be pointed out that \( u_{k1} \) and \( u_{k2} \) satisfy the following condition:
\[
0 \leq u_{k1} < u_{k2} \leq \pi, \quad k = 1, 2, \ldots v.
\]

Once \( \tilde{\Theta}, u_{k1}, \) and \( u_{k2} \) are obtained, the channel capacity can be expressed as follows:
\[
C|_f = 2 \sum_{k=1}^{V} \int_{f_{k1}}^{f_{k2}} \log \left[ 1 + \left( \frac{\Theta}{N_0} - \frac{H(f)}{|H(f)|^2} \right) \frac{|H(f)|^2}{N_0} \right] df
\]
\[
= 2 \sum_{k=1}^{V} \int_{f_{k1}}^{f_{k2}} \log \left( \Theta \frac{|H(f)|^2}{N_0} \right) df
\]
\[
= 2 \sum_{k=1}^{V} \int_{u_{k1}}^{u_{k2}} \log \left[ \tilde{\Theta} H(u) \right] \frac{B}{2\pi} du
\]
\[
= \frac{B}{\pi} \sum_{k=1}^{V} \int_{u_{k1}}^{u_{k2}} \log[\tilde{\Theta} H(u)] du. \quad (3.21)
\]

From Equations (3.19), (3.20), and (3.21) we can see that the channel capacity \( C|_f \) depends on \( B \) linearly for a given SNR \( \rho \). This fact will facilitate the simulation studies greatly.

### 3.4.2 The MIMO Case

Next consider the MIMO case. By defining \( \tilde{S}_X(u) = S_X(u B/2\pi) \), we can re-express the conditional mutual information between \( X \) and \( Y \) in Equation (3.11) as
\[
\mathcal{I}|_f = \frac{B}{\pi} \int_{0}^{\pi} \log \det \left\{ \mathbf{1} + \frac{1}{N_0} \tilde{H}(u) \tilde{S}_X(u) \tilde{H}^\dagger(u) \right\} du. \quad (3.22)
\]
In this case, the transmit power spectrum $S_X(f)$ (or equivalently $\tilde{S}_X(u)$) can be designed so that $C|\tilde{f}$ is maximized for each realization of $\tilde{f}$. In the following, we follow the procedure outlined in [244], but adapt the formulation to make it suitable to the UWB-MIMO channel. Since the matrix $\tilde{H}(u)\tilde{H}(u)$ is Hermitian, it can be diagonalized. Let

$$\tilde{H}(u)\tilde{H}(u) = U\Lambda(u)U^H,$$

where $U(u)$ is unitary and $\Lambda(u) = \text{diag}\{\lambda_1(u), \lambda_2(u), \ldots, \lambda_{N_T}(u)\}$. Notice that both $U(u)$ and $\Lambda(u)$ are the functions of the normalized frequency $u$. Using the matrix determinant identity $\text{det}(I + M_1M_2) = \text{det}(I + M_2M_1)$ for any two compatible matrices $M_1$ and $M_2$, we have

$$\mathcal{I}|_f = \frac{B}{\pi} \int_0^{\pi} \log \text{det} \left\{ I + \frac{1}{N_0} \tilde{S}_X(u)\tilde{H}(u)\tilde{H}(u) \right\} \, du$$

$$= \frac{B}{\pi} \int_0^{\pi} \log \text{det} \left\{ I + \frac{1}{N_0} \Lambda^{1/2}(u)\tilde{S}_X(u)\Lambda^{1/2}(u) \right\} \, du.$$

Notice that the matrix $\Lambda^{1/2}(u)\tilde{S}_X(u)\Lambda^{1/2}(u)$ is non-negative definite. Thus we have

$$\text{det} \left\{ I + \frac{1}{N_0} \Lambda^{1/2}(u)\tilde{S}_X(u)\Lambda^{1/2}(u) \right\} \leq \prod_{i=1}^{N_T} \left[ 1 + \frac{1}{N_0} q_i(u)\lambda_i(u) \right],$$

where $q_i(u)$, $i = 1, \ldots, N_T$, are the diagonal entries of $\tilde{S}_X(u)$, and the equality holds when $\tilde{S}_X(u)$ is diagonal. Therefore:

$$\mathcal{I}|_f \leq \frac{B}{\pi} \int_0^{\pi} \sum_{i=1}^{N_T} \log \left[ 1 + \frac{1}{N_0} q_i(u)\lambda_i(u) \right] \, du. \quad (3.23)$$

Now we proceed to find $q_i(u)$ such that the integral in inequality (3.23) is further maximized under the constraint (3.3). Let $\Theta^{-1}$ be the Lagrange multiplier and construct the functional

$$J(\tilde{S}_X) = \sum_{i=1}^{N_T} \log \left[ 1 + \frac{1}{N_0} q_i(u)\lambda_i(u) \right] - \Theta^{-1} \sum_{i=1}^{N_T} q_i(u),$$

where we have used the fact that $\frac{\partial}{\partial q_i} (S_X(f)) = \frac{\partial}{\partial q_i} (\tilde{S}_X(u))$. Notice that $\Theta$ is a constant which is independent of the frequency $u$. By setting $\frac{\partial}{\partial q_i} (\tilde{S}_X(u))/\partial q_i = 0$, it can be seen that the optimal solution for $q_i(u)$ which maximizes $\mathcal{I}|_f$ is given by

$$q_i(u) = \left[ \Theta - \frac{N_0}{\lambda_i(u)} \right]_+.$$

By definition, we set $q_j(u) = 0$ if some eigenvalue of $\tilde{H}(u)\tilde{H}(u)$, say $\lambda_j(u)$, is zero. The constant $\Theta$ is determined by the power constraint (3.3):

$$\int_{-B/2}^{B/2} \frac{B}{\pi} \int_0^{\pi} \log (\tilde{S}_X(u)) \, du = \frac{B}{\pi} \int_0^{\pi} \sum_{i=1}^{N_T} \left[ \Theta - \frac{N_0}{\lambda_i(u)} \right]_+ \, du = \tilde{S}.$$
The conditional channel capacity is given by

\[
C_{|f} = \frac{B}{\pi} \sum_{i=1}^{N_T} \int_{F_{i\Theta}} \log \left\{ \frac{\Theta}{N_0} \lambda_i(u) \right\} \, du,
\]

(3.24)

where \(F_{i\Theta}\) denotes the intervals of \(f\) in which \(N_0/\lambda_i(u) \leq \Theta, i = 1, \ldots, N_T\).

### 3.5 Special Case: SISO with Two Paths

In the preceding cases, we are not able to give a closed form expression for the channel capacity. In the extremely simple case where the multipaths consist of only two paths, we can give a closed form expression for the channel capacity. This is illustrated in the following.

Recall the definition \(\tilde{C}_{|f} = C_{|f} / B\). The distribution of \(C_{|f}\) can be easily obtained from the distribution of \(\tilde{C}_{|f}\). Thus we only deal with the latter.

In this subsection, we assume that \(\varrho = 1\), i.e., \(\Omega_1 = \Omega_2 := \Omega\). Since only two clusters are involved and \(\varrho\) is near to one, the bias caused by this simplification is negligible. The simulation results will also show this point. Applying \(N_T = N_R = 1\) and \(L = 2\) to Equation (3.16), we have:

\[
\tilde{C}_{|f} = \frac{1}{\pi} \int_0^{\pi} \log \left[ 1 + \rho (\alpha_1 + \alpha_2 \cos u)^2 + \rho \alpha_2^2 \sin^2 u \right] \, du \\
= \frac{1}{\pi} \int_0^{\pi} \log \left[ 1 + \rho \left( \alpha_1^2 + \alpha_2^2 \right) + 2 \rho \alpha_1 \alpha_2 \cos u \right] \, du \\
= \log \frac{1 + \rho \left( \alpha_1^2 + \alpha_2^2 \right) + \sqrt{\left[ 1 + \rho \left( \alpha_1^2 + \alpha_2^2 \right) \right]^2 - 4 \rho^2 (\alpha_1 \alpha_2)^2}}{2} \\
= \log \frac{1 + \rho (\eta_1 + \eta_2) + \sqrt{\left[ 1 + \rho (\eta_1 + \eta_2) \right]^2 - 4 \rho^2 \eta_1 \eta_2}}{2},
\]

(3.25)

where \(\alpha_1\) and \(\alpha_2\) are the channel fadings for the two taps, \(\eta_1 := \alpha_1^2\) and \(\eta_2 := \alpha_2^2\). The pdfs of \(\eta_1\) and \(\eta_2\) are as follows:

\[
p_{\eta l}(x) = \begin{cases} 
\left( \frac{k}{2\Omega} \right)^{k/2} \frac{1}{\Gamma(k/2)} x^{(k/2)-1} e^{-x/(2\Omega)} & \text{if } x \geq 0,

0 & \text{if } x < 0,
\end{cases} \quad l = 1, 2.
\]

The third equality of Equation (3.25) follows from [92, p. 527].

To find the distribution of \(\tilde{C}_{|f}\), we define the random vector \((\mu, \xi)\) as

\[
\begin{cases} 
\mu = \eta_1 + \eta_2, \\
\xi = \eta_1 \eta_2.
\end{cases}
\]

(3.26)

The domain in which \((\mu, \xi)\) is defined as

\[
\mathcal{R}_{(\mu, \xi)} = \{ (\mu, \xi) \in \mathbb{R}^2 : \mu \geq 0, \xi \geq 0, \mu^2 \geq 4\xi \}.
\]
The inverse transformation can be found as follows:

\[
\begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix} = \mathbf{g}_1(\mu, \xi) := \begin{bmatrix}
\frac{\mu + \sqrt{\mu^2 - 4\xi}}{2} \\
\frac{\mu - \sqrt{\mu^2 - 4\xi}}{2}
\end{bmatrix} \quad \text{if } (\eta_1, \eta_2) \in \mathcal{R}_{(\eta_1, \eta_2)}^1,
\]

\[
\begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix} = \mathbf{g}_2(\mu, \xi) := \begin{bmatrix}
\frac{\mu - \sqrt{\mu^2 - 4\xi}}{2} \\
\frac{\mu + \sqrt{\mu^2 - 4\xi}}{2}
\end{bmatrix} \quad \text{if } (\eta_1, \eta_2) \in \mathcal{R}_{(\eta_1, \eta_2)}^2,
\]

where

\[
\mathcal{R}_{(\eta_1, \eta_2)}^1 = \{(\eta_1, \eta_2) \in \mathbb{R}^2 : \eta_1 \geq 0, \eta_2 \geq 0, \eta_1 \geq \eta_2\},
\]

\[
\mathcal{R}_{(\eta_1, \eta_2)}^2 = \{(\eta_1, \eta_2) \in \mathbb{R}^2 : \eta_1 \geq 0, \eta_2 \geq 0, \eta_1 \leq \eta_2\}.
\]

Since \( \mathbf{g}_1^{-1}(\mathcal{R}_{(\eta_1, \eta_2)}^1) = \mathbf{g}_2^{-1}(\mathcal{R}_{(\eta_1, \eta_2)}^2) = \mathcal{R}_{(\mu, \xi)} \), we obtain:

\[
p_{(\mu, \xi)}(\mu, \xi) = p_{(\eta_1, \eta_2)}(\mathbf{g}_1(\mu, \xi)) \left| \det \left( \frac{\partial \mathbf{g}_1(\mu, \xi)}{\partial (\mu, \xi)} \right) \right| + p_{(\eta_1, \eta_2)}(\mathbf{g}_2(\mu, \xi)) \left| \det \left( \frac{\partial \mathbf{g}_2(\mu, \xi)}{\partial (\mu, \xi)} \right) \right|. \tag{3.27}
\]

It is easy to obtain the following

\[
\det \left( \frac{\partial \mathbf{g}_1(\mu, \xi)}{\partial (\mu, \xi)} \right) = \det \left[ \begin{array}{cc}
\frac{1}{2} + \frac{\mu}{2\sqrt{\mu^2 - 4\xi}} & -\frac{1}{\sqrt{\mu^2 - 4\xi}} \\
\frac{1}{2} - \frac{\mu}{2\sqrt{\mu^2 - 4\xi}} & \frac{1}{\sqrt{\mu^2 - 4\xi}}
\end{array} \right] = \frac{1}{\sqrt{\mu^2 - 4\xi}}, \tag{3.28}
\]

\[
\det \left( \frac{\partial \mathbf{g}_2(\mu, \xi)}{\partial (\mu, \xi)} \right) = -\frac{1}{\sqrt{\mu^2 - 4\xi}}. \tag{3.29}
\]

Since \( \alpha_1 \) and \( \alpha_2 \) are independent of each other, so are \( \eta_1 \) and \( \eta_2 \). Thus we have

\[
p_{(\eta_1, \eta_2)}(\eta_1, \eta_2) = \left\{ \begin{array}{ll}
\left[ \frac{\kappa}{2\mu} \right]^{\kappa/2} \frac{1}{\Gamma(\kappa/2)} \left( \eta_1 \eta_2 \right)^{(\kappa/2)-1} e^{-\kappa(\eta_1+\eta_2)/(2\Omega)} & \text{when } (\eta_1, \eta_2) \in \mathbb{R}^2_+,
\end{array} \right.
\]

\[
\text{otherwise,} \tag{3.30}
\]

where \( \mathbb{R}^2_+ := \{(\eta_1, \eta_2) \in \mathbb{R}^2 : \eta_1 \geq 0, \eta_2 \geq 0\} \). Substituting Equations (3.26), (3.28), (3.29), and (3.30) into Equation (3.27) yields

\[
p_{(\mu, \xi)}(\mu, \xi) = \left\{ \begin{array}{ll}
2 \left[ \frac{\kappa}{2\Omega} \right]^{\kappa/2} \frac{1}{\Gamma(\kappa/2)} \left[ \frac{1}{\sqrt{\mu^2 - 4\xi}} \right]^{(\kappa/2)-1} e^{-\kappa\mu/(2\Omega)} & \text{when } (\mu, \xi) \in \mathcal{R}_{(\mu, \xi)},
\end{array} \right.
\]

\[
\text{otherwise.} \tag{3.31}
\]
In terms of Equations (3.25) and (3.31), we can obtain the cumulative density function (cdf) of $\bar{C}_f$ as follows:

$$P_{\bar{C}_f}(x) = \Pr\{\bar{C}_f \leq x\} = \Pr\left\{\log\frac{1 + \rho \mu + \sqrt{(1 + \rho \mu)^2 - 4 \rho^2 \xi}}{2} \leq x\right\} = \Pr\{(\mu, \xi) \in G\},$$

where $\Pr\{\cdot\}$ denotes the probability of a random event, the region $G$ is defined by

$$G := \{(\mu, \xi) \in \mathbb{R}^2 : 0 \leq \mu \leq \frac{2e^x - 1}{\rho}, 0 \leq \xi \leq \frac{1}{4} \mu^2, \xi \geq \varpi(\mu, x)\},$$

$$\varpi(\mu, x) = \rho^{-1} e^x \mu - \rho^{-2} e^x (e^x - 1).$$

In the $(\mu, \xi)$ plane, the parabola $\xi = (1/4)\mu^2$ and the line $\xi = \varpi(\mu, x)$ have two intersections, the abscissas of which are $\mu_1(x) := [2(e^x - e^{x/2})]/\rho$ and $\mu_3(x) := (2(e^x + e^{x/2})/\rho$, respectively. Let us define $\mu_2(x) = (2e^x - 1)/\rho$ and $\mu_0(x) = (e^x - 1)/\rho$. The latter is the abscissa of the intersection of the line $\xi = \varpi(\mu, x)$ and the $\mu$-axis. It is clear that for all $x \geq 0$, we have $\mu_0(x) \leq \mu_1(x) < \mu_2(x) < \mu_3(x)$. Thus the region $G$ can also be expressed as

$$G = \left\{(\mu, \xi) \in \mathbb{R}^2 : \begin{array}{ll}
0 \leq \xi \leq (1/4)\mu^2 & \text{when } 0 \leq \mu \leq \mu_0(x) \\
\varpi(\mu, x) \leq \xi \leq (1/4)\mu^2 & \text{when } \mu_0(x) < \mu \leq \mu_1(x) \\
\end{array} \right\}. $$

Therefore,

$$P_{\bar{C}_f}(x) = \int_0^{\mu_0(x)} \int_0^{(1/4)\mu^2} p_{(\mu, \xi)}(\mu, \xi) \, d\xi \, d\mu + \int_{\mu_0(x)}^{\mu_1(x)} \int_{\varpi(\mu, x)}^{(1/4)\mu^2} p_{(\mu, \xi)}(\mu, \xi) \, d\xi \, d\mu. $$

Recalling the definition of the beta function $\beta(\upsilon_1, \upsilon_2)$ (cf. [92, p. 948])

$$\beta(\upsilon_1, \upsilon_2) = \int_0^1 t^{\upsilon_1-1}(1 - t)^{\upsilon_2-1} \, dt$$

and the hypergeometric function $F(\upsilon_1, \upsilon_2; \upsilon_3; z)$ (cf. [92, p. 1040])

$$F(\upsilon_1, \upsilon_2; \upsilon_3; z) = \frac{1}{\beta(\upsilon_2, \upsilon_3 - \upsilon_2)} \int_0^1 t^{\upsilon_2-1}(1 - t)^{\upsilon_3-\upsilon_2-1}(1 - tz)^{-\upsilon_1} \, dt,$$

we have

$$\int_0^{(1/4)\mu^2} \frac{\xi^{(\kappa/2)-1}}{\sqrt{\mu^2 - 4\xi}} \, d\xi = \int_0^1 \left(\frac{1}{4} \mu^2 \right)^{\kappa/2} t^{(\kappa/2)-1} \frac{1}{\mu \sqrt{1 - t}} \, dt = \frac{1}{2^\kappa} \mu^{\kappa-1} \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right),$$

(3.32)
A simple manipulation yields
\[ \int_0^\nu \frac{\xi^{(\kappa/2)-1}}{\sqrt{\mu^2 - 4\xi}} \, d\xi = \frac{\nu^{\kappa/2}}{\mu} \int_0^1 \frac{1}{\sqrt{1 - \frac{4}{\mu^2} \nu t}} \, dt \]
\[ = \frac{\nu^{\kappa/2}}{\mu} \beta\left(\frac{\kappa}{2}, 1\right) F\left(\frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2}; 1; \frac{4}{\mu^2} \nu\right) \]
\[ = \frac{\nu^{\kappa/2}}{\mu} \left(\frac{\kappa}{2}\right)^{-1} F\left(\frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2}; 1; \frac{4}{\mu^2} \nu\right). \quad (3.33) \]

where \( \nu \) is a positive number satisfying \( \nu < \frac{1}{4} \mu^2 \). In the above we have used the fact that
\[ \beta\left(\frac{\kappa}{2}, 1\right) = \int_0^1 t^{(\kappa/2)-1} \, dt = \left(\frac{\kappa}{2}\right)^{-1}. \]

Using Equations (3.32) and (3.33), we have
\[ \int_{\sigma(\mu, x)}^{(1/4)\mu^2} \frac{\xi^{(\kappa/2)-1}}{\sqrt{\mu^2 - 4\xi}} \, d\xi = \frac{1}{2^{\kappa}} \mu^{\kappa-1} \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right) \]
\[ - \frac{\sigma(\mu, x)^{\kappa/2}}{\mu} \left(\frac{\kappa}{2}\right)^{-1} F\left(\frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2}; 1; \frac{4\sigma(\mu, x)}{\mu^2}\right). \]

Therefore, we have
\[ P_{C_{\|}}(x) = 2 \left[ \left(\frac{\kappa}{2\Omega}\right)^{\kappa/2} \frac{1}{\Gamma\left(\frac{\kappa}{2}\right)} \right]^2 \left[ \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\mu^{\kappa-1}}{2^\kappa} \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right) d\mu \right] \]
\[ + \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\mu^{\kappa-1}}{2^\kappa} \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right) d\mu \]
\[ - \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\sigma(\mu, x)^{\kappa/2}}{\mu} \left(\frac{\kappa}{2}\right)^{-1} F\left(\frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2}; 1; \frac{4\sigma(\mu, x)}{\mu^2}\right) d\mu \]}
\[ = 2 \left[ \left(\frac{\kappa}{2\Omega}\right)^{\kappa/2} \frac{1}{\Gamma\left(\frac{\kappa}{2}\right)} \right]^2 \left[ \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\mu^{\kappa-1}}{2^\kappa} \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right) d\mu \right] \]
\[ - \left(\frac{\kappa}{2}\right)^{-1} \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\sigma(\mu, x)^{\kappa/2}}{\mu} F\left(\frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2}; 1; \frac{4\sigma(\mu, x)}{\mu^2}\right) d\mu \]. \quad (3.34)

A simple manipulation yields
\[ \int_0^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \mu^{\kappa-1} \, d\mu = \frac{(\kappa - 1)!}{(\kappa/(2\Omega))^{\kappa}} - e^{-\kappa \mu_1(x)/(2\Omega)} \sum_{l=0}^{k-1} \frac{(\kappa - 1)!}{l!} \frac{\mu_1^l(x)}{(\kappa/(2\Omega))^{k-l}}. \quad (3.35) \]

Using the fact that
\[ \beta\left(\frac{\kappa}{2}, \frac{1}{2}\right) = \frac{\sqrt{\pi} \Gamma(\kappa/2)}{\Gamma((\kappa + 1)/2)} \]
and substituting Equation (3.35) into Equation (3.34) gives

\[
P_{\bar{C}_{lf}}(x) = \frac{(\kappa - 1)!\sqrt{\pi}}{2^{\kappa-1} \Gamma(\kappa/2) \Gamma(\kappa + 1/2)} \left[ 1 - e^{-\kappa \mu_1(x)/(2\Omega)} \sum_{l=0}^{\kappa-1} \left( \frac{\kappa}{2\Omega} \right)^l \frac{\mu_1'(x)}{l!} \right] \\
- \frac{4}{\kappa} \left( \frac{\kappa}{2\Omega} \right)^{\kappa/2} \frac{1}{\Gamma(\kappa/2)} \int_{\mu_0(x)}^{\mu_1(x)} e^{-\kappa \mu/(2\Omega)} \frac{\varphi(\mu, x)^{\kappa/2}}{\mu} \right. \\
\times 
F \left( \frac{1}{2}, \frac{\kappa}{2}; \frac{\kappa}{2} + 1; \frac{4\varphi(\mu, x)}{\mu^2} \right) d\mu.
\]

Having the expression for \(P_{\bar{C}_{lf}}(x)\), the outage probability is simply given by

\[
P_{\text{out}}(R) = P_{\bar{C}_{lf}}(R) = P_{\bar{C}_{lf}}(R/B),
\]

where \(R\) is the communication rate supportable by the channel with a probability of \(1 - P_{\text{out}}(R)\), or in other words, with an outage probability of \(P_{\text{out}}(R)\).

### 3.6 Simulation Results

We study three cases: multiple transmit antennas and single receive antenna (MISO), single transmit antenna and multiple receive antennas (SIMO), and MIMO (we fix \(N_T = N_R\) in this case for the convenience of exposition). In all the simulations, it is assumed that \(\kappa = 4\), \(\Omega_1 = 1\), and \(\rho = 0.95\).

We first investigate the performance difference between the uniform power spectrum allocation (UPSA) and the optimal power spectrum allocation (OPSA) for the transmitted signal. The results are plotted in Figure 3.1 and Figure 3.2, where the ergodic capacity is

![Figure 3.1](image-url)  
**Figure 3.1** Efficiency of UPSA relative to OPSA (SISO: \(N_T = N_R = 1\)).
Figure 3.2  Efficiency of UPSA relative to OPSA (MIMO): (a) $N_T = N_R = 4$; (b) $N_T = N_R = 8$. used for illustration. Figure 3.1 is for the SISO case and Figure 3.2 is for the MIMO case. It can be seen that when the SNR is low, the OPSA scheme can increase the channel capacity somehow compared to the UPSA scheme, but when the SNR is high, the benefit of the OPSA can be hardly seen compared to the UPSA. Let $C_{\text{uni}}$ and $C_{\text{opt}}$ denote the ergodic capacity of the system using the UPSA and OPSA schemes respectively. We
use $C_{\text{uni}}/C_{\text{opt}}$ as an index for the efficiency of the UPSA relative to the OPSA. It is shown that the efficiency of the UPSA is lower than 0.61 when the SNR is lower than $-20$ dB. This says that the transmission rate can be increased roughly more than 1.6 times if the OPSA, instead of the UPSA, is adopted. But when $\rho \geq +10$ dB, $C_{\text{uni}}/C_{\text{opt}}$ almost approaches one. Surprisingly, the curves of $C_{\text{uni}}/C_{\text{opt}}$ as a function of SNR for both SISO and MIMO possess not only the same tendency, but also roughly the same numerical values for a given SNR.

The above property is also true for the capacity in the sense of outage probability. Figure 3.3 illustrates the result for the $8 \times 8$ MIMO case, and Figure 3.4 illustrates the result for the SISO case. It can be seen that the outage probabilities of the MIMO and SISO capacity possess the same tendency in the efficiency of the UPSA relative to the OPSA.

From the results shown in Figures 3.1–3.4, we can say that measures such as the water-filling algorithm should be considered to take full advantage of frequency selectivity of the UWB systems if the SNR is rather low.

In the rest of this chapter, all the simulation results are conducted for the UPSA case.

The results for the MISO case are depicted in Figure 3.5. It is seen that for a given $\rho$, the outage probability decreases with the number of transmit antennas when $R/B$ is lower than some value (denoted as $R_1$), but increases instead when $R/B$ is higher than another value (denoted as $R_2$). The corresponding outage probability is so high when $R/B$ is larger than $R_2$ that to transfer information at this rate is of little practical interest. Therefore, if the required transmission rate is indeed too high and the available power is limited, it is better to concentrate all power in one antenna, rather than to distribute the power in multiple antennas.

It is difficult to calculate $R_2$ exactly, which depends on the number of transmit antennas under comparison. However, $R_1$ can be calculated in the following way [287]. Noticing that the total signal power $\bar{S}$ is equally distributed among transmit antennas and uniformly distributed in the whole band for the MISO case, the received power, denoted as $\bar{S}_Y$, from the useful signals should be

$$\bar{S}_Y = \sum_{m=1}^{N_T} \sum_{l=1}^{L} \alpha_{l,m}^2 \frac{\bar{S}}{BN_T} = \frac{L}{N_T} \sum_{m=1}^{N_T} \alpha_{l,m}^2 \bar{S}.$$  

Therefore, it is clear that the larger the number $N_T$, the smaller the variance of the power of the received useful signals. In the extreme case, when $N_T$ approaches infinity, we have

$$\frac{\sum_{m=1}^{N_T} \alpha_{l,m}^2}{N_T} \rightarrow \Omega_l \text{ with probability 1, as } N_T \rightarrow \infty$$

according to the strong law of large numbers. Thus we further have

$$\bar{S}_Y \rightarrow \sum_{l=1}^{L} \frac{\Omega_l}{B} \bar{S} = \frac{(1-\rho L)\Omega_1}{1-\rho} \bar{S} \text{ with probability 1, as } N_T \rightarrow \infty.$$  

Therefore, in this extreme case, we obtain

$$\frac{R}{B} = \log \left[ 1 + \frac{(1-\rho L)\Omega_1}{1-\rho} \frac{\bar{S}}{BN_0} \right] = \log \left[ 1 + \frac{(1-\rho L)\Omega_1}{1-\rho} \frac{\rho}{BN_0} \right] = R_c.$$
The above says that the channel capacity will approach a constant $R_c$ when the number of transmit antennas approaches infinity. We call $R_c$ the critical transmission rate. From Figure 3.5, we can see that $R_t = R_c$. Substituting the corresponding parameters for Figure 3.5 into the above equation, we get $R_c = 4.68$. 

**Figure 3.3** Outage probability $P_{out}$ vs. transmission rate $R/B$ (in nats/s/Hz) for different $L$. Comparison between OPSA and UPSA (MIMO: $N_T = N_R = 8$): (a) $\rho = -10$ dB; (b) $\rho = +10$ dB.
To explain the calculation of $R_2$, let us express the outage probability illustrated in Figure 3.5 as a function of both $R/B$ and $N_T$. Denote this function as $P_{out}(R/B, N_T)$ for temporary abuse of notations. For any two positive integers $N_1$ and $N_2$ (supposing $N_1 < N_2$), there exists a rate $R_2(N_1, N_2)$ such that
Figure 3.5 Outage probability $P_{\text{out}}$ vs. transmission rate $R/B$ (in nats/s/Hz) for different $N_T$. Effect of the number of transmit antennas on the channel capacity ($\rho = +10$ dB, $L = 15$, $N_R = 1$): (a) panorama; (b) detail.

\[
\begin{align*}
&P_{\text{out}}(R/B, N_1) > P_{\text{out}}(R/B, N_2) \quad \text{if} \quad R/B < R_2(N_1, N_2); \\
&P_{\text{out}}(R/B, N_1) < P_{\text{out}}(R/B, N_2) \quad \text{if} \quad R/B > R_2(N_1, N_2). 
\end{align*}
\]

Therefore, the following conclusion can be drawn: using $N_2$, rather than $N_1$, transmit antennas yields better system performance in the sense of the outage probability when the
transmission rate \( R/B \) is less than \( R_2(N_1, N_2) \); on the other hand, when \( R/B \) is larger than \( R_2(N_1, N_2) \), using \( N_2 \), instead of \( N_1 \), transmit antennas yield worse system performance. As mentioned before, it is difficult to give an explicit expression for \( R_2(N_1, N_2) \), but it is clear from Figure 3.5 that \( R_2(N_1, N_2) \geq R_c \) for all \( N_1 \) and \( N_2 \). Therefore, \( R_c \) gives a conservative estimation on \( R_2(N_1, N_2) \) for any pair \( \{N_1, N_2\} \). From Figure 3.5 it is observed that the gap between \( R_2 \) and \( R_c \) is not big.

More clear pictures about the critical transmission rate can be seen from the case of the frequency-flat channels [286], where it is found that \( R_2 \) almost coincides with \( R_c \). The same phenomenon is also found in narrowband MISO systems in [244].

The results for the SIMO case are demonstrated in Figure 3.6, which clearly shows that, for a given transmission rate, the outage probability decreases considerably with the number of receive antennas in the range of whole transmission rate, making a striking contrast to the MISO case. In other words, we can say that the communication rate can be considerably increased for a given outage probability by increasing the number of receive antennas. To see the quantitative relationship between \( N_R \) and the increased amount of the communication rate, we view Figure 3.6 from another perspective. That is, first fix \( P_{\text{out}} \); then, for this \( P_{\text{out}} \), find the communication rate, \( R/B \), supportable by the channel corresponding to different \( N_R \); and finally plot the curves showing the relationship between \( R/B \) and \( N_R \) for different \( P_{\text{out}} \), which is depicted in Figure 3.7. Notice that we have deliberately used \( \log N_R \) as the abscissa of Figure 3.7. This figure clearly reveals that \( R \) increases with \( \log N_R \) almost linearly. This phenomenon can be observed more clearly in the scenario of high SNR. Basically, this is due to the linear increase in SNR with \( N_R \) under the optimal combining of the received signals at the receiver. It is noticed

![Figure 3.6](attachment:figure36.png)

**Figure 3.6** Outage probability \( P_{\text{out}} \) vs. transmission rate \( R/B \) (in nats/s/Hz) for different \( N_R \): effect of the number of receive antennas on the channel capacity \((\rho = +10 \text{ dB}, L = 15, N_T = 1)\).
that the variance of the channel capacity decreases with the number of receive antennas. This phenomenon is also observed in [153, 168].

Figure 3.8 shows the results for the MIMO case, from which we can again observe that, for a given transmission rate, the outage probability decreases with the numbers of transmit and receive antennas considerably in the range of the whole transmission rate. To investigate the quantitative relationship between $N_T$ (or $N_R$) and the increased amount of the communication rate, we plot Figure 3.8 from another perspective, similar to the SIMO case. The result is shown in Figure 3.9. From this figure it can be seen that $R$ increases with $N_T$ (or $N_R$) linearly. This is a well-known property for narrowband MIMO communication systems. This property also holds true for the UWB-MIMO systems because of the fact that the order of the integral and the summation inherited from $\log \det$ in Equation (3.16) is exchangeable. Suppose that the matrix $H(f)$ is of full rank.\textsuperscript{3} By using the singular value decomposition of the matrix $H(f)$, the matrix $H(f)H^\dagger(f)$ can be transformed into a diagonal form with the diagonal entries being the singular values of $H(f)$. Thus, from Equation (3.16) we can see that the capacity of a UWB-MIMO system is equal to that of the system with $N_R$ parallel channels, and hence is $N_R$-fold of the capacity of a UWB-SISO system.

Compare the results obtained by the Monte Carlo method to those obtained through the analytic method for the case of SISO and $L = 2$. The result is plotted in Figure 3.10, which shows that the difference between these two methods is negligible.

\textsuperscript{3} Since $H(f)$ is a random matrix, the probability that $H(f)$ is rank-deficient will be zero.
Figure 3.8 Outage probability $P_{\text{out}}$ vs. transmission rate $R/B$ (in nats/s/Hz) for different $N_T$ (or $N_R$): effect of the numbers of transmit and receive antennas on the channel capacity ($\rho = +10$ dB, $L = 15$, $N_T = N_R$, where $N_{TR} := N_T = N_R$).

Figure 3.9 The transmission rate $R/B$ (in nats/s/Hz) vs. $N_T$ (or $N_R$) for different outage probability $P_{\text{out}}$ ($\rho = +10$ dB, $L = 15$, $N_T = N_R$).
Figure 3.10 Outage probability $P_{\text{out}}$ vs. transmission rate $R/B$ (in nats/s/Hz) for different $\rho$: comparison between analytic method and Monte Carlo method ($L = 2$, $N_T = N_R = 1$): (a) for the case of low SNR; (b) for the case of high SNR.
3.7 Channel Correlation

In the preceding development, we have assumed that (1) all the matrices $A_l, l = 1, \ldots, L$, are independent of each other; and (2) all the entries of $A_l$ for a given $l$ are also independent of each other. Generally, the first assumption is reasonable since different matrices characterize the UWB propagations caused by different scatterers, but the second assumption is impractical since the matrix $A_l$ describes the UWB propagations caused by the same group of scatterers. In the following, we will investigate the channel capacity based on the correlation model (2.19), (2.20), and (2.21) with $\rho_{ct} = \rho_{ct} = \rho_c$.

Generally, the channel capacity of MIMO systems will decrease when the correlation of the spatial channels is strong because the diversity of the channels decreases due to the correlation. For narrowband MIMO systems, this result can be easily seen from the singular value decomposition of the channel matrix [244]. In the extreme case where the ranks of the correlation matrices $R_r$ and $R_t$ reduce to one, the MIMO channel will collapse to a SISO channel. This phenomenon also exists in UWB-MIMO systems. A simulation result is shown in Figure 3.11, where we fix $N_T = N_R = 8$ and $L = 50$. As shown in Figure 3.11, the channel capacity decreases significantly with $\rho_c$ if $\rho_c \geq 0.5$, while it almost keeps the same value as that of the uncorrelated channels if $\rho_c \leq 0.2$. Note that the correlation coefficient is mainly determined by the distance of two antennas. Combining the result illustrated in Figure 2.1 and the aforementioned observation, we can claim that the channel capacity of UWB-MIMO systems will not suffer degradation if the antennas are separated from each other by more than 10 cm.

![Figure 3.11](image)

**Figure 3.11** The effect of correlation coefficient of the spatial channels on the ergodic channel capacity of MIMO systems. Here $N_T = N_R = 8$ for MIMO, and $L = 50$ for both MIMO and SISO.
3.8 Measured Channel Capacity

In this section, the measured channel capacity by using the testbed in our laboratory as shown in Section 2.3 will be illustrated.

In measuring the channel capacity, an important issue is the definition and reliable estimation of the SNR at each receive antenna branch. For the purpose of system performance verification, there are typically two main approaches to attaining different SNR

![Figure 3.12](image)

**Figure 3.12** Measured ergodic capacity and outage capacity of UWB-MIMO systems: (a) ergodic capacity; (b) outage capacity.
levels. The first one is based on varying the power of the transmitted signal in predefined steps and repeatedly transmitting the same sequence of frames [31]. The drawback of this approach is that it is time consuming and requires huge storage capabilities of the processing computer.

The second approach is to transmit the whole sequence of generated MB-OFDM frames with the maximum allowed EIRP (equivalent isotropically radiated power), and then synthetically add colored band-limited noise to the received signal in order to vary the SNR [160]. In particular, we first estimate the actual received (maximal) SNR in each

![Figure 3.13 BER performance of MIMO VHDR MB-OFDM. Coding rate = 1/2, 16-QAM: (a) zero forcing; (b) MMSE.](image)

(a)

(b)
antenna branch using the PSD of the band-limited RF signal and then calculate the noise power necessary to achieve a desired SNR value. Since the received signal at each antenna is the mixture of all $N_T$ transmitted waveforms and thermal noise, we first estimate its PSD and then subtract the average PSD of the pure noise signal between each two transmitted frames for a given frequency range. This rough estimation of the SINR (signal-to-interference-plus-noise power ratio) has been verified using the MIMO VHDR MB-OFDM simulator under different channel models and proved reliable for the purpose of a MIMO MMSE (minimum mean square error) receiver. In the next step, we generate the colored noise of predetermined average power so as to reach a desired SNR, and finally estimate its value for further use during the MMSE detection and performance measurements. In this section, the second approach is used.

The measured channel capacity is plotted in Figure 3.12. As shown from both Figure 3.12a and b, both the ergodic capacity and outage capacity of $2 \times 2$ MIMO are approximately doubled, respectively, compared to those for the SISO case. As expected, for a fixed number of transmit antennas, each additional receive antenna provides a capacity gain that remains constant with increasing SNR.

The BER performance of the system is shown in Figure 3.13. Notice that the transmission rate for the cases of two transmit antennas is doubled compared to the rate of the cases of one transmit antenna, since multiplexing is used to transmit the data symbols. It is observed that the $2 \times 2$ spatially multiplexed system provides an additional array gain of approximately 3 dB compared to the SISO case. Moreover, each additional antenna at the RX is able to provide a further diversity gain, as is seen when comparing the $1 \times 1$ with $1 \times 2$, and the $2 \times 2$ with $2 \times 3$ configurations.

### 3.9 Summary

In this chapter, the channel capacity of UWB channels is studied. The results show that

- For both SISO and MIMO cases, when the transmit SNR is lower than some value, say $-20$ dB, using OPSA at the transmitter side can increase the reliable transmission rate considerably compared to UPSA, while when the transmit SNR is higher than some value, say $10$ dB, the benefit of OPSA is very limited. Therefore, measures such as the water-filling algorithm should be taken to make full use of frequency selectivity of the UWB channels if the transmit SNR is low.

- In the MISO case, the capacity outage probability decreases with the number of transmit antennas when the communication rate is lower than the critical transmission rate $R_c$, but increases when the rate is higher than another value $R_2$. The gap between $R_2$ and $R_c$ is not big for the case considered here. $R_c$ is determined by the fading power and the transmit SNR of the system. We can roughly say that it is not beneficial to use multiple transmit antennas if the required transmission rate (normalized by the system bandwidth) is higher than the critical transmission rate.

- In the SIMO case, the communication rate supportable by the channel with a given outage probability increases approximately logarithmically with the number of receive antennas.

- In the MIMO case with equal numbers of transmit and receive antennas, the communication rate supportable by the channel with a given outage probability increases linearly with the number of transmit or receive antennas.
It should be pointed out that the channel correlation issue is not sufficiently discussed in this chapter. It is still waiting for further investigation on how to evaluate the channel capacity when the correlation among the elements of the fading matrices is considered from the viewpoint of the fading pdf matrices. The difficulty lies in that we need to specify the distribution, rather than the correlation matrix, of an $N_R \times N_T$-dimensional random matrix for UWB systems. While the random matrix theory for the case of each entry being Gaussian is well established (see, for example, [161]), very few reports on random matrix theory for other kinds of multivariate distributions have appeared in the literature; see [75] for an account. Even if such a multivariate distribution were at one’s hand, it would be still challenging how to generate, by using the Monte Carlo method, the random matrices which are of such a kind of distribution.
4

UWB-MIMO Space–Time Coding

4.1 Introduction

An essential objective for using MIMO is to increase the data rate by its inherent multiplexing gain and/or diversity gain. Different from a narrowband communication channel, an IR-based UWB channel may exhibit much less severe fading. Therefore, STC for IR-based UWB systems might not be so attractive as predicted by the theoretical diversity order existing in this kind of system, while MB-OFDM-based UWB systems can achieve a multiplexing gain as high as wideband OFDM systems do. In this chapter, we will first discuss a simple yet widely used STC scheme for IR-based UWB systems, i.e., Alamouti coding. Detailed performance comparison between two kinds of Alamouti STC for $2 \times 1$ UWB-MIMO and UWB-SISO systems will be provided. After that, we will present a general approach for the STC of IR-based UWB systems with arbitrary numbers of transmit and receive antennas. Antenna selection for UWB systems will also be discussed briefly. Finally, we will address the issue of the STC technique for MB-OFDM-based UWB systems.

A few papers have addressed the issue of exploiting transmit diversity for IR-based UWB systems; for example, see [8, 258, 260, 273]. In [260], the performance of pulse-amplitude-modulated signals in a UWB-MIMO channel is evaluated. In [273], the authors innovated an STC scheme for both binary pulse-amplitude modulation (PAM)-based and pulse-position modulation (PPM)-based UWB systems with two transmit antennas. The STC scheme in [273] is similar to the Alamouti STC scheme for narrowband systems, which is easy to implement. In [8], a general STC scheme for $M$-ary PAM and PPM UWB systems is proposed based on cyclic division algebra theory. However, it was shown that energy-efficient codes are possible only for two transmit antennas. In [258], a scheme of time-switched transmit diversity for IR-based UWB systems was proposed and its performance was investigated. It was shown that using multiple transmit antennas in a UWB channel can improve the system performance by reducing signal variations. However, using multiple transmit antennas does not provide diversity gain in the strict sense. This is in contrast to the result obtained in [273], where it was illustrated both analytically and via simulations that the diversity order of the space–time-coded UWB...
system can be increased. These seemingly contradictory results can be explained by the fact that the diversity order is generally helpful in achieving better BER performance in the high SNR regime, while the UWB systems are required to operate in the low SNR range. Thus, the diversity order pointed out in [273] is beneficial only in the high SNR regime. The simulation results illustrated in Figure 4.5 give a quantitative description for this idea.

### 4.2 A Revisit of Alamouti Space–Time Coding for Narrowband Systems

Alamouti coding is one of the most popularly used STC schemes in narrowband MIMO systems. In this section, a brief revisit for the Alamouti coding scheme will be provided to show its basic idea.

As shown in Figure 4.1, we consider a narrowband system with two transmit antennas and one receive antenna. The channel fading gains from the two transmit antennas to the receive antenna are denoted as $h_1$ and $h_2$ respectively.

When discussing the Alamouti STC, the following assumptions are generally required.

**Assumption 4.1** The channel impulse responses are frequency-flat and independent of each other.

**Assumption 4.2** The channel fadings $h_1$ and $h_2$ do not change across two consecutive symbol transmissions.

![Alamouti STC scheme for 2 × 1 narrowband MIMO.](image)
Assumption 4.3 The channel state information (CSI) $h_1$ and $h_2$ is available at the receiver.

According to the Alamouti STC scheme, two information symbols, denoted $X_1$ and $X_2$, are transmitted at two consecutive time instants in the following way [14]. At time $t$, $X_1$ and $X_2$ are transmitted from antenna 1 and antenna 2 respectively; at time $t + 1$, $-X_2^*$ and $X_1^*$ are transmitted from antenna 1 and antenna 2 respectively. At the receiver, the received signals, denoted $r(t)$ and $r(t + 1)$ at the two consecutive time instants, are combined as follows:

$$Y_1 = h_1^* r(t) + h_2 r^*(t + 1),$$
$$Y_2 = h_2^* r(t) - h_1 r^*(t + 1).$$

Denote by $n(t)$ the receiver noise at time instant $t$. Then it is easy to show that

$$\begin{align*}
Y_1 &= (|h_1|^2 + |h_2|^2) X_1 + h_1^* n(t) + h_2 n^*(t + 1), \\
Y_2 &= (|h_1|^2 + |h_2|^2) X_2 + h_2^* n(t) - h_1 n^*(t + 1).
\end{align*}$$

(4.1)

Therefore, the information symbols $X_1$ and $X_2$ are decoupled from each other and can be easily decoded from the combined receiver outputs $Y_1$ and $Y_2$ respectively.

Let us compare the above system with the system with a single transmit and single receive antenna. The latter system can be modelled as

$$Y = h X + n,$$

(4.2)

where $X$, $Y$ and $h$ are the transmitted symbol, received signal and channel fading respectively. Denote by $\text{SNR}_0 := \mathbb{E}(|X|^2)/\sigma_n^2$ the SNR at the transmitter side, where $\sigma_n^2$ is the variance of the noise $n$. Then the conditional SNR (conditioned on the CSI $h$) at the receiver side for system (4.2) is

$$\text{SNR}_{\text{SISO}} = |h|^2 \text{SNR}_0.$$ 

(4.3)

For an STC system, it is typically required that a symbol uses the same energy as that in the uncoded system. Therefore, for the system as illustrated in Figure 4.1, we assume that $\mathbb{E}(|X_1|^2) = \mathbb{E}(|X_2|^2) = \mathbb{E}(|X|^2)/2$. Then the conditioned SNR (conditioned on the CSI $h_1$ and $h_2$) at the receiver side for system (4.1) is

$$\text{SNR}_{2 \times 1} = \frac{|h_1|^2 + |h_2|^2}{2} \text{SNR}_0.$$ 

(4.4)

The Alamouti STC idea for the $2 \times 1$ MISO can be easily extended to the case of $2 \times N_R$ MIMO, as illustrated in Figure 4.2.

The information symbols $X_1$ and $X_2$ are also transmitted in the same way as in the case of $2 \times 1$ MISO. Now denote the received signals at receive antenna $i$ as $r_i(t)$ and $r_i(t + 1)$ ($i = 1, \ldots, N_R$) respectively at time instants $t$ and $t + 1$. Similarly, the receiver noises at receive antenna $i$ are denoted as $n_i(t)$ and $n_i(t + 1)$ ($i = 1, 2$) respectively at time instants $t$ and $t + 1$. Let the channel fading gain from transmit antenna $i$ to receive antenna $j$ be
Figure 4.2 Alamouti STC scheme for $2 \times N_R$ narrowband MIMO.

$h_{ji}$ ($i = 1, 2, j = 1, 2, \ldots, N_R$). Combine the received signals in the following way [14]:

\[
Y_1 = \sum_{j=1}^{N_R} h_{j1}^* r_j(t) + h_{j2}^* r_j^*(t + 1),
\]

\[
Y_2 = \sum_{j=1}^{N_R} h_{j2}^* r_j(t) - h_{j1}^* r_j^*(t + 1).
\]

Then we have

\[
\begin{align*}
Y_1 &= \sum_{j=1}^{N_R} \sum_{i=1}^{2} |h_{ji}|^2 X_1 + \sum_{j=1}^{N_R} h_{j1}^* n_j(t) + h_{j2}^* n_j^*(t + 1) \\
Y_2 &= \sum_{j=1}^{N_R} \sum_{i=1}^{2} |h_{ji}|^2 X_2 + \sum_{j=1}^{N_R} h_{j2}^* n_j(t) - h_{j1}^* n_j^*(t + 1)
\end{align*}
\]

(4.5)

Again, it is observed that the information symbols $X_1$ and $X_2$ are decoupled from each other and, hence, can be decoded from the combined receiver outputs $Y_1$ and $Y_2$ respectively. The conditioned SNR (conditioned on the CSI $h_{ji}$ ($i = 1, 2, j = 1, \ldots, N_R$))
at the receiver side for system (4.5) is
\[
\text{SNR}_{2 \times N_R} = \frac{\sum_{j=1}^{N_R} \sum_{i=1}^{2} |h_{ji}|^2}{2} \text{SNR}_0. \tag{4.6}
\]

Suppose that each channel undergoes independent and identically distributed (i.i.d.) Rayleigh fading, i.e., \(h_{ji}\) are i.i.d. complex Gaussian with zero mean and unit variance. Consider the binary phase-shift keying (BPSK) modulation. Let \(f(h) = \sum_{j=1}^{N_R} \sum_{i=1}^{2} |h_{ji}|^2\). Then the average BER, denoted as \(p_e\), is
\[
p_e = \mathbb{E}_f(h) \left[ Q \left( \sqrt{2f(h) \frac{\text{SNR}_0}{2}} \right) \right] = \left( \frac{1 - \mu}{2} \right)^{2N_R - 1} \sum_{k=0}^{2N_R - 1} \binom{2N_R - 1 + k}{k} \left( \frac{1 + \mu}{2} \right)^k, \tag{4.7}
\]
where \(Q(\cdot)\) is the \(Q\)-function and
\[
\mu := \sqrt{\frac{\text{SNR}_0/2}{1 + \text{SNR}_0/2}}.
\]

In the second equality of Equation (4.7) we have used the result in [248, p. 62]. At high SNR, Equation (4.7) reads as [248, p. 63]
\[
p_e \approx \left( \frac{4N_R - 1}{2N_R} \right) \frac{1}{(2\text{SNR}_0)^{2N_R}}. \tag{4.8}
\]

From Equations (4.1), (4.4)–(4.6) and (4.8), we can see the following properties of the Alamouti STC scheme:

1. The Alamouti STC fully exploits the spatial multiplexing rate. By using two transmit antennas, the system transmits two symbols in two time slots.
2. The Alamouti STC fully exploits the diversity order (or diversity gain). Physically, the diversity order can be considered as the number of independent fading gains that are utilized by the system receiver to decode the transmitted data. From Equations (4.1) and (4.5), one can see that the receiver has used all the available fading gains in two noninterfering parallel channels. Mathematically, the diversity order is defined as the (negative) slope of the BER curve (on a dB-log scale) at the high SNR regime. From Equation (4.8), one can see that the diversity order of the Alamouti STC for \(2 \times N_R\) MIMO is \(2N_R\).
3. Additional (receive) array gain can be obtained for \(2 \times N_R\) MIMO systems by using the Alamouti STC when \(N_R > 1\).

Notice that the array gain is always beneficial for improving the BER performance of a system, but the transmit diversity gain is only beneficial in the high SNR regime for improving the BER performance. When the SNR of a system is too low, it is better to use less transmit antennas (see [286] for narrowband systems and [118, 287] for UWB systems).
For the BER simulation results of the Alamouti STC for narrowband systems, readers are referred to [14].

4.3 Alamouti Space–Time Coding for UWB Systems

In the following, we will briefly introduce the approach of [273]. Afterwards, further performance investigation will be conducted.

Let \( w(t) \) be the waveform of the monopulse used to carry information in a UWB system. Its duration \( T_w \) is typically between 0.2 and 2 ns. The energy carried by \( w(t) \) is normalized, i.e., \( \int_0^{T_w} w^2(t) \, dt = 1 \).

The information symbol \( S \) can be conveyed by \( N_f \) pulses:

\[
 s(t) = \sum_{n_f=0}^{N_f-1} a_{n_f}(S) w(t - n_f T_f - b_{n_f}(S) \Delta), \tag{4.9}
\]

where \( a_{n_f}(S) \) and \( b_{n_f}(S) \) are the functions of both information symbol \( S \) and frame index \( n_f \). \( \Delta \) is the position modulation index, \( T_f \) is the frame duration and \( s(t) \) is the signal to be transmitted. Here, we consider the case of only one user. If multiple users are considered, then either the amplitude or the position of the pulse can be further changed, but independently from the information symbol \( S \), using an appropriate spreading code.

A general assumption for STC is that the channel fading will not change for some period. Here, we maintain this assumption, i.e., that the channel fading will not change in \( N_f \) and \( 2N_f \) consecutive encoding frames, for the two encoding schemes addressed below.

Now let us consider the case of two transmit antennas and one receive antenna. In [273], two STC schemes are proposed for this case.

4.3.1 The 1S/2A Coding Scheme

In this scheme, one information symbol \( S \) is transmitted simultaneously from the two transmit antennas. The signals to be transmitted at antennas numbered 0 and 1 respectively are as follows:

\[
 \begin{align*}
 s_0(t) &= \sqrt{\frac{E_0}{2N_f}} \sum_{n_f=0}^{N_f-1} (-1)^{n_f} S w(t - n_f T_f), \\
 s_1(t) &= \sqrt{\frac{E_0}{2N_f}} \sum_{n_f=0}^{N_f-1} S w(t - n_f T_f),
\end{align*} \tag{4.10}
\]

where \( E_0 \) is the relative symbol energy, i.e., \( \int_0^{N_f T_f} s_0^2(t) \, dt = \int_0^{N_f T_f} s_1^2(t) \, dt = E_0/2 \). It is easy to cast Equation (4.10) into the form of Equation (4.9). Since one symbol is encoded across two antennas per \( N_f \) frames in this coding scheme, we call it a 1S/2A coding scheme.

The schematic diagram for the decoding system is illustrated in Figure 4.3.
As shown in Figure 4.3, the whole decoding system is divided into four steps. First, the received signal $r(t)$ passes through a matching filter whose impulse response is $w(-t)$, and the sampled (with sampling rate being $1/(2T_w)$) signal is divided into two branches, namely an evenly indexed frame, denoted as $x_e(l)$, and an odd indexed frame, denoted as $x_o(l)$. Second, the signals $x_e(l)$ and $x_o(l)$ are fed to the Rake receiver (with $L$ fingers) and combined with the maximum ratio combining (MRC) algorithm, producing signals $y_e(n_f)$ and $y_o(n_f)$ respectively. Third, the signals $y_e(n_f)$ and $y_o(n_f)$ are summed up over the $N_f$ frames corresponding to the same symbol $S$. It is in this step that the SNR is strengthened due to spatial processing. Finally, the signal $z$ is fed to the decision maker to decode the symbol $S$.

Let the CIRs from transmit antenna $i$ to the receive antenna be

$$h_i(t) = \sum_{l=1}^{L} \alpha(i)_l \delta(t - \tau(i)_l), \quad i = 0, 1,$$

where $\alpha(i)_l$ and $\tau(i)_l$ are the amplitude fading and arrival time of the $l$th resolvable cluster for the $i$th transmit and receive antenna pair, and $L$ is the number of resolvable clusters. Then the signals $y_e(n_f)$, $y_o(n_f)$ and $z$ can be expressed as [273]

$$\begin{align*}
y_e(n_f) &= \sum_{l=1}^{L} \left[ \alpha(0)_l + \alpha(1)_l \right] x_e(l) = \sqrt{E_0} \sum_{l=1}^{L} \left[ \alpha(0)_l + \alpha(1)_l \right]^2 S + \xi_e(n_f), \\
y_o(n_f) &= \sum_{l=1}^{L} \left[ \alpha(1)_l - \alpha(0)_l \right] x_o(l) = \sqrt{E_0} \sum_{l=1}^{L} \left[ \alpha(0)_l - \alpha(1)_l \right]^2 S + \xi_o(n_f), \\
z &= \sum_{n_f=0}^{N_f/2-1} [y_o(n_f) + y_e(n_f)] \\
&= \sqrt{N_f E_0} \sum_{l=1}^{L} \left[ \alpha(0)_l \right]^2 + \left[ \alpha(1)_l \right]^2 S + \sum_{n_f=0}^{N_f/2-1} [\xi_e(n_f) + \xi_o(n_f)],
\end{align*}
$$

(4.11)

where the new equivalent amplitude fading $\alpha(i)_l (i = 0, 1)$ is derived from the physical amplitude fading of the channel but reformed by the correlator function of $w(t)$ and the arrival time delays $\tau(i)_l$ of the CIR (see [273, Equation (5)]), $\xi_e(n_f)$ and $\xi_o(n_f)$ are the derived measurement noises, which are independent of each other, of zero mean, and of
variances of $\sigma^2 \sum_{l=1}^{L} [a_l^{(0)} + a_l^{(1)}]^2$ and $\sigma^2 \sum_{l=1}^{L} [a_l^{(0)} - a_l^{(1)}]^2$ respectively, and $\sigma^2$ is the variance of the measurement noise contained in $r(t)$.

### 4.3.2 The 2S/2A Coding Scheme

In this scheme, two information symbols $S_a$ and $S_b$ are transmitted simultaneously from the two transmit antennas, but encoded across two consecutive frames in the following way \cite{273}:

$$
\begin{align*}
    s_0(t) &= \sqrt{\frac{E_0}{4N_f}} \sum_{n_f=0}^{N_f-1} [S_a w(t - 2n_fT_f) - S_b w(t - 2n_fT_f - T_f)] , \\
    s_1(t) &= \sqrt{\frac{E_0}{4N_f}} \sum_{n_f=0}^{N_f-1} [S_b w(t - 2n_fT_f) + S_a w(t - 2n_fT_f - T_f)].
\end{align*}
$$

(4.12)

It can also be shown that Equation (4.12) can be cast into the form of Equation (4.9). Since two symbols are simultaneously encoded across the two antennas per $N_f$ frames in this coding scheme, we call it the 2S/2A coding scheme.

The schematic diagram for the encoding–decoding system is given in Figure 4.4.

As shown in Figure 4.4, the whole decoding system is divided into five steps:

1. The received signal $r(t)$ passes through a matching filter whose impulse response is $w(-t)$ and the sampled signal (with sampling rate being $1/(2T_w)$) in the first $N_f$ frames is divided into two branches, namely an even-indexed frame, denoted as $x_e(l)$, and an odd-indexed frame, denoted as $x_o(l)$.
2. The signals $x_e(l)$ and $x_o(l)$ are fed to the Rake receiver (with $L$ fingers) and combined with the MRC algorithm, producing signals $y_a(n_f)$ and $y_b(n_f)$ respectively.
3. $y_a(n_f)$ and $y_b(n_f)$ are each summed up over the first $N_f$ frames, producing signals $z_{a1}$ and $z_{b1}$ respectively.

![Figure 4.4](image)

**Figure 4.4** Schematic diagram for the space–time encoding–decoding scheme 2S/2A.

\footnote{Note that the scaling coefficient $\sqrt{E_0/4N_f}$ is different from that in \cite{273}. We choose the scaling coefficient as shown here in order to make the energy consumption per symbol per frame be the same as that for the 1S/2A scheme.}
4. The same procedure as in steps 1–3 is repeated for the second $N_f$ frames, producing signals $z_{a2}$ and $z_{b2}$ respectively. Sum up signals $z_{a1}$ and $z_{a2}$, and signals $z_{b1}$ and $z_{b2}$, yielding signals $z_a$ and $z_b$ respectively.

5. The signals $z_a$ and $z_b$ are fed to the decision maker to decode the symbols $S_a$ and $S_b$. Denote the amplitude fading of the channel from transmit antenna $i$ to the receive antenna as $\{\alpha_i^{(0)}, l = 0, \ldots, L\}$, and $\{\tilde{\alpha}_i^{(1)}, l = 0, \ldots, L\}$, respectively for the first and second $N_f$ frames. Then the signals $y_a(n_f)$, $y_b(n_f)$ and $z$ can be expressed as [273]:

$$
\begin{align*}
\begin{cases}
    y_a(n_f) &= \sum_{l=1}^{L} \left[ x_c(l)\alpha_i^{(0)} + x_o(l)\alpha_i^{(1)} \right] \\
    &= \sqrt{\frac{E_0}{4N_f}} \sum_{l=1}^{L} \left\{ \left[ \alpha_i^{(0)} \right]^2 + \left[ \alpha_i^{(1)} \right]^2 \right\} S_a + \xi_a(n_f), \\
    y_b(n_f) &= \sum_{l=1}^{L} \left[ x_c(l)\alpha_i^{(1)} - x_o(l)\alpha_0^{(1)} \right] \\
    &= \sqrt{\frac{E_0}{4N_f}} \sum_{l=1}^{L} \left\{ \left[ \alpha_i^{(0)} \right]^2 + \left[ \alpha_i^{(1)} \right]^2 \right\} S_b + \xi_b(n_f), \\
    z_a &= \sum_{n_f=0}^{N_f-1} y_{a1}(n_f) + \sum_{n_f=0}^{N_f-1} y_{a2}(n_f) \\
    &= \sqrt{\frac{N_f E_0}{16}} \sum_{l=1}^{L} \left\{ \left[ \alpha_i^{(0)} \right]^2 + \left[ \alpha_i^{(1)} \right]^2 + \left[ \tilde{\alpha}_i^{(0)} \right]^2 + \left[ \tilde{\alpha}_i^{(1)} \right]^2 \right\} S_a + \tilde{\xi}_a(n_f) + \bar{\xi}_a(n_f), \\
    z_b &= \sum_{n_f=0}^{N_f-1} y_{b1}(n_f) + \sum_{n_f=0}^{N_f-1} y_{b2}(n_f) \\
    &= \sqrt{\frac{N_f E_0}{16}} \sum_{l=1}^{L} \left\{ \left[ \alpha_i^{(0)} \right]^2 + \left[ \alpha_i^{(1)} \right]^2 + \left[ \tilde{\alpha}_i^{(0)} \right]^2 + \left[ \tilde{\alpha}_i^{(1)} \right]^2 \right\} S_b + \bar{\xi}_b(n_f), \\
\end{cases}
\end{align*}
$$

(4.13)

where $\xi_a(n_f)$ and $\bar{\xi}_b(n_f)$ are the derived measurement noises for the first $N_f$-frame processing, which are independent of each other, of zero mean and of variances

$$
\sigma^2 \sum_{l=1}^{L} \left\{ \left[ \alpha_i^{(0)} \right]^2 + \left[ \alpha_i^{(1)} \right]^2 \right\},
$$

$\tilde{\xi}_a(n_f)$ and $\bar{\xi}_b(n_f)$ are the noises similar to $\xi_a(n_f)$ and $\bar{\xi}_b(n_f)$ but for the second $N_f$-frame processing, and $y_{a1}(n_f)$ (or $y_{b1}(n_f)$) and $y_{a2}(n_f)$ (or $y_{b2}(n_f)$) represent the signal $y_a(n_f)$ (or $y_b(n_f)$) obtained at the first $N_f$- and second $N_f$-frame processing respectively.

From Equations (4.11) and (4.13) we can see that the inherent diversity of UWB signals in the temporal domain is collected by the Rake, while the diversity in the spatial domain is harvested by the use of the STC.
4.3.3 Simulation Results

The performance of the aforementioned STC schemes is illustrated in Figure 4.5.

In the simulation, the standard UWB channel model [77, 164] is used, where the exponentially decaying parameters of the power delay profile for clusters and rays are chosen as 33 ns and 5 ns respectively; the cluster arrival rate and ray arrival rate are set to be 0.5 (ns)^{-1} and 2 (ns)^{-1} respectively [273]. The sampling period is chosen as 0.7 ns and we set \( N_f = 2 \). The second derivative of the Gaussian function is chosen as the information-carrying monopulse, i.e.,

\[
w(t) = c_{\text{norm}} \left[ 1 - 4\pi \left( \frac{t}{\tau_p} \right)^2 \right] \exp \left[ -2\pi \left( \frac{t}{\tau_p} \right)^2 \right],
\]

where the constant \( c_{\text{norm}} \) is to normalize the energy of the pulse \( w(t) \) and \( \tau_p = 0.1225 \) ns. The autocorrelation \( R_w(\tau) \) of \( w(t) \) is given by [108]

\[
R_w(\tau) = c_{\text{norm}}^2 \left[ 1 - 4\pi \left( \frac{\tau}{\tau_p} \right)^2 + 4\pi^2 \left( \frac{3}{\tau_p} \right)^3 \right] \exp \left[ -\pi \left( \frac{\tau}{\tau_p} \right)^2 \right].
\]

From Figure 4.5 it can be observed that the 1S/2A scheme outperforms the SISO systems in the BER performance according to per-symbol-per-frame-per-unit-energy data transmission, while the 2S/2A scheme outperforms considerably both the 1S/2A scheme and SISO in the BER performance when the SNR is high, but it yields (marginally or moderately) poorer BER performance than both the 1S/2A scheme and SISO when the SNR is low. Note that the 2S/2A scheme requires the longest decoding delay among the three kinds of system. Notice also that the ranges of the aforementioned low and high SNRs depend on the number of fingers in the Rake receiver. Table 4.1 gives a detailed comparison among the SISO, 1S/2A and 2S/2A schemes in terms of data rate, energy consumption and BER performance. Therefore, we recommend using the 1S/2A coding scheme if the number of fingers in the Rake receiver is not too large, say less than 16, considering the factors of both BER performance and decoding delay.

From Figure 4.5 and the channel capacity for the MISO case (Figure 3.5), we can draw a basic conclusion about the diversity and system performance: if the available SNR is too low, then it is better to concentrate the available power on a single antenna or a single symbol to transmit the information data than to distribute the power across multiple antennas or symbols to gain the diversity; while the diversity gain can only be realized if the available SNR is sufficiently high. In other words, the transmit power should be somewhat peaky (in the sense addressed in [171, 245]) if the available SNR is too low. The reason behind this fundamental compromise can be deduced from an extreme case where the available SNR is so low that we cannot correctly receive one single bit in a given period if we distribute the low energy across multiple antennas or symbols, while we can correctly receive several bits in the given period if we use only one antenna or spend the whole power on one single symbol to transmit. In practice, it is important to give some quantitative characterizations for this fundamental compromise, such as the critical transmission rate given in Chapter 3. Unfortunately, it does not seem easy to provide such a quantitative characterization for the STC schemes discussed here. We can just state that it depends on the channel length \( L \), the channel fading parameters and the STC schemes.
Figure 4.5 BER performance comparison of SISO and MISO with STC schemes 1S/2A and 2S/2A. (From [118]. Reproduced by permission of © IEEE 2009.)
Table 4.1 Performance comparison among the SISO, 1S/2A and 2S/2A schemes. (From [118]. Reproduced by permission of © IEEE 2009.)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Energy consumption per $N_f$ frames</th>
<th>Symbol rate per $N_f$ frames</th>
<th>Decoding delay</th>
<th>BER</th>
</tr>
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<tr>
<td>SISO</td>
<td>$E_0 N_f$</td>
<td>$N_f$ symbols</td>
<td>1 frame</td>
<td>$&gt;$ BER$_{1S/2A}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$&lt;$ BER$_{2S/2A}$ at low SNR</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td>$&gt;$ BER$_{2S/2A}$ at high SNR</td>
</tr>
<tr>
<td>1S/2A</td>
<td>$E_0$</td>
<td>1 symbol</td>
<td>$N_f$ frames</td>
<td>$&lt;$ BER$_{SISO}$</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$&lt;$ BER$_{2S/2A}$ at low SNR</td>
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<td>$&gt;$ BER$_{2S/2A}$ at high SNR</td>
</tr>
<tr>
<td>2S/2A</td>
<td>$E_0$</td>
<td>1 symbol</td>
<td>$2N_f$ frames</td>
<td>$&gt;$ BER$_{SISO}$ at low SNR</td>
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<td></td>
<td></td>
<td></td>
<td>$&lt;$ BER$_{1S/2A}$ at high SNR</td>
</tr>
</tbody>
</table>

4.4 General Space–Time Coding for UWB Systems

Let us first review the real orthogonal design (ROD) proposed by Tarokh et al. in [242].

**Definition 4.1** [242] A real orthogonal design $G$ of size $m$ is an $m \times p$ matrix with entries $0, \pm S_1, \pm S_2, \ldots, \pm S_k$ such that $GG^T = D$, where $D$ is a diagonal matrix with diagonal entries being $D_{ii} = l_{i1} S_1^2 + l_{i2} S_2^2 + \cdots + l_{ik} S_k^2$, $i = 1, 2, \ldots, m$, and the coefficients $l_{i1}, l_{i2}, \ldots, l_{ik}$ are strictly positive integers.

In some cases we need to specify the arguments of $G$ explicitly. In these cases, the ROD will be denoted as $G(S_1, S_2, \ldots, S_k)$, where $S_1, S_2, \ldots, S_k$ are the arguments of $G$.

The construction of general RODs can be found in [242]. For completeness, the RODs for the cases of $m = 2, 3, \ldots, 8$, denoted as $G^{(2)}$, $G^{(3)}$, $\ldots$, $G^{(8)}$ respectively, are listed as follows:

$$G^{(2)} = \begin{bmatrix} S_1 & -S_2 \\ S_2 & S_1 \end{bmatrix},$$

$$G^{(3)} = \begin{bmatrix} S_1 & -S_2 & -S_3 & -S_4 \\ S_2 & S_1 & S_4 & -S_3 \\ S_3 & -S_4 & S_1 & S_2 \end{bmatrix},$$

$$G^{(4)} = \begin{bmatrix} S_1 & -S_2 & -S_3 & -S_4 \\ S_2 & S_1 & S_4 & -S_3 \\ S_3 & -S_4 & S_1 & S_2 \\ S_4 & S_3 & -S_2 & S_1 \end{bmatrix},$$

(4.15) (4.16) (4.17)
A matrix satisfying the following equation:

Definition 4.2 A CROD of a ROD $G(S_1, S_2, \ldots, S_k)$, denoted as $G_c(\alpha_1, \alpha_2, \ldots, \alpha_m)$, is a matrix satisfying the following equation:

$$
G(S_1, S_2, \ldots, S_k) = [S_1 S_2 \ldots S_k] G_c(\alpha_1, \alpha_2, \ldots, \alpha_m).
$$

For the RODs as shown in Equations (4.15)–(4.21), their CRODs are

$$
G_c^{(2)} = \begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_2 & -\alpha_1
\end{bmatrix},
$$

$$
G_c^{(3)} = \begin{bmatrix}
\alpha_1 & \alpha_2 & \alpha_3 & 0 \\
\alpha_2 & -\alpha_1 & 0 & \alpha_3 \\
\alpha_3 & 0 & -\alpha_1 & -\alpha_2 \\
0 & -\alpha_3 & \alpha_2 & -\alpha_1
\end{bmatrix},
$$
\[ \mathcal{G}_c^{(4)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 \\ \alpha_3 & -\alpha_4 & -\alpha_1 & -\alpha_2 \\ \alpha_4 & -\alpha_3 & -\alpha_2 & -\alpha_1 \end{bmatrix}, \quad (4.24) \]

\[ \mathcal{G}_c^{(5)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & 0 & 0 & 0 \\ \alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 & -\alpha_5 & 0 & 0 \\ \alpha_3 & -\alpha_4 & -\alpha_1 & \alpha_2 & 0 & 0 & -\alpha_5 & 0 \\ \alpha_4 & -\alpha_3 & -\alpha_2 & -\alpha_1 & 0 & 0 & 0 & -\alpha_5 \\ \alpha_5 & 0 & 0 & 0 & -\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \alpha_5 & 0 & 0 & -\alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 \\ 0 & 0 & \alpha_5 & 0 & -\alpha_3 & \alpha_4 & -\alpha_3 & -\alpha_2 \\ 0 & 0 & 0 & \alpha_5 & -\alpha_4 & -\alpha_3 & \alpha_2 & -\alpha_1 \end{bmatrix}, \quad (4.25) \]

\[ \mathcal{G}_c^{(6)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & 0 & 0 \\ \alpha_2 & -\alpha_1 & \alpha_4 & -\alpha_3 & \alpha_6 & -\alpha_5 & 0 & 0 \\ \alpha_3 & -\alpha_4 & -\alpha_1 & \alpha_2 & 0 & 0 & -\alpha_5 & -\alpha_6 \\ \alpha_4 & \alpha_3 & -\alpha_2 & -\alpha_1 & 0 & 0 & \alpha_6 & -\alpha_5 \\ \alpha_5 & \alpha_6 & 0 & 0 & -\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_6 & \alpha_5 & 0 & 0 & -\alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 \\ 0 & 0 & \alpha_5 & -\alpha_6 & -\alpha_3 & \alpha_4 & -\alpha_3 & -\alpha_2 \\ 0 & 0 & \alpha_6 & \alpha_5 & -\alpha_4 & -\alpha_3 & \alpha_2 & -\alpha_1 \end{bmatrix}, \quad (4.26) \]

\[ \mathcal{G}_c^{(7)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & 0 \\ \alpha_2 & -\alpha_1 & \alpha_4 & -\alpha_3 & \alpha_6 & -\alpha_5 & 0 & \alpha_7 \\ \alpha_3 & -\alpha_4 & -\alpha_1 & \alpha_2 & \alpha_7 & 0 & -\alpha_5 & -\alpha_6 \\ \alpha_4 & \alpha_3 & -\alpha_2 & -\alpha_1 & 0 & -\alpha_7 & \alpha_6 & -\alpha_5 \\ \alpha_5 & \alpha_6 & -\alpha_7 & 0 & -\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_6 & \alpha_5 & 0 & \alpha_7 & -\alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 \\ \alpha_7 & 0 & \alpha_5 & -\alpha_6 & -\alpha_3 & \alpha_4 & -\alpha_1 & -\alpha_2 \\ 0 & -\alpha_7 & \alpha_6 & \alpha_5 & -\alpha_4 & -\alpha_3 & \alpha_2 & -\alpha_1 \end{bmatrix}, \quad (4.27) \]

\[ \mathcal{G}_c^{(8)} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 \\ \alpha_2 & -\alpha_1 & \alpha_4 & -\alpha_3 & \alpha_6 & -\alpha_5 & -\alpha_8 & \alpha_7 \\ \alpha_3 & -\alpha_4 & -\alpha_1 & \alpha_2 & \alpha_7 & \alpha_8 & -\alpha_5 & -\alpha_6 \\ \alpha_4 & \alpha_3 & -\alpha_2 & -\alpha_1 & \alpha_8 & -\alpha_7 & \alpha_6 & -\alpha_5 \\ \alpha_5 & -\alpha_6 & -\alpha_7 & -\alpha_8 & -\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_6 & \alpha_5 & -\alpha_8 & \alpha_7 & -\alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 \\ \alpha_7 & \alpha_8 & \alpha_5 & -\alpha_6 & -\alpha_3 & \alpha_4 & -\alpha_1 & -\alpha_2 \\ \alpha_8 & -\alpha_7 & \alpha_6 & \alpha_5 & -\alpha_4 & -\alpha_3 & \alpha_2 & -\alpha_1 \end{bmatrix}, \quad (4.28) \]

For a given ROD, its CROD can be generally calculated in the following way.

Suppose that the \( m \times p \) matrix \( \mathcal{G}(S_1, S_2, \ldots, S_k) \) is a ROD. Let us represent \( \mathcal{G} \) as follows:

\[ \mathcal{G}(S_1, S_2, \ldots, S_k) = \sum_{l=1}^{k} S_l M_l, \quad (4.29) \]
where $M_l$ is the coefficient matrix for the variable $S_l$. Clearly, the entry of $M_l$ takes a value from $\pm 1$ and 0. Define

$$\alpha := [\alpha_1 \alpha_2 \cdots \alpha_m].$$

According to Equation (4.29) we have

$$\alpha G = \alpha \sum_{l=1}^{k} S_l M_l = \sum_{l=1}^{k} S_l \alpha M_l = [S_1 \ S_2 \ \cdots \ S_k].$$

Thus

$$G_c(\alpha) = \begin{bmatrix} \alpha M_1 \\ \alpha M_2 \\ \vdots \\ \alpha M_k \end{bmatrix}.$$

An interesting question is whether or not a CROD itself is a ROD. For the CRODs as defined in Equations (4.22)–(4.28), the answer to this question is yes. Actually, it can be easily checked that the equality

$$G_c[G_c]^T = \sum_{j=1}^{m} \alpha_j^2 \cdot I$$

holds true for the CRODs defined in Equations (4.22)–(4.28). The answer to the above question for the general case is not clear yet. We guess the answer is also yes, but currently we are not able to provide a proof. So we state the following claim as a conjecture.

**Conjecture 4.1** A CROD as defined in Definition 4.2 is also a ROD.

Using RODs and the corresponding CRODs, a general STC scheme for IR-based UWB-MIMO can be worked out.

Consider a UWB-MIMO with $N_T$ transmit antennas and $N_R$ receive antennas. Let $G$ be a ROD in variables $S_1, S_2, \ldots, S_K$, where $S_1, S_2, \ldots, S_K$ are the symbols to be transmitted at the $N_T$ transmit antennas in one STC frame. Suppose $G$ is of dimension $N_T \times K$. Define

$$w(t, n_f) = \begin{bmatrix} w(t - n_f K T_f) \\ w(t - n_f K T_f - T_f) \\ \vdots \\ w[t - n_f K T_f - (K - 1)T_f] \end{bmatrix},$$

where $w(t)$ is the waveform of the monopulse with duration $T_w$, $n_f$ is the frame index and $T_f$ is the frame duration. Then the transmitted signal across the $N_T$ transmit antennas
at time slot $t$ is

$$s(t) = \sqrt{\frac{E_0}{N_T K N_f}} \sum_{n_f=0}^{N_f-1} \mathcal{G}(S_1, S_2, \ldots, S_K) w(t, n_f), \quad (4.31)$$

where $N_f$ is the number of frames used to transmit one group of the space-time coded symbols. Notice that the scaling coefficient $\sqrt{E_0/(N_T K N_f)}$ is to normalize the overall energy consumption per frame at the transmitter side to unity. By using Equation (4.31), the spatio-temporal structure of the transmitted signal is illustrated in Figure 4.6.

Let the CIR matrix, including the effect of the matching filter (matched to the monopulse $w(t)$), of the UWB-MIMO system be

$$H(t) = \sum_{l=1}^{L} A_l \delta(t - (l - 1) \Delta \tau),$$

where $\Delta \tau$ is the sampling interval and $A_l$, $l = 1, \ldots, L$, are the amplitude fading matrices, whose statistical properties are described in Chapter 2. It is assumed that the channel matrix does not change across the $N_f$-frame transmission. To avoid intersymbol interference (ISI), we choose a sufficient large $T_f$ such that $T_f \geq L \Delta \tau + T_w$. Then the received signal after sampling can be expressed as

$$x(n_f, l) = \sqrt{\frac{E_0}{N_T K N_f}} A_l \mathcal{G}(S_1, S_2, \ldots, S_K) + n(n_f, l), \quad (4.32)$$

where $x(n_f, l)$ is the received signal sampled at the $n_f$th frame and the $l$th tap, and $n(n_f, l)$ is the receiver noise (a matrix) at the corresponding time instant. Notice that $x(n_f, l)$ is of dimension $N_R \times K$, since one frame of the transmitted signal contains the pulses of $K$ time slots.

Denote by $[\mathbf{M}]_j$ the $j$th row of a matrix $\mathbf{M}$. Let us consider the $j$th row of the matrix $x(n_f, l)$, which is the received signal at the $j$th antenna for the $n_f$th frame and the

![Figure 4.6](image.png)

**Figure 4.6** Structure of the transmitted signal in the space–time domain for UWB-MIMO, where $G_k$ is the $k$th column of the ROD $\mathcal{G}$. 
Let

\[
[A_l]_j = \begin{bmatrix}
\alpha_{j1}(l) & \alpha_{j2}(l) & \cdots & \alpha_{jN_T}(l)
\end{bmatrix}.
\]

Since the transmitted signal is space–time coded, the entries in \([x(n_f, l)]_j\) should be related to each other somehow. By right-hand multiplying both sides of Equation (4.32) with the matrix

\[
[G_c(\alpha_{j1}(l), \alpha_{j2}(l), \ldots, \alpha_{jN_T}(l))]^T,
\]

we have

\[
y_j(n_f, l) := [x(n_f, l)]_j[G_c(\alpha_{j1}(l), \alpha_{j2}(l), \ldots, \alpha_{jN_T}(l))]^T
= \sqrt{\frac{E_0}{N_T K N_f}} [A_l]_j G(S_1, S_2, \ldots, S_K) + [n(n_f, l)]_j
\times [G_c(\alpha_{j1}(l), \alpha_{j2}(l), \ldots, \alpha_{jN_T}(l))]^T
\]

\[
= \sqrt{\frac{E_0}{N_T K N_f}} \sum_{k=1}^{N_T} [\alpha_{jk}(l)]^2 [S_1 \ S_2 \ \cdots \ S_K] + [n(n_f, l)]_j
\times [G_c(\alpha_{j1}(l), \alpha_{j2}(l), \ldots, \alpha_{jN_T}(l))]^T.
\]

From Equation (4.34) we can see that the transmitted symbols \(S_1, S_2, \ldots, S_K\) are decoupled from each other in the processed signal \(y_j(n_f, l)\) through the processing algorithm (4.33).

To collect all the diversities provided by multiple receive antennas, multiframes, and multipaths, we define

\[
z := \sum_{j=1}^{N_R} \sum_{n_f=1}^{N_f} \sum_{l=1}^{L} y_j(n_f, l)
= \sqrt{\frac{E_0}{N_T K N_f}} \sum_{j=1}^{N_R} \sum_{n_f=1}^{N_f} \sum_{l=1}^{L} \sum_{k=1}^{N_T} [\alpha_{jk}(l)]^2 [S_1 \ S_2 \ \cdots \ S_K] + \sum_{j=1}^{N_R} \sum_{n_f=1}^{N_f} \sum_{l=1}^{L} \sum_{k=1}^{N_T} [n(n_f, l)]_j [G_c(\alpha_{j1}(l), \alpha_{j2}(l), \ldots, \alpha_{jN_T}(l))]^T.
\]
The equality (4.36) follows from the assumption that the channel is static across the $N_f$ frames. Actually it is sufficient to assume that the channel is static across one STC frame. In this case, the matrix $A_l$, and consequently the variable $\alpha_{jk}(l)$, will be a function of the frame index $n_f$, denoted as $A_l(n_f)$ and $\alpha_{jk}(n_f,l)$ respectively. Following the same line as the argument above, we can see that the final receiver output $z$ can be expressed as Equation (4.35) with $\alpha_{jk}(l)$ being substituted by $\alpha_{jk}(n_f,l)$.

Finally, the transmitted symbols $S_1, S_2, \ldots, S_K$ can be independently decoded from the output $z$.

The schematic diagram of the above space–time encoding–decoding scheme is illustrated in Figure 4.7.

From Equation (4.36) we can see the system can achieve a diversity order of $N_R N_T L$ if the channel is static across $N_f$ frames, while the system can achieve a diversity order of $N_R N_T N_f L$ if the channel is static in only one STC frame but randomly changing in the subsequent STC frames. Notice that the benefit of such a high diversity order can only be exploited in the high SNR regime. Notice also that the benefit provided by the diversity order does not increase proportionally with the tap number $L$ due to the exponentially decaying power delay profile in UWB channels.

When the channel is not severely changing across frames, the main role of transmitting the same group of symbols in $N_f$ frames is to increase the received SNR by adding the signals coherently, as illustrated in Equation (4.36), or to enable multiuser communications.

### 4.5 Performance of Antenna Selection

The gains obtained by deploying multiple antennas come at the price of hardware complexity. The radio front end has a complexity, size and cost that scale with the number of antennas [207]. A possible way to reduce the cost and complexity while not sacrificing the gains too much is to use antenna selection schemes. For narrowband MIMO
communication systems, there are a lot of research results about antenna selection; see the comprehensive surveys [169, 207] and references cited therein. However, for UWB-MIMO systems, few results are currently available.

A general setup of the antenna selection is to choose \( \bar{N}_T \) transmit and \( \bar{N}_R \) receive antennas out of \( N_T \) transmit and \( N_R \) receive antennas respectively. For a fixed \( \bar{N}_T \) and \( \bar{N}_R \), the selection is normally based on two kinds of performance criteria, which are the BER performance and channel capacity [207]. Generally, the selection based on the BER performance will coincide with the selection based on the received SNR. On the other hand, the selections based on the BER performance and channel capacity do not agree with each other. As shown in [170] for narrowband MIMO, only 50% of the selections based on the two criteria in all the fading channel realizations agree with each other.

In the extreme case when \( \bar{N}_T \) is not fixed in advance and \( \bar{N}_R = N_R \), the selection based on the channel capacity will yield the water-filling algorithm across the frequency–spatial domain [287]. The transmit power will be distributed in patches on the surface of the frequency and spatial domains. Thus, in a certain frequency band, only some of the antennas are selected for transmission. As shown in Figures 3.1 and 3.2, this kind of antenna selection can yield a considerable gain in capacity in the low SNR range. However, if the antenna selection is based only on the spatial domain, whether or not the selection can yield a performance gain will be determined by the scattering environments and the relative locations of the antennas.

Currently, there are no reports to address the antenna selection problem for UWB-MIMO systems in the sense of the aforementioned general setup. In the following, let us consider the simple case where \( N_T = 2 \) and \( N_R = 1 \). Therefore, we have \( \bar{N}_T = \bar{N}_R = 1 \). In this case, antenna selection actually reduces to antenna switching. As mentioned before, it was pointed out in [258] that antenna switching in MISO systems does not provide diversity gain in the strict sense. Examining the simulation scenario investigated in [258], the two channels from the transmitters to the receiver experience the same scattering environment and thus possess the same pdf in their fading statistics. Generally, the spatial diversity exists for rich scattering environments and sufficiently separated antennas for narrowband systems, and antenna selection can be used to exploit such diversity even though the channels possess the same fading pdf. For UWB-MIMO channels, the fading is distributed over many channel taps and the Rake receiver is normally used to collect the received energy across these taps. Even though every channel tap may experience fading and spatial diversity may exist among these single taps for different channels, the Rake receiver actually smoothes the fading by the (possibly weighted) summation of the signals from the selected channel taps. Therefore, the antenna selection will not yield diversity gain in the strict sense if the spatial channels share the same fading pdf. On the other hand, if the antennas are separated sufficiently far, then different scattering environments may be experienced by the spatial channels. For example, when one channel is in LOS and another channel is in NLOS, the channels will have different fading pdfs (in either forms or parameters) and different power delay profiles. In such a scenario, the spatial diversity exists even after the Rake processing, and the antenna selection can be used to improve the system performance.

To illustrate the above idea, let us consider the following experiment. There are two transmit antennas and one receive antenna in a typical office environment, with one
transmit antenna in LOS and another in NLOS, as shown in Figure 4.8a. All the antennas are omni-directional UWB conical antennas with 3 dB gain. The height of the room is 3.02 m and the height of transmit and receive antennas is 1.5 m above ground. The channel transfer functions between the receiver and each transmitter are measured at the sampled frequencies from 1 to 11 GHz with a frequency sampling step of $\Delta f = 6.25$ MHz (so 1602 frequency points are measured for each channel). The CIRs from each transmitter to the receiver are obtained by the inverse Fourier transform of the corresponding channel transfer functions. The binary PPM is used to transmit the information symbols and a selective Rake [184, 263] receiver with 10 strongest taps being selected is used to collect the signal energy. Two kinds of receivers are compared. The first is based on the antenna

![Floor plan diagram](image)

![BER Performance](image)

**Figure 4.8** Performance comparison between antenna selection and equal gain combiner: (a) floor plan; (b) BER performance. (From [118]. Reproduced by permission of © IEEE 2009.)
selection scheme, where one of the transmit antennas from which the receiver receives a stronger summation signal in the selective Rake is selected for symbol transmission. The second is based on an equal gain combining receiver, where both transmitters are used to transmit symbols. Note that, in the second case, the symbol energy per transmit antenna is halved compared with that in the first case. The performance comparison for the two cases is illustrated in Figure 4.8b. It is seen that the antenna selection can provide better BER performance than the equal gain combiner.

The reason for the above phenomenon is that the two channels exhibit some kind of diversity. To have a deep insight into this phenomenon, let us return to the general UWB channel model proposed in [164, Appendix A]. There are two kinds of fading in the CIR, with one characterizing the small-scale fading for each channel tap and another the large-scale shadowing, which applies to all the channel taps. Clearly, the spatial diversity among the channels in UWB-MIMO systems is characterized by the large-scale shadowing, while the temporal diversity among the taps of each single channel is characterized by the small-scale fading. Generally, the Rake receivers can remove the micro diversity caused by the small-scale fading, but they cannot remove the macro diversity caused by the large-scale shadowing if the shadowing statistics are different.

4.6 Spatio-Frequency Multiplexing in MB-OFDM-Based UWB Systems

The basic idea of MB-OFDM-based UWB systems is to divide the whole UWB frequency band (3.1–10.6 GHz) into multiple smaller frequency bands, which are referred to as sub-bands, and then to use multiple carrier frequencies to multiplex the information symbols into different sub-bands. It is required that each sub-band is of bandwidth of greater than 500 MHz, to comply with the FCC definition of the UWB signals. According to the WPAN standard IEEE 802.15.3a, the bandwidth of 528 MHz for each sub-band is adopted. The transmitted OFDM symbols are time-interleaved across the sub-bands. An advantage of this approach is that the transmitted power density per megahertz can be well designed to satisfy the power density requirement imposed by regulatory authorities such as the FCC and EC. Another merit of this approach is that the very high sampling rate in the receiver, as required by the IR-based UWB receiver, is greatly relaxed. Other pros and cons of MB-OFDM systems and the treatment of some specific issues existing in MB-OFDM systems can be found in the literature [21, 103, 144, 183, 215, 223, 224, 226, 227, 272, 290].

The transmitter (TX) and receiver (RX) structures for a typical MB-OFDM system are illustrated in Figure 4.9a and 4.9b respectively [21].

As seen from Figure 4.9, the TX and RX architectures for an MB-OFDM system are very similar to those of a conventional wireless OFDM system. The main difference is that the centre frequency of the MB-OFDM system changes with time according to a specific pattern. It is the functional unit time–frequency kernel [21] that specifies the hopping pattern of the centre frequencies of sub-bands. For an MB-OFDM system consisting of three sub-bands, the time–frequency changing pattern is illustrated in Figure 4.10, which shows that the first, second and third OFDM block symbols are transmitted in sub-band 1 (channel 1), sub-band 3 (channel 3) and sub-band 2 (channel 2) respectively. The subsequent OFDM block symbols will be transmitted by repeating the same pattern.
Figure 4.9  The transmitter and receiver structure of a typical MB-OFDM system, where GI, LNA, LPF, VGA, AGC, FEQ, and FFT denote guard interval, low-noise amplifier, low-pass filter, voltage gain amplifier, automatic gain control, frequency-domain equalizer, and fast Fourier transform respectively: (a) transmitter; (b) receiver. (From [21]. Reproduced by permission of © IEEE 2004.)

Figure 4.10  Time–frequency hopping pattern of an MB-OFDM system with three sub-bands. (From [21]. Reproduced by permission of © IEEE 2004.)

The hopping pattern across the sub-bands in MB-OFDM-based UWB systems can be exploited to provide frequency diversity, multiple access or both.

For the details about the design of each functional unit in the aforementioned MB-OFDM-based UWB system, readers are referred to the literature [21, 224].
Multiple antennas can be directly combined with the MB-OFDM modulation to form MIMO MB-OFDM communication systems. Figure 4.11 shows such a combination. In such a system, the information symbols are jointly encoded across transmit antennas, OFDM subcarriers (in a sub-band) and time.

Let us consider an MB-OFDM system with two transmit antennas and one receive antenna and adopting Alamouti STC, as shown in Figure 4.12.

Let the number of subcarriers of the OFDM modulation in a given sub-band be $N_o$. Let the vectors $x_i (i = 1, 2)$ denote respectively the symbols to be transmitted in one OFDM block in the branch of antenna $i$, but before inverse discrete-time Fourier transform (IDFT) processing, as shown in Figure 4.12. Suppose

$$x_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ S_{iN_o} \end{bmatrix}, \quad i = 1, 2.$$  

(4.37)

![Figure 4.11](image1)  
**Figure 4.11** Schematic diagram of an MB-OFDM MIMO system.

![Figure 4.12](image2)  
**Figure 4.12** A $2 \times 1$ MB-OFDM MIMO system adopting Alamouti STC.
Denote by vectors $\bar{x}_i (i = 1, 2)$ the symbols to be transmitted in one OFDM block at antenna $i$. Then $\bar{x}_i$ and $x_i$ are related by

$$\bar{x}_i = Wx_i,$$  \hspace{1cm} (4.38)

where $W = \frac{1}{\sqrt{N_o}} \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & \sigma & \sigma^2 & \ldots & \sigma^{(N_o-1)} \\ 1 & \sigma^2 & \sigma^4 & \ldots & \sigma^{2(N_o-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \sigma^{(N_o-1)} & \sigma^{2(N_o-1)} & \ldots & \sigma^{(N_o-1)^2} \end{bmatrix}$  \hspace{1cm} (4.39)

where $\sigma = e^{j2\pi/N_o}$ and $j = \sqrt{-1}$. Let $H_1$ and $H_2$ be the Fourier transforms of the CIRs $h_1$ and $h_2$ (which are in the discrete-time domain) respectively. Then $H_i(k)$ is the frequency-domain channel fading from transmit antenna $i$ to the receive antenna at subcarrier $k$. Define

$$\Lambda_i = \text{diag}\{H_i(0), H_i(2), \ldots, H_i(N_o-1)\}, \quad i = 1, 2,$$

where $\text{diag}$ denotes a diagonal matrix with the diagonal entries being specified by the arguments correspondingly. Let $\tilde{y}$ denote the received signal in one OFDM block, with each entry, say $\tilde{y}(k)$, representing the received signal at subcarrier $k$, $k = 0, 1, \ldots, N_o - 1$, i.e., $\tilde{y}$ is the received OFDM block symbol expressed in the frequency domain. Then we have

$$\tilde{y} = \Lambda_1\bar{x}_1 + \Lambda_2\bar{x}_2 + \tilde{n},$$  \hspace{1cm} (4.40)

where $\tilde{n}$ is the receiver noise. Let $y$ be the signal after the discrete-time Fourier transform (DFT) processing, i.e., $y$ is the received OFDM block symbol expressed in the time domain, as shown in Figure 4.12. Then we have

$$y = W^\dagger \tilde{y},$$  \hspace{1cm} (4.41)

where the superscript $\dagger$ denotes the conjugate transpose of a complex vector or matrix. Combining Equations (4.38), (4.40) and (4.41) and noticing the fact that

$$W^{-1} = W^\dagger = W^\ast,$$  \hspace{1cm} (4.42)

where the superscript $\ast$ denotes the element-wise conjugate (without transpose) of a complex vector or matrix, we have

$$y = W^\dagger \Lambda_1 Wx_1 + W^\dagger \Lambda_2 Wx_2 + W^\dagger \tilde{n}$$

$$= \bar{H}_1x_1 + \bar{H}_2x_2 + n,$$  \hspace{1cm} (4.43)

where

$$\bar{H}_i := W^\dagger \Lambda_i W, \quad i = 1, 2$$  \hspace{1cm} (4.44)

$$n := W^\dagger \tilde{n}.$$  \hspace{1cm} (4.45)
Now consider Alamouti STC at the transmitter side and decoding at the receiver side. Therefore, we consider the transmission and reception of two consecutive OFDM blocks. Suppose the CSI \( \mathbf{H}_i \) or \( \mathbf{A}_i \) does not change during the transmission of two consecutive OFDM blocks. According to the Alamouti encoding scheme, two OFDM block symbols, denoted \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \) (as shown in Equation (4.37)), are transmitted at two consecutive time slots in the following way.

**Algorithm 4.1** Alamouti STC for MB-OFDM-based UWB systems:

- at time slot \( t \), the symbol vectors \( \mathbf{W}^\dagger \mathbf{x}_1 \) and \( \mathbf{W}^\dagger \mathbf{x}_2 \) are sent to the OFDM modulation unit at the branches antenna 1 and antenna 2 respectively for processing and then are transmitted through antenna 1 and antenna 2 respectively;
- at time slot \( t + 1 \), the symbol vectors \( -\mathbf{W}^\dagger \mathbf{x}_2^* \) and \( \mathbf{W}^\dagger \mathbf{x}_1^* \) are sent to the OFDM modulation unit at the branches antenna 1 and antenna 2 respectively for processing and then are transmitted through antenna 1 and antenna 2 respectively.

Denote the received OFDM block signals, after DFT processing, as \( \mathbf{y}(t) \) and \( \mathbf{y}(t + 1) \). Then, according to Equation (4.43), we have

\[
\mathbf{y}(t) = \tilde{\mathbf{H}}_1(\mathbf{W}^\dagger \mathbf{x}_1) + \tilde{\mathbf{H}}_2(\mathbf{W}^\dagger \mathbf{x}_2) + \mathbf{n}(t),
\]

\[
\mathbf{y}(t + 1) = \tilde{\mathbf{H}}_1(-\mathbf{W}^\dagger \mathbf{x}_2^*) + \tilde{\mathbf{H}}_2(\mathbf{W}^\dagger \mathbf{x}_1^*) + \mathbf{n}(t + 1),
\]

where \( \mathbf{n}(t) \) and \( \mathbf{n}(t + 1) \) are the receiver noise after the OFDM processing unit at time slots \( t \) and \( t + 1 \) respectively. For convenience, denote the corresponding noise vectors before the OFDM processing unit as \( \tilde{\mathbf{n}}(t) \) and \( \tilde{\mathbf{n}}(t + 1) \) respectively, where \( \tilde{\mathbf{n}} \) and \( \mathbf{n} \) are related to each other through Equation (4.45). Combine \( \mathbf{y}(t) \) and \( \mathbf{y}(t + 1) \) in the following way:

\[
\begin{align*}
\mathbf{z}_1 &= \mathbf{A}_1^\dagger \mathbf{W} \mathbf{y}(t) + \mathbf{A}_2^\dagger \mathbf{W} \mathbf{y}(t + 1))^*, \\
\mathbf{z}_2 &= \mathbf{A}_2^\dagger \mathbf{W} \mathbf{y}(t) - \mathbf{A}_1^\dagger \mathbf{W} \mathbf{y}(t + 1))^*.
\end{align*}
\]

From Equations (4.42), (4.44), (4.46) and (4.47) we have

\[
\begin{align*}
\mathbf{z}_1 &= \Lambda_1^\dagger \mathbf{W}[\mathbf{W}^\dagger \mathbf{A}_1 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_1) + \mathbf{W}^\dagger \mathbf{A}_2 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_2) + \mathbf{n}(t)] \\
&\quad + \Lambda_2^\dagger \mathbf{W}[-\mathbf{W}^\dagger \mathbf{A}_1 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_2^*) + \mathbf{W}^\dagger \mathbf{A}_2 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_1^*) + \mathbf{n}(t + 1))^* \\
&= \Lambda_1^\dagger \mathbf{A}_1 \mathbf{x}_1 + \Lambda_2^\dagger \mathbf{A}_2 \mathbf{x}_2 - \Lambda_2 \Lambda_1^\dagger \mathbf{x}_2 + \Lambda_2 \Lambda_2^\dagger \mathbf{x}_1 + \Lambda_1^\dagger \tilde{\mathbf{n}}(t) + \Lambda_2^\dagger \tilde{\mathbf{n}}(t + 1) \\
&= (|\Lambda_1|^2 + |\Lambda_2|^2)\mathbf{x}_1 + \Lambda_1^\dagger \tilde{\mathbf{n}}(t) + \Lambda_2^\dagger \tilde{\mathbf{n}}(t + 1),
\end{align*}
\]

\[
\begin{align*}
\mathbf{z}_2 &= \Lambda_2^\dagger \mathbf{W}[\mathbf{W}^\dagger \mathbf{A}_1 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_1) + \mathbf{W}^\dagger \mathbf{A}_2 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_2) + \mathbf{n}(t)] \\
&\quad - \Lambda_1^\dagger \mathbf{W}[-\mathbf{W}^\dagger \mathbf{A}_1 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_2^*) + \mathbf{W}^\dagger \mathbf{A}_2 \mathbf{W}(\mathbf{W}^\dagger \mathbf{x}_1^*) + \mathbf{n}(t + 1))^* \\
&= \Lambda_2^\dagger \mathbf{A}_1 \mathbf{x}_1 + \Lambda_1^\dagger \mathbf{A}_2 \mathbf{x}_2 + \Lambda_1 \Lambda_1^\dagger \mathbf{x}_2 - \Lambda_1 \Lambda_2^\dagger \mathbf{x}_1 + \Lambda_2^\dagger \tilde{\mathbf{n}}(t) - \Lambda_1^\dagger \tilde{\mathbf{n}}(t + 1) \\
&= (|\Lambda_1|^2 + |\Lambda_2|^2)\mathbf{x}_2 + \Lambda_2^\dagger \tilde{\mathbf{n}}(t) - \Lambda_1^\dagger \tilde{\mathbf{n}}(t + 1),
\end{align*}
\]

where

\[
|\Lambda_i|^2 = \text{diag}\{||\mathbf{H}_i(0)||^2, ||\mathbf{H}_i(1)||^2, \ldots, ||\mathbf{H}_i(N_0 - 1)||^2\}, \quad i = 1, 2.
\]
From Equations (4.49) and (4.50) we can see that the transmitted symbols $S_{ij}$, $i = 1, 2$, $j = 0, 1, \ldots, N_0 - 1$, are decoupled from each other in the processed receive signals $z_1$ and $z_2$. Thus, the symbols $\{S_{ij}\}$ can be easily decoded from $z_1$ and $z_2$.

In summary, the encoding and decoding schemes for Alamouti MB-OFDM UWB systems are illustrated in Figure 4.13.

Other STC designs for narrowband communication systems can be similarly extended to MB-OFDM-based UWB systems.

4.7 Summary

In this chapter we have discussed a simple, yet widely used, STC scheme for IR-based UWB systems, i.e., Alamouti coding. A detailed performance comparison among two kinds of Alamouti STC for $2 \times 1$ UWB-MIMO and UWB-SISO systems has been provided. Simulation results show that it is beneficial to employ a space–time code with a longer coding block length by collecting more diversities when the system SNR is sufficiently high, while it is detrimental to use a code of long coding block length when the system SNR is low. A general design approach for the STC of IR-based UWB systems with arbitrary numbers of transmit and receive antennas is presented. This design approach is based on the concept of ROD proposed in [242]. To ease the design of decoding at the receiver, we have proposed the concept of CROD. A general calculation method for CROD is given. The space–time decoding algorithm for IR-based UWB systems can be elegantly formulated based on CRODs. Antenna selection for UWB systems is also briefly discussed. Finally, we have addressed the issue of the STC technique for MB-OFDM-based UWB systems.
5

UWB Beamforming and Localization

5.1 Introduction

Array processing and smart antenna concepts offer a promising solution to the significant increasing of data rates in wireless transmission systems. There have been widespread interests in array processing over more than four decades, especially in radar and deep space exploration, while the interest in the application of array processing in wireless communication systems started two decades ago and booms rapidly nowadays. Perhaps the delayed surging of the interest in the latter field is due to, on one hand, the lack of a need for the array technology in the wireless communication market (imagine that wireless communications have become popular only in the recent two decades) and, on the other hand, the difficulty in dealing with multipaths of the propagation signals in wireless communications, which will cause a fundamental trouble for the array processing.

Beamforming for UWB impulse signals has some special properties that are quite different from the narrowband beamformers. For example, the use of unequal weighting filters for the individual antenna branch will increase the side-lobe level in a UWB beamformer, and there are no grating lobes in the UWB beampattern [204]. In this chapter, we will investigate how these properties appear in UWB beamformers.

Localization can be considered as a possible application of array processing. If the antenna array is a uniform linear array, combining the direction-of-arrival (DoA) and time-of-arrival (ToA) information obtained by the array will uniquely decide the target position. If the antenna elements in the ‘array’ are distributed irregularly but sufficiently far from each other, like the case of sensor networks (in this case, the word ‘array’ just means multiple sensors located at different places), the ToA information obtained at several elements in the ‘array’ can jointly give a unique solution for the target position. It has been proved that the positioning accuracy by using a radio system is in general proportional to the reciprocal of the effective bandwidth of the transmitted signal. Since UWB systems possess a huge bandwidth, using UWB technology can provide extremely high

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1 A part of Sections 5.1–5.5 of this chapter is reproduced with permission from [118]. Reproduced by permission of © IEEE 2009.
positioning accuracy in principle. However, there is a great challenge in fully exploiting the potential in the localization problem offered by the UWB technology because of abundant multipaths and the NLOS issue. Another aim of this chapter is to discuss relevant issues in the UWB localization problem.

5.2 Ideal UWB Impulse Beamforming

An ideal UWB impulse beamforming system is illustrated in Figure 5.1, where $N$ receive antennas are aligned linearly and identically separated by a distance $d$, which is several centimeters in typical usage. All the antennas are assumed to be omni-directional. As in the narrowband case, we assume that the receive antenna array is located in the far field, i.e., the distances from the transmit antenna to each receive antenna are much larger than $d$, so that the rays from the transmit antenna to each receive antenna can be considered parallel to each other. In Figure 5.1, $\beta_i(t), i = 0, \ldots, N - 1$, are the impulse responses of the pre-filters and $\tau_i, i = 0, \ldots, N - 1$, are the additional time delays, which are applied to each branch to control the steering direction of the beamformer.

Let $c$ be the propagation speed of the UWB impulse wave, $\theta$ the angle between the incidence ray and broadside direction of the antenna array, and $x(t)$ the transmitted signal. Generally, $x(t)$ is a train of monopulses,\(^2\) i.e.,

$$x(t) = \sum_{n=-\infty}^{+\infty} w(t - nT_f), \quad (5.1)$$

where $w(t)$ is the waveform of the UWB monopulse and $T_f$ is the pulse repetition period. The output of the beamformer can be expressed as

$$y(t; \theta) = \sum_{k=0}^{N-1} y_k(t - \tau_k), \quad y_k(t) = x(t - k\frac{d}{c}\sin\theta) \ast \beta_k(t), \quad k = 0, \ldots, N - 1,$$

Figure 5.1 The schematic diagram for an ideal UWB impulse beamforming system.

\(^2\)Notice that the interframe coding is not considered in this section, as is usual for the beamforming problem.
where * denotes the convolution. For the case of $\beta_k(t) = \frac{1}{N} \delta(t)$ (i.e., constant and equal weighting), $\tau_k = (k - 1) \Delta \tau$ ($\Delta \tau$ is a constant), and $w(t)$ is as given in Equation (4.14), the output $y(t; \theta)$ is shown in Figure 5.2. From this figure, we can see that

- the main power of the beamformer output is focused on a certain direction;
- the focused direction can be controlled by adjusting the delays $\tau_i, i = 0, \ldots, N - 1$.

Figure 5.2  Beamformer output as a function of time and spatial angle, where $N = 8$, $d = 3.63$ cm, $\tau_p = 0.1225$ ns: (a) $\Delta \tau = 0$; (b) $\Delta \tau = 0.1052$ ns.
Now we consider the beampattern problem, which is a key issue for a beamformer. In the literature, there are three types of definitions for the beampattern [204]. These are

Type I

\[
BP_I(\theta) = \frac{\int_{-\infty}^{+\infty} |y(t; \theta)|^2 dt}{\int_{-\infty}^{+\infty} |x(t)|^2 dt}
\]  

(5.2)

Type II

\[
BP_{II}(\theta) = \max_{t \in (-\infty, +\infty)} \frac{|y(t; \theta)|^2}{\max_{t \in (-\infty, +\infty)} |x(t)|^2}
\]  

(5.3)

Type III

\[
BP_{III}(\theta) = \frac{\max_{t \in (-\infty, +\infty)} \int_{t-T/2}^{t+T/2} |y(t; \theta)|^2 dt}{\max_{t \in (-\infty, +\infty)} \int_{t-T/2}^{t+T/2} |x(t)|^2 dt}
\]  

(5.4)

In using the above definitions, a monopulse, instead of a train of monopulses, should be adopted in Equation (5.1) to avoid the trivial singularity.

For narrowband beamformers, the three kinds of definitions for the beampattern are equivalent, but for UWB beamformers, these definitions give different results. Clearly, definition (5.4) reduces to (5.2) when \(T\) approaches infinity, and definition (5.4) reduces to (5.3) when \(T\) approaches zero. Therefore, (5.4) gives a more general definition. In general, definition (5.2) is more suitable for theoretical analysis, while (5.4) is more convenient for practical beampattern calculation. For the case shown in Figure 5.2, the corresponding beampattern is illustrated in Figure 5.3.

### 5.3 The Main Lobe Beamwidth of UWB Beamformers

It is difficult to calculate the beamwidth of the main lobe of a UWB beamformer, according to either Equation (5.2), (5.3) or (5.4), because of the complexity of the UWB waveform. This is different from the narrowband case. In this section, we will give a coarse estimation for the main lobe beamwidth of the UWB beamformer.

To address this problem, we assume that \(\beta_k(t) = \delta(t), \Delta \tau = 0,\) and \(N\) is sufficiently large. Let \(T_w\) denote the duration of the UWB impulse. Let \(E_w\) be the energy of the monopulse \(w(t)\). First, consider the case that the UWB impulse signal arrives from the broadside of the array (i.e., \(\theta = 0\)). In this case, all the signals from each array branch will arrive at the same time, resulting in a coherent summing up and producing a maximum
output. Thus we have
\[ \int_{-\infty}^{+\infty} |y(t; 0)|^2 \, dt = N^2 E_w \quad \text{and} \quad \mathcal{BP}_I(\theta) = N^2. \]

Now suppose that the signal ray arrives from the direction of some angle \( \theta_{-3dB} \) such that \( N/\sqrt{2} \) pulses are contained in the time window of duration \( T_w \). Then we have
\[ \int_{-\infty}^{+\infty} |y(t; 0)|^2 \, dt \approx \frac{N^2}{2} E_w \quad \text{and} \quad \mathcal{BP}_I(\theta_{-3dB}) \approx \frac{N^2}{2}. \]

Therefore, the beamwidth of the main lobe of the beamformer, denoted as \( \theta_{bw} \), is \( 2\theta_{-3dB} \). The corresponding \( \theta_{-3dB} \) angle is related to other parameters of the beamformer as follows
\[ \frac{N}{\sqrt{2}c} \sin \theta_{-3dB} = T_w, \]
which gives
\[ \theta_{bw} = 2\theta_{-3dB} = 2 \arcsin \frac{\sqrt{2}cT_w}{Nd}. \quad (5.5) \]

For the case in Figure 5.3, if we choose \( T_w \) as the width between the two points at which \( w(t) \) is across zero (\( T_w = 0.0691 \) ns), Equation (5.5) gives \( \theta_{bw} = 11.6^\circ \), which is in quite congruity with the numerical result.
5.4 Optimal Beamforming

In the preceding section, we derived the beamwidth for the main lobe under the condition that all the branches in the array are equally weighted. In this section, we will explain the reason for making such a choice. Now let us consider the general setup of the beamformer illustrated in Figure 5.1. Assume that all the weighting functions $\beta_k(t)$ are also of short duration, i.e., $\beta_k(t) = 0$ for all $|t| \geq T_\beta$ and $k \in \{0, 1, \ldots, N - 1\}$. For the convenience of discussion, we assume that $T_\beta \leq T_w$. It is also assumed in this section that $\tau_k = 0, \forall k \in \{0, 1, \ldots, N - 1\}$ and only a monopulse is adopted in the transmitted signal, i.e., $x(t) = w(t)$. Similar to the argument in the preceding section, we can conclude that the main lobe is obtained when $\theta = 0$. Hence the strength of the main lobe is

$$BP_{ml} = BP_1(0) = \frac{1}{E_w} \int_{-\infty}^{+\infty} |y(t; 0)|^2 dt = \frac{1}{E_w} \int_{-\infty}^{+\infty} \left[ \sum_{k=0}^{N-1} w(t) * \beta_k(t) \right]^2 dt.$$  \hspace{1cm} (5.4)

For the side lobe to appear, the ray incidence angle should be wide enough such that the two pulses from two adjacent branches are separated. This happens if

$$\frac{d}{c} \sin \theta \geq T_w + T_\beta.$$  \hspace{1cm} (5.5)

Because of the equi-distance property in the array, the pulses from all the branches will be separated from each other in the time domain if the above condition holds. Thus the strength of the side lobes reads as

$$BP_{sl}(\theta) = BP_1(\theta) = \frac{1}{E_w} \int_{-\infty}^{+\infty} |y(t; \theta)|^2 dt = \frac{1}{E_w} \int_{-\infty}^{+\infty} \left[ \sum_{k=0}^{N-1} w\left(t - k\frac{d}{c} \sin \theta\right) * \beta_k(t) \right]^2 dt.$$  \hspace{1cm} (5.5)

A general design objective for a beamformer is to maximize the strength of the main lobe while minimizing the strength of the side lobes simultaneously. This objective can be mathematically formulated as

$$\left\{ \begin{array}{l} \max_{\beta_k(t), k=0, \ldots, N-1} BP_{ml} = \int_{-\infty}^{+\infty} \left[ \sum_{k=0}^{N-1} w(t) * \beta_k(t) \right]^2 dt \\
\text{such that } BP_{sl}(\theta) = \sum_{k=0}^{N-1} \int_{-\infty}^{+\infty} \left[ w\left(t - k\frac{d}{c} \sin \theta\right) * \beta_k(t) \right]^2 dt \leq 1. \end{array} \right.$$  \hspace{1cm} (5.7)

The above criterion is equivalent to maximizing the ratio $BP_{ml}/BP_{sl}(\theta)$. About this problem, we have the following proposition.

**Proposition 5.1** Suppose that the weighting filters $\beta_k(t), k = 0, \ldots, N - 1$, are of short duration. The optimal beamformer is obtained if all the weighting filters $\beta_k(t)$ are identical.
For the optimal beamformer, the amplitude of the side lobe is independent of the ray incidence angle $\theta$ if (5.6) is satisfied, and the ratio between the amplitudes of the main lobe and side lobes is

$$\frac{BP_{ml}}{BP_{s1}(\theta)} = N.$$  

**Proof.** Even though the problem is formulated in a form different from the one in [204], we can use the same idea in [204] to solve the constrained maximization problem. Let us denote by $W(f)$ and $\gamma_k(f)$, respectively, the Fourier transforms of the signals $w(t)$ and $\beta_k(t)$. Then, by the Parseval theorem, we have

$$BP_{ml} = \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)|^2 |\gamma_{k_1}(f)\gamma_{k_2}^*(f)| \, df,$$

$$BP_{s1}(\theta) = \sum_{k=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)|^2 |\gamma_k(f)|^2 \, df.$$

For the integral in (5.8), applying the Schwarz inequality yields

$$BP_{ml} = \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \int_{-\infty}^{+\infty} W(f)W^*(f)\gamma_{k_1}(f)\gamma_{k_2}^*(f) \, df,$$

$$\leq \left[ \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)\gamma_{k_1}(f)|^2 \, df \int_{-\infty}^{+\infty} |W^*(f)\gamma_{k_2}^*(f)|^2 \, df \right]^{1/2},$$

where the equality holds if and only if

$$W(f)\gamma_{k_1}(f) = c_{k_1,k_2} W(f)\gamma_{k_2}(f)$$

for almost all $f \in (-\infty, +\infty)$, with $c_{k_1,k_2}$ being a constant depending only on the indices $k_1$ and $k_2$. For the summation in (5.9), applying the Cauchy inequality yields

$$\left[ \sum_{k=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)\gamma_k(f)|^2 \, df \right]^{1/2} \leq \sum_{k=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)\gamma_k(f)|^2 \, df \sum_{k=0}^{N-1} 1,$$

$$= N \sum_{k=0}^{N-1} \int_{-\infty}^{+\infty} |W(f)\gamma_k(f)|^2 \, df,$$

where the equality holds if and only if

$$\int_{-\infty}^{+\infty} |W(f)\gamma_k(f)|^2 \, df = \varsigma,$$
where $\varsigma$ is a constant and independent of $k$. Combining Equations (5.10) and (5.12) gives $c_{k_1k_2} = 1$ for all $k_1$ and $k_2$. Therefore, the maximal strength for the main lobe is achieved if all the branches have the same weighting function. From Equation (5.12), we can see that the constraint on the side lobes is satisfied if $\Upsilon_k(f)$ is so designed such that

$$\int_{-\infty}^{+\infty} |W(f)\Upsilon_k(f)|^2 df \leq \frac{1}{N}. \quad (5.13)$$

This completes the proof. $\square$

It is easy to understand the result in Proposition 5.1. Notice that all the weighting filters $\beta_k(t)$ $(k = 0, 1, \ldots, N - 1)$ play an equal role in the optimization problem (5.7). Because of this symmetry, the solution to problem (5.7) will be that all the weighting filters are in the same form. A less obvious observation is as follows. Suppose that all the weighting filters are equal before the time instant under consideration. Now we start to increase the value of a given weighting filter. Then the increased value in this weighting filter will contribute more to the side lobes than to the main lobe of the beamformer.

### 5.5 Various Aspects of UWB Beamformers

Similar to the narrowband case, UWB beamformers can be used for both estimation of DoA of UWB rays and beam steering for UWB communications.

For beam steering, suppose that the steering angle is $\theta_0$. Then $\Delta\tau$ should be chosen such that

$$\Delta\tau = \frac{d}{c} \sin(\theta_0).$$

For DoA estimation, the procedure is quite different from the narrowband case. To address this issue, let us investigate the beamformer output in more detail. This is illustrated in Figure 5.4, where we fix the beam steering angle, which is controlled by $\Delta\tau$, and investigate the beamformer output from the viewpoint of impinging angle and time respectively. Denote now the beam steering angle as $\theta_{\text{DoA}}$, which is also called the look direction. From Figure 5.4a, we can see that the levels of the beamformer output for different impinging angles are almost identical when the steering angle deviates from the impinging angle by some amount. This kind of relationship between $\theta$ and $y$ excludes the possibility of solving $\theta$ directly from $y(t; \theta)$. From Figure 5.2b, we can see that the difference among the time-patterns for different look directions is also marginal when the look direction is not locked in some range of the impinging angle. This also weakens the possibility of solving $\theta$ from the history of $y(t; \theta)$, $t \in [t_0, t_1]$, where $t_0$ and $t_1$ are two given observation time instants.

To solve the above problem, a DoA estimation approach is shown in Figure 5.5.

In Figure 5.5, there are $M$ outputs, each of which aiming to find a direction with the help of the time delay vector $[\tau_{i,0}, \tau_{i,1}, \ldots, \tau_{i,N-1}] = \Delta\tau_i[0, 1, \ldots, N - 1]$, $i = 1, 2, \ldots, M$. Therefore, if the $i$th output is maximum among all $\{y_1, \ldots, y_M\}$, the DoA of the incidence ray will read as

$$\theta_{\text{DoA}} = \arcsin \frac{c\Delta\tau_i}{d}. $$
Figure 5.4  Beamformer output for the purpose of DoA estimation: (a) impinging angle-pattern for a fixed look direction $\theta_{\text{DoA}}$, where $t = 0$, and $\Delta \tau = 0.1047$ ns, implying $\theta_{\text{DoA}} = 60^\circ$; (b) time-pattern for four different fixed look directions $\theta_{\text{DoA}}$, where $\theta = 60^\circ$ and $\Delta \tau = 0.1047$ ns, 0.0777 ns, 0.0604 ns, and 0.0413 ns, implying $\theta_{\text{DoA}} = 60^\circ$, 40$^\circ$, 30$^\circ$, and 20$^\circ$, respectively.
The design of the difference between $\Delta \tau_i$ and $\Delta \tau_j$, $i \neq j$, depends on the resolution requirement for DoA estimation and system complexity. However, the following relationship

$$\min_{j=1,...,M, j \neq i} \left| \arcsin \frac{c \Delta \tau_i}{d} - \arcsin \frac{c \Delta \tau_j}{d} \right| \leq \theta_{bw}, \quad \forall i \in \{1, \ldots, M\}$$

should be satisfied. Otherwise, it is possible that all the outputs will produce small signals of almost equal amplitude, resulting in a failure to find the DoA.

Noticing that the delays among different branches in Figure 5.5 are fixed and well structured, the $M$ outputs in one antenna branch can be realized by one chip using an analog radio frequency-domain FIR (finite impulse response) filter as proposed in [174, 175, 228]. This technology will make the beamformer in Figure 5.5 less expensive to implement. If the delays in Figure 5.5 could be electronically adjusted as changing the phase of the signals for the case of narrowband beamformers, it would be possible to further simplify the architecture illustrated in Figure 5.5, but it seems difficult to use the current technology to electronically adjust the delay.

For the element distance, there is a limit on the maximal distance if interframe coding (which is often used in UWB systems, as illustrated in Chapter 4) is adopted in the transmitted signal. In this case, to avoid catastrophic superposition of the signals coming from different frames, the distance between two nearest elements $d$ should be chosen such that

$$d \leq cT_f.$$ 

In order to implement the beamformer mentioned in Figure 5.4, one should be able to adjust the delay between the pulse trains on the order of several tens of picoseconds. Currently, the arbitrary waveform generator (AWG) available in the market, for example,
Figure 5.6  The space–time response (a) and beam pattern (b) of the beamformer using the first derivative of the Gaussian monopulse, where $\tau_p = 0.1225$ ns, $N = 8$, $d = 3.63$ cm, and $\Delta \tau = 0$. 
Figure 5.7  The space–time response (a) and beam pattern (b) of the beamformer using the monopulse in (5.14), where $N = 8$, $d = 3.63$ cm, $\Delta \tau = 0$, $\tau_p = 0.25$ ns, and $\xi = 1.5$. 
AWG7102 by Tektronix,\(^3\) can be used to generate two pulse trains with adjustable delay between each other in 100 ps. Using an analog radio frequency-domain FIR filter proposed in [174, 175, 228] is another possibility to realize the required delay.

In UWB communication systems, other kinds of monopulses are also adopted. A popularly used form is the first derivative of the Gaussian monopulse:

\[
w(t) = c'_{\text{norm}} t \exp \left[-2\pi \left(\frac{t}{\tau_p}\right)^2\right],
\]

where the constant \(c'_{\text{norm}}\) is to normalize the peak amplitude of the pulse \(w(t)\). The space–time response and beam pattern of the beamformer are illustrated in Figure 5.6.

Another kind of less popular but interesting monopulse is proposed by Hussain [109, 110]:

\[
w(t) = \frac{1}{1 - \xi} \left\{ \exp \left[-4\pi \left(\frac{t}{\tau_p}\right)^2\right] - \exp \left[-4\pi \left(\frac{\xi t}{\tau_p}\right)^2\right] \right\}, \quad \xi \neq 1, \tag{5.14}
\]

where \(\xi\) is a scaling parameter and \(\tau_p\) is to control the spread-in-time of the monopulse, as for the case of the Gaussian monopulse. The space–time response and beam pattern of the beamformer are illustrated in Figure 5.7.

### 5.6 UWB Localization

Localization based on radio technology has been a long-standing issue, which can be traced back to the invention of radar and is still a hot research field for different applications. Recently, localization by using wireless networks or sensors has found applications in logistics, security tracking, medical services (monitoring of patients), search and rescue operations, control of home appliances, automotive safety, location-sensitive billing, military systems and so on [61, 85, 208]. In the USA, it was required by the FCC that all wireless service providers should report the accurate (within 100 m) location information of mobile stations for emergency calls from October 2001 onwards [5, 208]. In Europe, a similar requirement was also issued in 2002 [53]. Several comprehensive surveys on the localization issue in the context of UWB or narrowband radios are available [61, 85, 84, 187, 208, 208].

The main challenge for localization by using wireless networks or sensors is caused by two factors: NLOS and multipaths. Owing to the NLOS, the measured ranging information is always positively biased from the true range between the target to be localized and the sensor, and the bias is difficult to estimate. Owing to the multipaths, it is difficult to identify which path comes from the target to be localized when the target is not in LOS.

For localization based on wideband or UWB wireless technologies, the approaches can be broadly categorized into two classes [85]:

- The mapping (or fingerprinting) approach. The basic assumption of this approach is that the signal characteristics, such as the channel impulse response (CIR) or power delay

\(^3\) See the website http://www.tek.com/site/ps/0,,76-19779-INTRO_EN,00.html.
profile, as a function of the measuring location, have a one-to-one mapping relationship
with the locations of the target (often acting as a transmitter) and receiver. The position
of the target is solved from the mapping.

• The geometric or parameterized approach. In this approach, the position of the tar-
get is associated with the received-signal-strength (RSS), angle-of-arrival (AoA)\(^4\) or
the ToA of the received signal at the sensor and solved from the RSS, AoA and/or
ToA information.

It can be easily seen that, in the mapping approach, the multipaths act as a positive
role. It is the rich multipaths that make this approach possible. The more scatters the
environment has, or the richer the multipaths, the better localization accuracy the system
can offer. The NLOS is not an issue in this approach, whereas the multipaths and NLOS
generally play a negative role with the geometric or parameterized approach.

It has been proved [195] (see also [61, 85, 84]) that in the ToA-based geometric
approach, the localization accuracy is generally inversely proportional to the effective (or
root-mean-square) signal bandwidth defined by

\[
\bar{B} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 \, df}{\int_{-\infty}^{\infty} |S(f)|^2 \, df}},
\]

where \(S(f)\) is the Fourier transform of the transmitted signal.

Considering the huge bandwidth of UWB signals and the induced rich multipaths, we
use UWB wireless systems to achieve extremely high localization accuracy either in
the mapping approach or in the geometric approach.

There have been several studies on the localization problem by using UWB systems.
In [33], the Cramer–Rao bound (CRB) for the ToA estimate was obtained. In [157], the
CRBs for the AoA estimate and for the hybrid method using both AoA and ToA infor-
mation were derived. In [211], a method for the detection of NLOS was proposed, where
the key idea is to make use of the sudden change information in the received SNR which
will likely appear when the transmitter goes from LOS to NLOS or vice versa. Obviously,
the method only applies to the localization of moving targets. In [68, 213], the so-called
BeamLoc approach was used to solve the localization problem under general scenarios,
including both LOS and NLOS cases. In [139], a generalized maximum-likelihood esti-
mation algorithm for the ToA in dense multipath environments was presented. In [233],
the ToA estimation by using the low-complexity noncoherent energy-collection receiver
was examined through experimental results. In [86], a two-step searching policy was
proposed to solve the complexity issue in the ToA estimation of UWB systems caused
by rich multipaths. In [16], the power delay profile of UWB channels was exploited to
obtain a coarse estimate of the transmitter. In [68], the received signal strength (RSS)
information and the characteristic of electric field polarization were exploited to tackle
the NLOS problem. In [140], the effect of multiple users on the UWB ranging accuracy
was analysed.

Localization methods based on the RSS in UWB systems are largely ignored. Even
though the RSS is easy to measure, it provides less accurate positioning [187]. On the

\(^4\) In the literature, AoA is also often referred to as DoA. In this book, we use both terms interchangeably.
other hand, it needs accurate channel models for the RSS approach [218, 217, 219, 221, 220]. For the UWB indoor wireless localization problem, these models are difficult to obtain accurately, since the power decay exponent changes dramatically with the scattering environments.

In the subsequent sections of this chapter, we will discuss the corresponding techniques by using UWB technology. Some measures for combating the NLOS and multipaths will be also briefly summarized.

In this chapter, we confine our focus to the localization in the two-dimensional space. It is not difficult to extend the illustrated approaches to the case of the three-dimensional space.

For the localization problem in wireless networks or sensors, it is often assumed that there are some fixed elements, which may be the base stations or sensors, in the infrastructure and the target to be located is often equipped with a transmitter, which is different from the conventional radar ranging problem. Hence, the target to be located is referred to as a tag [34] in the rest of this chapter, to show the subtle difference between the localization problem considered here and the ranging problem in the conventional radar engineering.

5.6.1 Beamforming Approach

The basic idea in the beamforming approach is to use two beamformers to find the position of a tag, as shown in Figure 5.8. It can be clearly seen from this figure that the position of the tag is uniquely decided by the rays from each of the two directions. The positioning accuracy is determined by the main lobe beamwidth of the beamformer.

If the ToA can also be obtained in the beamformers, the information can be integrated together with the AoA information to increase the positioning accuracy. In the following, we discuss some details about how to solve the tag position by using relevant information.

![Figure 5.8](image)

**Figure 5.8** Localization via the AoA measurement. The dark nodes are the UWB beamformers and the light node is the tag.
Let the coordinates of the two beamformers be \((x_i, y_i), i = 1, 2\), the coordinate of the tag be \((x, y)\), the AoA of the signals received at the two beamformers be \(\theta_1\) and \(\theta_2\) respectively, and the ToA of the signals received at the two beamformers be \(t_1\) and \(t_2\) respectively. Then we have

\[
\frac{y - y_1}{x - x_1} = \tan(\theta_1),
\]

\[
\frac{y - y_2}{x - x_2} = \tan(\theta_2),
\]

\[
r_1^2 = (x - x_1)^2 + (y - y_1)^2 = [c(t_1 - t^0)]^2,
\]

\[
r_2^2 = (x - x_2)^2 + (y - y_2)^2 = [c(t_2 - t^0)]^2,
\]

where \(c\) is the speed of light \((c = 3 \times 10^8 \text{ m/s})\), \(t^0\) is the time instant at which the tag begins transmission and \(r_i\) is the distance between the tag and the \(i\)th beamformer.

If there is only AoA information available, the unknown position \((x, y)\) can be solved from Equations (5.15) and (5.16). Rewrite both equations in the following matrix form:

\[
M_1 p = b_1,
\]

where

\[
p = \begin{bmatrix} x \\ y \end{bmatrix}, \quad M_1 = \begin{bmatrix} \sin(\theta_1) & -\cos(\theta_1) \\ \sin(\theta_2) & -\cos(\theta_2) \end{bmatrix}, \quad b_1 = \begin{bmatrix} x_1 \sin(\theta_1) - y_1 \cos(\theta_1) \\ x_2 \sin(\theta_2) - y_2 \cos(\theta_2) \end{bmatrix}.
\]

The solution is then

\[
p = M_1^{-1} b_1.
\]

It is clear that the position \(p\) can be uniquely decided if and only if \(\theta_1 - \theta_2 \neq k\pi\), where \(k\) is an integer.

If the ToA information is also available, Equations (5.17) and (5.18) can be combined with Equations (5.15) and (5.16) to increase the position estimation accuracy. The first approach is to directly solve a nonlinear least-squares (LS) problem based on Equations (5.15)–(5.18), which is not easy. Another major drawback of this approach is that the drift in the tag clock (the unknown bias in \(t^0\)) will make indoor wireless localization nonsense, since the clock of a mobile device might have a drift that can reach a few microseconds, which will cause a distance error of several hundred metres. The second approach is to use the time-difference-of-arrival (TDoA) information between two beamformers. In the following we will present the solution to the problem following the method as outlined in [208]. Define

\[
d_{21} := r_2 - r_1 = c(t_2 - t^0) - c(t_1 - t^0) = c(t_2 - t_1).
\]

From Equations (5.17) and (5.18), we have

\[
r_2^2 = (d_{21} + r_1)^2 = (x - x_1 + x_1 - x_2)^2 + (y - y_1 + y_1 - y_2)^2
\]

\[
= -2(x_2 - x_1)x + 2(y_2 - y_1)y + r_1^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2.
\]
Expanding the above equation yields

\[(x_2 - x_1)x + (y_2 - y_1)y = -d_{21}r_1 - \frac{1}{2}(d_{21}^2 + x_2^2 - x_1^2 + y_2^2 - y_1^2). \tag{5.21}\]

Combining Equations (5.19) and (5.21) gives

\[M_2p = b_3r_1 + b_2, \tag{5.22}\]

where

\[
M_2 = \begin{bmatrix}
\sin(\theta_1) & -\cos(\theta_1) \\
\sin(\theta_2) & -\cos(\theta_2) \\
x_2 - x_1 & y_2 - y_1
\end{bmatrix},
\tag{5.23}\]

\[
b_2 = \begin{bmatrix}
x_1 \sin(\theta_1) - y_1 \cos(\theta_1) \\
x_2 \sin(\theta_2) - y_2 \cos(\theta_2) \\
\frac{1}{2}(x_2^2 - x_1^2 + y_2^2 - y_1^2 - d_{21}^2)
\end{bmatrix},
b_3 = \begin{bmatrix}
0 \\
0 \\
-d_{21}
\end{bmatrix}. \tag{5.24}\]

Equation (5.22) is overdetermined. The LS intermediate solution for Equation (5.22) is

\[\hat{p} = (M_2^TM_2)^{-1}M_2^T(b_3r_1 + b_2). \tag{5.25}\]

Substituting the solved \(\hat{p}\) into

\[r_1^2 = (x - x_1)^2 + (y - y_1)^2 \tag{5.26}\]

leads to a quadratic equation in \(r_1\). Solving for \(r_1\) and substituting the positive root back into Equation (5.25) gives the final solution for \(p\).

A more accurate estimate \(\hat{p}\) than that provided by Equation (5.25) can be theoretically obtained if the statistical information about the measurement noises contained in \(\theta_i\) \((i = 1, 2)\) and \(d_{21}\) is available. In this case, the maximum likelihood estimation (MLE) method can be exploited to estimate \(p\). However, it can be seen from Equations (5.23) and (5.24) that the nonlinear transforms are performed upon the measurement noises in the final estimation model (5.22), which will make it very difficult to obtain the MLE estimate of \(p\).

### 5.6.2 More Than Two Beamformers

The results in the preceding subsection can be easily extended to the case of more than two beamformers. Suppose that there are now \(M(M \geq 3)\) UWB beamformers which can measure the AoA and ToA of the tag. Let the coordinates of the \(M\) beamformers be \((x_i, y_i)\), the AoA of the signals received at the \(i\)th beamformer be \(\theta_i\), and the ToA of the signals received at the \(i\)th beamformer be \(t_i\), \(i = 1, 2, \ldots, M\). Similarly, these variables are related to each other according to the following equations:

\[
\frac{y - y_i}{x - x_i} = \tan(\theta_i),
\]

\[r_i^2 = (x - x_i)^2 + (y - y_i)^2 = [c(t_i - t^0)]^2, \quad i = 1, 2, \ldots, M.
\]
In the case where the available information is only the AoA, the measurement model can be written as the following matrix form:

\[ \mathbf{M}_3 \mathbf{p} = \mathbf{b}_4, \quad (5.27) \]

where

\[
\mathbf{M}_3 = \begin{bmatrix}
\sin(\theta_1) & -\cos(\theta_1) \\
\sin(\theta_2) & -\cos(\theta_2) \\
\vdots & \vdots \\
\sin(\theta_M) & -\cos(\theta_M)
\end{bmatrix}, \quad \mathbf{b}_4 = \begin{bmatrix}
x_1 \sin(\theta_1) - y_1 \cos(\theta_1) \\
x_2 \sin(\theta_2) - y_2 \cos(\theta_2) \\
\vdots \\
x_M \sin(\theta_M) - y_M \cos(\theta_M)
\end{bmatrix}.
\]

Equation (5.27) is overdetermined. Its LS solution is given by

\[
\hat{\mathbf{p}} = (\mathbf{M}_3^T \mathbf{M}_3)^{-1} \mathbf{M}_3^T \mathbf{b}_4. \quad (5.28)
\]

In the case where the information about the ToA, besides the AoA, is also available, we can proceed as follows. Define

\[
d_{i1} := r_i - r_1 = c(t_i - t^0) - c(t_1 - t^0) = c(t_i - t_1), \quad i = 2, \ldots, M.
\]

Then following the same procedure as the preceding subsection, we have

\[
(x_i - x_1)x + (y_i - y_1)y = -d_{i1}r_1 - \frac{1}{2}(d_{i1}^2 + x_i^2 - x_1^2 + y_i^2 - y_1^2). \quad (5.29)
\]

Combining Equations (5.27) and (5.29) gives

\[ \mathbf{M}_4 \mathbf{p} = \mathbf{b}_5 r_1 + \mathbf{b}_6, \quad (5.30) \]

where

\[
\mathbf{M}_4 = \begin{bmatrix}
\sin(\theta_1) & -\cos(\theta_1) \\
\sin(\theta_2) & -\cos(\theta_2) \\
\vdots & \vdots \\
\sin(\theta_M) & -\cos(\theta_M) \\
x_2 - x_1 & y_2 - y_1 \\
\vdots & \vdots \\
x_M - x_1 & y_M - y_1
\end{bmatrix}, \quad \mathbf{b}_5 = \begin{bmatrix}
x_1 \sin(\theta_1) - y_1 \cos(\theta_1) \\
x_2 \sin(\theta_2) - y_2 \cos(\theta_2) \\
\vdots \\
x_M \sin(\theta_M) - y_M \cos(\theta_M) \\
\frac{1}{2}(x_2^2 - x_1^2 + y_2^2 - y_1^2 - d_{21}^2) \\
\vdots \\
\frac{1}{2}(x_M^2 - x_1^2 + y_M^2 - y_1^2 - d_{M1}^2)
\end{bmatrix}, \quad \mathbf{b}_6 = \begin{bmatrix}
0 \\
0 \\
\vdots \\
-d_{21} \\
\vdots \\
-d_{M1}
\end{bmatrix}.
\]
The LS intermediate solution for Equation (5.30) is
\[ \hat{p} = (M_4^T M_4)^{-1} M_4^T (b_5 r_1 + b_6). \] (5.31)
Substituting the solved \( \hat{p} \) into Equation (5.26) leads to a quadratic equation in \( r_1 \). Solving for \( r_1 \) and substituting the positive root back into Equation (5.31) gives the final solution for \( p \).

The approach outlined above can be easily extended to the case where only the ToA information is available. For completeness, we repeat the solution as follows. The equivalent measurement equation is
\[ M_5 p = b_7 r_1 + b_8, \] (5.32)
where
\[
M_5 = \begin{bmatrix}
x_2 - x_1 & y_2 - y_1 \\
x_3 - x_1 & y_3 - y_1 \\
\vdots & \vdots \\
x_M - x_1 & y_M - y_1
\end{bmatrix},
\]
\[
b_7 = \begin{bmatrix}
\frac{1}{2} (x_2^2 - x_1^2 + y_2^2 - y_1^2 - d_{21}^2) \\
\frac{1}{2} (x_3^2 - x_1^2 + y_3^2 - y_1^2 - d_{31}^2) \\
\vdots \\
\frac{1}{2} (x_M^2 - x_1^2 + y_M^2 - y_1^2 - d_{M1}^2)
\end{bmatrix},
b_8 = \begin{bmatrix}
-d_{21} \\
-d_{31} \\
\vdots \\
-d_{M1}
\end{bmatrix}.
\]
The LS intermediate solution for Equation (5.32) is
\[ \hat{p} = (M_5^T M_5)^{-1} M_5^T (b_7 r_1 + b_8). \] (5.33)
Substituting the solved \( \hat{p} \) into Equation (5.26) leads to a quadratic equation in \( r_1 \). Solving for \( r_1 \) and substituting the positive root back into Equation (5.33) gives the final solution for \( p \).

5.6.3 The BeamLoc Approach

The BeamLoc approach was first proposed in [213]. In the BeamLoc approach, both the transmitter (tag) and the receiver (sensor) are equipped with UWB beamformers. When the angle-of-departure (AoD) of the transmitted signal at the tag coincides with the steering direction of the beamformer of the receiver, the receiver produces a maximal output signal among all the signals received at different steering directions. Thus, the direction of the tag can be determined. Since the beamformer is deployed at both sides, a high resolution in the signal direction finding can be achieved.

In the following, the simulation results in [213] will be presented.

The simulation scenario is shown in Figure 5.9. Three rooms with a size of 5 m × 7 m are considered. A receiver (RX) equipped with a UWB beamformer is located in the centre of each of the two rooms, and the transmitter (TX), also equipped with a UWB beamformer, is randomly located in the third room. Fifty arbitrary TX positions are
The numbers of antennas in all three of the beamformers are chosen to be equal and are denoted as \( N \). The uniform linear arrays are used with an equal array size \( D \) for all three beamformers. The TX beamformer changes its AoD with a step size in angle \( \Delta \theta \). The array size will be multiples of \( \lambda_c = 4.4 \text{ cm} \), where \( \lambda_c \) is the wavelength of a sinusoidal wave whose frequency equals to \( f_c = 6.85 \text{ GHz} \), the centre frequency of general UWB signals.

According to the different resources which are used to estimate the TX position, different kinds of average positioning errors are obtained, as shown in Table 5.1.

The simulation results are illustrated in Figures 5.10–5.13.

From these figures, the following conclusions can be drawn:

- The average positioning error decreases with the number of antennas used in the beamformers. This is because the undesired multipath components (due to the reflected paths) are better suppressed because of a reduced side-lobe level of the beampattern when the number of antennas is larger.
### Table 5.1 Possible combinations of positioning resources.

<table>
<thead>
<tr>
<th>Types of resource</th>
<th>AoA of RX1</th>
<th>ToA of RX1</th>
<th>AoA of RX2</th>
<th>ToA of RX2</th>
<th>Notation of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td>$\varepsilon_{\text{AoA}}$</td>
</tr>
<tr>
<td>RX1</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>$\varepsilon_1$</td>
</tr>
<tr>
<td>RX2</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td>$\varepsilon_2$</td>
</tr>
</tbody>
</table>

![Graph](image)

**Figure 5.10** Average positioning error: comparison between $D = 2\lambda_c$ (solid line) and $D = 10\lambda_c$ (dotted line). $N = 8$, $\tau_p = 0.5$ ns and $\Delta\theta = 1^\circ$. Legend: $\varepsilon_1(\ast)$, $\varepsilon_2(\Diamond)$ and $\varepsilon_{\text{AoA}}(\circ)$. (From [213]. Reproduced by permission of © IEEE 2006.)

- The average positioning error decreases when the pulse width decreases or the array size increases. This is because the direct path can be better filtered as the main lobe beamwidth is correspondingly reduced, as seen from Equation (5.5).
- The average positioning error decreases with the step size $\Delta\theta$; this is obvious, since a smaller step size provides a higher spatial resolution.
- The above performance difference for different parameters used is more conspicuous when the SNR is lower.
- When the SNR is high, the AoA-related approach outperforms the approach using a single receiver.

### 5.7 NLOS Issue

The NLOS issue has been extensively studied in the localization problem for narrowband wireless systems. The main studies are focused on two problems: (1) how to detect the NLOS and (2) how to correct the error caused by the NLOS.
Figure 5.11  Average positioning error: comparison between $\tau_p = 0.5$ ns (solid line) and $\tau_p = 1.8$ ns (dotted line). $N = 8$, $D = 2\lambda_c$ and $\Delta\theta = 1^\circ$. Legend: $\varepsilon_1 (*)$, $\varepsilon_2 (\diamond)$ and $\varepsilon_{\text{AoA}} (\circ)$. (From [213]. Reproduced by permission of © IEEE 2006.)

Figure 5.12  Average positioning error: comparison between $N = 8$ (solid line) and $N = 4$ (dotted line). $\tau_p = 0.5$ ns, $D = 2\lambda_c$ and $\Delta\theta = 1^\circ$. Legend: $\varepsilon_1 (*)$, $\varepsilon_2 (\diamond)$ and $\varepsilon_{\text{AoA}} (\circ)$. (From [213]. Reproduced by permission of © IEEE 2006.)

An early account for the NLOS issue was given by Wylie and Holtzman [269], where the history of the standard deviation of ranging measurements from different receivers is compared. If some receivers show a consistently large standard deviation of measurements compared with other receivers, then an NLOS for the former receivers can be reliably
declared and the ranging bias can be found from the history of the standard deviation of measurements for those receivers. Clearly, a prerequisite for this approach is that there are at least some LOS measurements and the accuracies of all the measurements are roughly in the same range. The advantage of this approach is that the probability of false alarm of an NLOS is low and the resultant bias estimate is accurate. The drawback is that it needs a lot of measurements for one spot.

In [34], a robust estimator was proposed to deal with the NLOS, where the least-median-of-squares technique is used to detect and reject NLOS measurements and a minimum number of necessary measurements are selected to obtain the estimate of the tag position. In [38], a residual test was proposed to detect the NLOS and the NLOS-related measurements were removed from the localization purpose. In [54], the residual ranking algorithm for NLOS detection, first proposed in [42], was modified by considering the asymmetric property of the distribution of the NLOS error. The NLOS error mitigation algorithms for three typical cases, namely the cases with known NLOS statistics, with limited a priori information, and with no knowledge of the NLOS error, are well documented in [54].

In [138], Le et al. proposed increasing the variance corresponding to the NLOS branch of the Kalman filter, thus reducing the weighting of NLOS-related measurements, to mitigate the NLOS effect on the positioning error. In [145], two parallel Kalman filters were employed at each base station for range estimation. The two Kalman filters were configured in correspondence with the two switching modes, i.e., LOS mode and NLOS mode, with the switching between the two modes being further governed by a Markov process. The final estimate of the range was obtained by smoothing the outputs of the two Kalman filters. While to achieve robustness is the main purpose of the approach, the robustness itself becomes an issue of the approach, since the models involved contain too many parameters to be tuned and the performance and convergence of the Kalman filter

Figure 5.13  Average positioning error: comparison between $\Delta \theta = 1^\circ$ (solid line) and $\Delta \theta = 5^\circ$ (dotted line). $N = 8$, $D = 2\lambda_c$ and $\tau_p = 0.5$ ns. Legend: $\varepsilon_1 (\ast)$, $\varepsilon_2 (\diamond)$ and $\varepsilon_{\text{AoA}} (\circ)$. (From [213]. Reproduced by permission of © IEEE 2006.)
itself are very sensitive to model parameters [158, 205]. In [208, 255], knowledge about
the geometry between transmitters and receivers was incorporated into an optimization
problem for solving the ToA from measurement data, and the NLOS bias can be corre-
spondingly suppressed. Although the NLOS error can be mitigated somehow by using this
approach, it remains unclear about how much the localization accuracy can be improved.
In [13], the multipath scattering model was exploited to reduce the NLOS error.

Basically, the measures exploited to deal with the NLOS issue in narrowband wireless
systems also apply to UWB systems. In the following, we will introduce the NLOS
detection algorithm proposed in [42] and its modified version in [54].

Generally, the measurement equation can be expressed as

$$r_i = f(p) + n_i + e_i, \quad i = 1, \ldots, N,$$

(5.34)

where $r_i$ is the measured data of the $i$th sensor, $p$ is the position vector of the tag, $f$ is
a function which relates the position of the tag to the measured data, $n_i$ is the ranging
noise (which is the noise after the data processing unit, derived from the RF front-end
noise), $e_i$ is the bias caused by the NLOS and $N$ ($N \geq 3$) is the number of sensors. The
measured data can be either the AoA or the ToA.

If $r_i$ is the AoA, then

$$f(p) = \arctan \frac{y - y_i}{x - x_i}.$$ 

If $r_i$ is the ToA (suppose $r_i = c(t_i - t^0)$), then

$$f(p) = \sqrt{(x - x_i)^2 + (y - y_i)^2}.$$ 

If $r_i$ is the TDoA (suppose $r_i = c(t_i - t_1)$ with $i \geq 2$), then

$$f(p) = \sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2}.$$ 

In the case of LOS measurement, $e_i = 0$. In the case of NLOS measurement, $e_i > 0$
and it typically holds true that $e_i \gg n_i$ almost surely. It is the latter property that is used
to detect NLOS or mitigate the effect of NLOS.

If the NLOS issue is not taking into account, then the LS estimate of the position $p$ is

$$\hat{p} = \arg \min_p \sum_{i=1}^{N} (r_i - f(p))^2.$$ (5.35)

Equation (5.35) can be solved by using Equation (5.25), (5.28), (5.31), or (5.33), according
to the corresponding situations as presented in Sections 5.6.1 and 5.6.2.

For the purpose of removing/mitigating NLOS errors, we can select a subset of the $N$
measurements to formulate a new position estimate $\hat{p}$. Let $S_k$, $k = 1, \ldots, K$, be all the
possible subsets of $N$ measurements which can be used to decide the position $p$, where

$$K = \sum_{i=3}^{N} \binom{N}{i}.$$
For a given subset $S_k$ and a given position $p$, define the related residual squares as
\[ R(p, S_k) = \sum_{i \in S_k} (r_i - f(p))^2. \] (5.36)

Now calculate the LS estimate of $p$ based on Equation (5.36) as follows (denoted as $\hat{p}_k$):
\[ \hat{p}_k = \arg \min_p R(p, S_k), \]
and the corresponding residue (normalized by the size of the subset used):
\[ \tilde{R}(\hat{p}_k, S_k) = \frac{1}{|S_k|} \sum_{i \in S_k} (r_i - f(\hat{p}_k))^2, \] (5.37)

where $|S_k|$ stands for the size of $S_k$, i.e., the number of measurements contained in the subset $S_k$.

The final estimate of $p$ is given by the weighted linear combination of the above intermediate estimate with the weights being inversely proportional to the corresponding residue $\tilde{R}(\hat{p}_k, S_k)$ [42]:
\[ \hat{p} = \frac{\sum_{k=1}^{K} \left[ \tilde{R}(\hat{p}_k, S_k) \right]^{-1} \hat{p}_k}{\sum_{k=1}^{K} \left[ \tilde{R}(\hat{p}_k, S_k) \right]^{-1}}. \]

An immediate modification of the above algorithm is to remove directly those subsets whose residues, as calculated in Equation (5.37), are too large compared with the residues of other subsets.

Simulation results in [42] show that substantial performance improvement by using the above algorithm has been achieved compared with the traditional LS estimation approach even when the NLOS measurements are not distinguishable. If the standard deviation of the measurement noise is small, the above algorithm performs so well as if only LOS measurements were used.

The prerequisite for this approach, similar to the approach in [269], is that there are at least three LOS measurements. The advantage of this approach is that there are no requirements on any a priori information about the scenario. The drawback is its computation burden. When the number $N$ is large, the computation burden is very heavy since the LS estimation algorithm should be performed for all possible $K$ subsets.

A possible remedy for the above drawback is first to remove some NLOS measurements by using a modified residual algorithm, as shown in [54].

As argued in [54], the residue defined in Equation (5.36) has three limitations. First, the information about the variance of measurement noises, which is often known, is not exploited. Second, the property that the residue is not symmetric with respect to the origin is not considered. Since $e_i \geq 0$, it is less likely that the $i$th measurement belongs to an NLOS if $r_i - f(p) < 0$ considering that the measurement noise is relatively small, while the square operation in Equation (5.36) removes the information indicated by the
sign of $r_i - f(p)$. Finally, the summation form in Equation (5.36) can only indicate how likely a group of measurements contains NLOS, but not which one(s). Considering the aforementioned limitations, [54] modified the residue as the following:

$$R(r_i, p) = 1 - Q\left(\frac{r_i - f(p)}{\sigma_i}\right), \quad (5.38)$$

where $\sigma_i^2$ is the variance of the $i$th measurement noise $n_i$, and $Q(\cdot)$ denotes the $Q$-function.

For a fixed $p$, $R(r_i, p)$ is a monotonously increasing function of $r_i$. Let $p$ be the true position of the tag. If $r_i = f(p) - 3\sigma_i$, which indicates that this measurement most likely belongs to a LOS, then we have $R(r_i, p) = 0.0013$. On the other hand, if $r_i = f(p) + 3\sigma_i$, which indicates that this measurement most likely belongs to an NLOS, then we have $R(r_i, p) = 0.9987$. Therefore, the function $R(r_i, p)$ provides a good measure about if a measurement belongs to an NLOS. The larger the residue $R(r_i, p)$, the more likely the measurement belongs to an NLOS.

To proceed with the above modification, one needs to find an initial estimate of the position $p$. This can be done by using all the measurements.

According to the above idea, the following localization algorithm was proposed in [54].

**Algorithm 5.1 Residue-based localization algorithm.** Initial data: given all measurement data $\{r_i, i = 1, \ldots, N\}$ and a threshold $\lambda$.

**Step 1** Calculate an initial estimate of the position $p$, denoted as $\hat{p}$.

**Step 2** Calculate the residues $R(r_i, \hat{p})$ according to Equation (5.38).

**Step 3** Remove those measurement data whose residues are larger than the predefined threshold $\lambda$, and use the remaining measurement data to calculate the new position estimate.

**Step 4** If the sum of the residues based on the new position estimate is smaller than the sum of the residues based on the previously obtained position estimate, then stop. Otherwise increase the threshold $\lambda$ to admit more measurement data and go to Step 2.

The design of parameter $\lambda$ is a critical issue for Algorithm 5.1, which affects the false alarm probability about NLOS, the convergence speed of the algorithm and the position estimation accuracy.

### 5.7.1 NLOS Mitigation Based on a Priori Information

If a priori information about LOS/NLOS is available, then we can also make use of the information to mitigate NLOS errors.

The information can be statistical or deterministic (geometric). The statistical a priori information includes the a priori probability of the tag at the LOS or NLOS position, the distribution of NLOS errors and the distribution of measurement noises. It is common to assume that the ranging noises are Gaussian distributed,\(^5\) which is valid when the SNR is high enough.

\(^5\) When discussing the ranging noise, it would be helpful to differentiate it from the receiver noise. The latter refers to the noise at the RF front end and is widely accepted as Gaussian distributed, while the former refers to the noise in the signal processing chain.
is high. It is also reasonable to assume that the NLOS error $e_i$ and ranging noise $n_i$ are independent of each other; $e_i$ and $e_j$ are independent of each other for any different $i$ and $j$, and so are $n_i$ and $n_j$.

Let $p_{\text{LOS}}$ be the probability that the tag is located at the LOS of the $i$th sensor. Denote by $p_{n,i}$ and $p_{e,i}$ the probability density functions of $e_i$ and $n_i$ respectively. Let

$$p_{en,i} = p_{e,i} \ast p_{n,i},$$

where $\ast$ stands for the convolution of the two functions involved. Define the measurement vector $\mathbf{r}$ as $\mathbf{r} = [r_1 r_2 \cdots r_N]^T$. Then the conditional probability density function of $\mathbf{r}$, conditioned on a given position $\mathbf{p}$, is

$$p_{\mathbf{r}}(\mathbf{r} | \mathbf{p}) = \prod_{i=1}^{N} \left[ p_{n,i}(r_i - f(\mathbf{p})) p_{\text{LOS}}(r_i - f(\mathbf{p}))(1 - p_{\text{LOS}}) \right].$$

(5.39)

Based on Equation (5.39), a maximum likelihood (ML) estimate of parameter $\mathbf{p}$ can be obtained, that is:

$$\hat{\mathbf{p}} = \arg \max_{\mathbf{p}} p_{\mathbf{r}}(\mathbf{r} | \mathbf{p}).$$

Various kinds of numerical methods [122, 235] can be resorted to to solve the ML estimate of parameter $\mathbf{p}$ based on Equation (5.39). An important issue in finding the numerical solution of the ML estimate of $\mathbf{p}$ is the local maxima problem, as shown in [54, Figures 2 and 3].

The deterministic (geometric) information includes the information about the infrastructure of the localization environment. In principle, if the detailed infrastructure about the environment is available, the position of the tag can be uniquely decided from the received signals at several places or even the received signal at a single place.

As an illustration of this approach, we consider the localization problem for the scenario shown in Figure 5.14. The tag and receiver are located in two rooms that are separated by a wall and connected via a door. For the transmitted UWB monopulse from the tag at time $t = 0$, suppose three spectacular rays are received at the receiver, with the ToAs being $t_1$, $t_2$ and $t_3$, which correspond to the rays transmitted through the wall, diffracted by the door and reflected by the wall respectively.

From the room geometry, we have

$$\begin{align*}
\sqrt{(x - x_1) + (y - y_1)} &= ct_1, \\
\sqrt{(x - x_2) + (y - y_2)} + \sqrt{(x_1 - x_2) + (y_1 - y_2)} &= ct_2, \\
\sqrt{(x - x_3) + (y - y_3)} + \sqrt{(x_1 - x_3) + (y_1 - y_3)} &= ct_3, \\
\frac{y-y_3}{x-x_3} &= \frac{y_1-y_3}{x_1-x_3}.
\end{align*}$$

(5.40)

to the noise exhibited in Equation (5.34) at the ranging data processing unit, which is derived from the receiver noise. Therefore, the distribution property of the ranging noise can be rigorously derived from that of the receiver noise. In Appendix 5.A we present a way to find the exact distribution of the ranging noises. Generally, it is not Gaussian, since the overall effect of the noise and NLOS bias should be in the range where the measured distance should be of a positive value. Therefore, a truncated Gaussian distribution might give a good approximation for the distribution of the ranging noise. However, to maintain the clarity of argument, the noise $n_i$ in Equation (5.34) is generally modelled as Gaussian distributed in the literature.
In the first equation, we have neglected the difference between the travel times of the rays propagating through the wall and through free space.

In Equation (5.40), there are four equalities and four unknown variables \( x, y, x_3 \) and \( y_3 \). Therefore, the solution for these four unknown variables can be obtained from Equation (5.40) in general cases.

If more rays can be identifiable, then the corresponding ToA information can be incorporated into Equation (5.40) to improve the position estimation accuracy.

The critical issue for this approach is that the rays coming from different ways should be identifiable and the mapping between the rays and the ToA is known. Thanks to the extremely narrow pulse width of the UWB monocycle, the ToAs \( t_1, t_2, t_3 \) etc. for different rays are identifiable. However, finding the aforementioned mapping is not an easy task.

The advantage of the approaches discussed in this subsection is that, to obtain the location estimate, it is sufficient to use only a single sensor. Therefore, the cost in hardware can be minimized compared with the approaches discussed in the preceding sections. The drawback of the two approaches presented in this subsection is that it is not easy to get the
required a priori information accurately; and how often to update the a priori information is also an issue. Basically, we can rely on either the resource of hardware (by increasing the number of sensors) or the resource of a priori information on the environment, or both, to reduce the NLOS errors. Without these resources, we can do little about the NLOS errors.

5.8 Multipath Issue

Essentially, the first path of the received multipath signal is considered as the ToA if the tag is at the LOS of the sensor. Therefore, multipaths will not affect the localization accuracy if the first path can be identified correctly. This observation is indirectly confirmed by the results of [195], where it is theoretically proved that the CRB of a wireless localization system is determined only by the LOS part of the measurements. But due to channel fading, the first path might not be the strongest one. If the amplitude of the first path falls below the detection threshold, then a ToA error will occur, which will lead to a localization error. Figure 5.15 shows such a case for a UWB channel. In this case, the missed detection of the first (direct) path leads to a distance error of about 6 m.

Note that the event for the undetected direct path does not happen accidentally. It occurs frequently in indoor UWB channels.

Therefore, first-path detection algorithms are very important for accurately estimating ToA. Several algorithms have been proposed; see [86, 99, 139, 275] for UWB systems and [98, 163] for wideband systems. Generally, the existence of multipaths in wideband

![Figure 5.15](image)

**Figure 5.15** An illustration of the ranging error caused by missed detection of the first (direct) path in a multipath environment.
channels is detrimental for the localization problem, but if the channel contains rich multipaths, such as UWB channels, then it also contains rich information which can be exploited to find the location of the transmitter. In the extreme case, if the timing information of each path of a UWB channel is exploited as shown in Section 5.7.1, where the timing information for only three taps is used for illustrative purposes, then the position of the transmitter can be accurately estimated by using even a single UWB sensor. In the following, we will consider an approach where both the timing information and amplitude information of all multipaths of a UWB channel are exploited to solve the localization problem.

Let us fix the sensor to be somewhere in a Cartesian coordinate system. Suppose the position of the transmitter (tag) is \((x, y)\) in this coordinate system. First, consider that both the sensor and transmitter are each equipped with a single antenna. We consider uniform sampling for the CIR. Hence, the CIR \(h(t) = \sum_{l=1}^{L} h_l \delta[t - (l - 1)\Delta \tau]\) can be equivalently described by a vector

\[
\mathbf{h} = \begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_L \\
\end{bmatrix},
\]

(5.41)

where \(h_l\) is the amplitude fading at the \(l\)th tap and \(\Delta \tau\) is the sampling interval. To indicate that \(h(t)\) depends on the position of the tag, we write \(\mathbf{h}\) as \(\mathbf{h}(\mathbf{p})\) instead, where \(\mathbf{p} = [x \ y]^T\).

In the following approach, the environment under consideration is first divided into sufficiently small grids. Each grid is represented by the coordinate, say \(\mathbf{p}_i = [x_i \ y_i]^T\), of the centre of that grid in the Cartesian coordinate system. For each grid \(\mathbf{p}_i\), define its \(\varepsilon\)-neighbour, denoted as \(B_\varepsilon(\mathbf{p}_i)\), as follows:

\[
B_\varepsilon(\mathbf{p}_i) = \{\mathbf{p}_j \in \mathcal{D} \subset \mathbb{R}^2 : \|\mathbf{p}_j - \mathbf{p}_i\| \leq \varepsilon\},
\]

where \(\mathcal{D}\) denotes the domain under consideration, which is a subset of the two-dimensional real space \(\mathbb{R}^2\), \(\|\cdot\|\) is any norm in \(\mathbb{R}^2\) and \(\varepsilon\) is a given positive number.

For any two vectors \(\mathbf{v}_1\) and \(\mathbf{v}_2\) of the same dimension, define their correlation coefficient as

\[
\rho(\mathbf{v}_1, \mathbf{v}_2) = \frac{\mathbf{v}_1^T \mathbf{v}_2}{\sqrt{\mathbf{v}_1^T \mathbf{v}_1} \sqrt{\mathbf{v}_2^T \mathbf{v}_2}}.
\]

Before the start of the algorithm, we suppose that the CIRs \(\{\mathbf{h}(\mathbf{p}_i) : \mathbf{p}_i \in \mathcal{D}\}\) for all the grids are available. They are pre-measured and stored in the database.

The location estimating method is detailed in the following algorithm.

**Algorithm 5.2 CIR-based localization algorithm.** Initial data: CIR database \(\{\mathbf{h}(\mathbf{p}_i) : \mathbf{p}_i \in \mathcal{D}\}\) and a measured CIR \(\mathbf{r}\) at some grid. Here, \(\mathbf{r}\) is of the same structure as \(\mathbf{h}\).

**Step 1 Centring.** Calculate an initial estimate of the position of the tag by solving

\[
\hat{\mathbf{p}} = \arg \max_{\mathbf{p}_i} \{\rho(\mathbf{r}, \mathbf{h}(\mathbf{p}_i)) : \mathbf{p}_i \in \mathcal{D}\}.
\]

(5.42)
**Step 2 Association and smoothing.** Calculate the smoothed estimate of the position of the tag by using the information of the associated $\varepsilon$-neighbour of $\hat{p}$ according to

$$ \bar{p} = \hat{p} + \sum_{p_i \in B_\varepsilon(\hat{p})} \rho(r, h(p_i))(p_i - \hat{p}). $$  \hspace{1cm} (5.43)

The purpose of Step 2 in Algorithm 5.2 is to improve the robustness in the estimation of the tag position. If the robustness is not an issue, Step 2 can be omitted.

It is straightforward to extend the above approach to the case of multiple receive antennas. In this case, the CIR vector in Equation (5.41) is revised as

$$ h = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{N_R} \end{bmatrix}, $$

where $N_R$ is the number of receive antennas at the sensor and $h_i$ is the CIR from the tag (equipped with a single antenna) to the $i$th antenna of the sensor. Then the position estimate can be obtained by applying Algorithm 5.2 to the newly defined CIR vector $h$.

This approach belongs to the class of the mapping or fingerprinting approach. In this approach, a CIR of rich multipaths is actually considered as the signature of a position. The advantage of this approach is that the information contained in the CIR of multipaths is fully exploited and the information about the infrastructure of the environment, which provides an indication about LOS/NLOS, is not required. Its disadvantage is that the CIR database should be provided and updated once the environment changes before the start of the localization, which generally is a heavy work burden.

### 5.9 Summary

In this chapter we have discussed the UWB beamforming problem. It has been shown that the optimal beamformer is obtained if all the weighting filters $\beta_k(t)$ are identical. Regarding the optimal beamformer, the amplitude of the side lobe is independent of the ray incidence angle if a mild condition is satisfied, which is different from the narrowband case, and the amplitude of the main lobe is increased by a fold of the element number in the array, which is similar to the narrowband case. Three kinds of beam patterns have been defined, and the main lobe beamwidth is obtained.

It should be pointed out that the beamformer results presented here are obtained under rather ideal conditions, i.e., the multipath nature inherited by the UWB channel is not considered. It is reported in several studies [32, 57, 250] that in typical UWB indoor environments the multipaths occur in a clustered way, meaning that the transmitted signal impinges the receiver from a few main directions. Based on this observation, some type of cluster beamformers can be constructed such that the main lobe width fits the cluster width so as to collect enough relevant energy, while the undesired clusters are suppressed with the array gain. In general multipath environments, challenges exist for how to design the corresponding beamformer properly.
Several approaches to the UWB localization problem have been presented, namely beamforming, ToA-based, and mapping-based approaches. The methods for dealing with NLOS and multipaths, which prevail in UWB channels, have been reviewed and some new ideas are proposed. However, further experimental or simulation examinations about these ideas need to be performed.

Currently, it is fair to say that robust precise ToA-based localization in rich-multipath indoor areas still remains a challenge. In fact, the lack of understanding of the complexities of indoor radio propagation has been the main source of the failure for precise indoor geolocation over the past decade [187]. The UWB channels provide far more information than narrowband channels. Maximally exploiting the rich information contained in the UWB channels may provide an effective tool for overcoming the aforementioned lack of understanding of complexities.

Appendix 5.A Distribution of Ranging Noise

In this appendix, we present a way for deriving the probability distribution of the ranging noise.

The basic ranging problem can be formulated as follows. A monopulse signal \( s(t) \) is transmitted from a tag. At the sensor, a time-delayed version of \( s(t) \), contaminated by the receiver noise, is received. So the received signal can be expressed as

\[
\tilde{r}(t) = s(t - \tau) + \tilde{n}(t),
\]

where the delay \( \tau \) is determined by the distance between the tag and the sensor and \( \tilde{n} \) is the receiver noise. It is commonly accepted that \( \tilde{n} \) is Gaussian distributed. Suppose further that \( \tilde{n} \) is of zero mean and constant double-sided power spectral density \( N_0 \).

Generally, a correlator receiver is used to find an estimate of the time delay \( \tau \):

\[
\hat{\tau}_{\text{ToA}} = \arg \max_{\tilde{\tau}} \int \tilde{r}(t) s(t - \tilde{\tau}) \, dt,
\]

where a template of the transmitted signal \( s(t - \tilde{\tau}) \) with a variable delay is correlated with the received signal and the time instant at which the correlator reaches its peak is taken as the estimate of \( \tau \). In general, the estimate \( \hat{\tau}_{\text{ToA}} \) can be represented as

\[
\hat{\tau}_{\text{ToA}} = \tau + n,
\]

where \( n \) is called ranging noise and \( \tilde{n} \) in Equation (5.44) is referred to as receiver noise. Our purpose is to derive the distribution of \( n \) from that of \( \tilde{n} \).

The ranging information \( r_i \) in Equation (5.34) is equal to \( c\hat{\tau}_{\text{ToA}} \). Therefore, the distribution of the noise \( n_i \) in Equation (5.34) is of the same distribution as that of \( c\hat{\tau}_{\text{ToA}} \) found in Equation (5.45), except for a shift in the mean and a scaling by constant \( c \) in the amplitude.

Let

\[
y(\tilde{\tau}) = \int \tilde{r}(t)s(t - \tilde{\tau}) \, dt = \int s(t - \tau)s(t - \tilde{\tau}) \, dt + \int \tilde{n}(t)s(t - \tilde{\tau}) \, dt.
\]
Then we have

\[
\Pr\{\hat{\tau}_{\text{ToA}} \leq \xi \mid \xi \geq 0\} = \Pr\{y(\hat{\tau}) \leq y(\xi), \forall \hat{\tau} \in [0, \xi] \mid \xi \geq 0\} = \Pr\{y(\hat{\tau}) - y(\xi) \leq 0, \forall \hat{\tau} \in [0, \xi] \mid \xi \geq 0\},
\]

(5.47)

where \(\Pr\) denotes the probability of an event. We resort to a limiting process to calculate the probability in (5.47). Let us partition the interval \([0, \xi]\) into

\[
[0, \xi] = \bigcup_{i=1}^{N} [t_i, t_{i+1}] \quad \text{with} \quad \max_{i \in \{1, \ldots, N\}} |t_{i+1} - t_i| \leq \Delta, \ t_1 = 0, \ t_{N+1} = \xi.
\]

Then we have

\[
\Pr\{y(\hat{\tau}) - y(\xi) \leq 0, \forall \hat{\tau} \in [0, \xi] \mid \xi \geq 0\} = \lim_{\Delta \to 0} \Pr\left\{ \bigcap_{i=0}^{N} \{y(t_i) - y(\xi) \leq 0 \mid \xi \geq 0\} \right\}.
\]

First, let us define the vector \(z\) as

\[
z = \begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_N \\
z_{N+1}
\end{bmatrix} = \begin{bmatrix}
y(t_1) \\
y(t_2) \\
\vdots \\
y(t_N) \\
y(\xi)
\end{bmatrix}.
\]

Since \(y(\hat{\tau})\), as a stochastic processes, is a linear transform of a Gaussian process \(\tilde{n}\), it is also a Gaussian process. Therefore, the vector \(z\) will be multivariate (\(N + 1\)-dimensional) Gaussian distributed. Denote by \(R_s(\nu)\) the correlation function (in the context of deterministic signals), i.e.,

\[
R_s(\nu) = \int s(t)s(t - \nu) \, dt.
\]

Suppose that the integration interval involved is sufficiently large, so that \(R_s(\nu)\) will be only a function of the time difference in the two integrands. Therefore, the mean vector and covariance matrix of the random vector \(z\) are given by

\[
E(z) = \begin{bmatrix}
R_s(t_1 - \tau) \\
R_s(t_2 - \tau) \\
\vdots \\
R_s(t_N - \tau) \\
R_s(\xi - \tau)
\end{bmatrix} := m_z
\]

and

\[
V := E\{[z - E(z)][z - E(z)]^T\}
\]
respectively, with the \((i, j)\)th entry of \(V\) given by

\[
V_{ij} = \mathbb{E}\left\{ \left[ \int \tilde{n}(u_1)s(u_1 - t_i) \, du_1 \right] \left[ \int \tilde{n}(u_2)s(u_2 - t_j) \, du_2 \right] \right\}
\]

\[
= \iint N_0 \delta(u_1 - u_2) s(u_1 - t_i) s(u_2 - t_j) \, du_1 \, du_2
\]

\[
= \iint N_0 \delta(v_1) s(v_1 - t_i) s(v_2 - t_j) \, dv_1 \, dv_2
\]

\[
= N_0 R_s(t_j - t_i).
\]

Now define another random vector

\[
\tilde{z} = \begin{bmatrix}
z_1 - z_{N+1} \\
z_2 - z_{N+1} \\
\vdots \\
z_N - z_{N+1}
\end{bmatrix}
= \begin{bmatrix}
y(t_1) - y(\xi) \\
y(t_2) - y(\xi) \\
\vdots \\
y(t_N) - y(\xi)
\end{bmatrix} = Az,
\]

where

\[
A = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & -1 \\
0 & 1 & 0 & \cdots & 0 & -1 \\
0 & 0 & 1 & \cdots & 0 & -1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & -1
\end{bmatrix}
\]

Since \(\tilde{z}\) is a linear transform of \(z\), \(\tilde{z}\) is multivariate (\(N\)-dimensional) Gaussian distributed. The mean vector and covariance matrix of the random vector \(\tilde{z}\) are

\[
\mathbb{E}(\tilde{z}) = Am_z = \begin{bmatrix}
R_s(t_1 - \tau) - R_s(\xi - \tau) \\
R_s(t_2 - \tau) - R_s(\xi - \tau) \\
\vdots \\
R_s(t_N - \tau) - R_s(\xi - \tau)
\end{bmatrix} := m_{\tilde{z}},
\]

\[
W := \mathbb{E}\{[\tilde{z} - \mathbb{E}(\tilde{z})][\tilde{z} - \mathbb{E}(\tilde{z})]^T\}
\]

\[
= \mathbb{E}\{[\tilde{z}]^T\} - \mathbb{E}(\tilde{z})[\mathbb{E}(\tilde{z})]^T
\]

\[
= A\mathbb{E}[zz^T]A^T - m_z m_z^T
\]

\[
= A[V + m_z m_z^T]A^T - Am_z m_z A^T
\]

\[
= AVA^T
\]

respectively. Thus, the distribution of random vector \(\tilde{z}\) is given by

\[
p_{\tilde{z}}(x) = \frac{1}{(2\pi)^{N/2}\sqrt{\det W}} \exp\left[ -\frac{1}{2}(x - m_{\tilde{z}})^T W^{-1} (x - m_{\tilde{z}}) \right].
\]
Therefore

$$\Pr\{\hat{\tau}_{\text{ToA}} \leq \xi \mid \xi \geq 0\} = \lim_{\Delta \to 0} \int_{-\infty}^{0} \cdots \int_{-\infty}^{0} \tilde{p}_{z}(x) \, dx$$

$$= \lim_{\Delta \to 0} \int_{-\infty}^{0} \cdots \int_{-\infty}^{0} \frac{1}{(2\pi)^{N/2} \sqrt{\det W}} \exp \left[ -\frac{1}{2}(x - m_{\tilde{z}})^{T}W^{-1}(x - m_{\tilde{z}}) \right] \, dx. \tag{5.48}$$

On the other hand, it is easily seen that

$$\Pr\{\hat{\tau}_{\text{ToA}} \leq \xi \mid \xi \leq 0\} = 0. \tag{5.49}$$

Combining Equations (5.48) and (5.49) together gives the distribution of the ranging noise.
6

Time-Reversal UWB Systems

6.1 Introduction

In a UWB system, the dense multipath components can be resolved for the purpose of both data communications and fine positioning. However, in order to harvest even half of the energy distributed in the entire impulse response, Rake receivers with at least 20 taps, but potentially much more, must be constructed. For many wireless devices, such a design is not low cost. The dense multipath also causes great difficulty/complexity for the UWB synchronizer and equalizer. The time-reversal (TR) technique combined with UWB offers a new possibility for decreasing the cost/complexity of the UWB receiver. It may also provide a solution to multiuser and/or secure communications.

TR is a technique to focus broadband signals tightly in space and time [142, 179, 234], where the multipath channel with rich scattering is exploited by actively modulating the signal at the transmitter side using the channel information, instead of being processed at the receiver by equalizers or Rake combiners as in the traditional communication systems. This technique has been extensively used in acoustic, medical applications and underwater communications [64, 76, 229, 230]. A report by Derode et al. [62] demonstrated that in the ultrasonic frequency regime it is possible to provide error-free communications with five receivers simultaneously. The first experiment with electromagnetic waves in [142] proved that a TR system is able to compensate for multiple scattering and recreate a short electromagnetic pulse at the destination and also achieve the spatial focusing advantage of the TR system. In wireless communications, several experiments were carried out to prove the spatial and temporal focusing properties of TR systems, for example, [132, 178, 179, 196, 288].

The main advantages of the TR technology are:

- **Temporal focusing.** The received signal is compressed in the time domain. Owing to this property, the intersymbol interference (ISI) caused by the original multipath channel is greatly reduced.

- **Spatial focusing.** The received signal is focused on the intended user at some specific position, which is determined by the transmitter or user that uses the corresponding channel to pre-filter the intended data signal. This is very useful in realistic environments where the interference from co-channel users limits the user capacity. If the transmitter is able to focus precisely, an ideal space-division multiple access (SDMA) technique and the location-based security might be enabled.
An illustration for the above properties is shown in Figure 6.1 [288], where each of two independent users is equipped with a transmit antenna array with four elements and the receiver has one antenna. The two users are separated by 0.2 m. As shown in Figure 6.1, the signal for the target user is very strong at the peak instant, while the signal at the instants other than the peak instant for the target user and the signal from an undesired user with a short distance of 0.2 m away are almost immersed in the background noise. As shown in [288, Fig. 8], the energy drop can be more than 15 dB for the aforementioned scenario. The capabilities of the TR system in temporal focusing and spatial focusing are clearly witnessed.

Because of the simplicity in principle and aforementioned advantages of the TR technology, the idea of applying the TR technique in wireless communications has gained much attention recently. The number of publications in this area has increased in recent years and several research centres have been formed, including at Aalborg University, Stanford University and Tennessee Technological University.

The research group at Stanford University led by A. Paulraj first proposed applying the TR technique to UWB systems [69, 234]. In [234], a minimum mean-square error (MMSE)-based equalizer, which is located at the receiver side, was proposed for UWB TR systems. In [69], it was proved that the optimal pre-filter is of an MMSE type, which converges to a TR filter at low SNR range.

The research group at Aalborg University has carried out valuable experimental studies on TR- and multi-antenna-based communications [132, 178, 179]. These experimental studies were mainly performed in the ISM (industrial, scientific, and medical) band 5 GHz radio systems with a bandwidth from 20 to 100 MHz [131, 132, 178, 179, 289], where remarkable temporal focusing (delay spread reduction) and spatial focusing (low spatial interference) in the context of wideband MISO systems are demonstrated. In [133, 134, 135], several equalizers, designed in both time domain and frequency domain, were proposed. In [180], a new space–time multiplexing scheme was proposed to reduce the

![Figure 6.1](attachment:image.png)
multiuser interference (MUI) by carefully choosing the offset time in each branch of the TR filter of the MISO system.

The research group at Tennessee Technological University has made extensive simulation and experimental studies on UWB TR systems [94, 95, 96, 97, 196, 288], including the first report on UWB TR experimental results [196], where the channel reciprocity is also verified in UWB channels. In [97], a detailed description for the UWB TR testbed was reported. In [94, 96], the BER performance of a UWB MISO TR spatial multiplexing (SM) system was shown via simulation results by using experimentally obtained channels. In [288], the temporal focusing and spatial focusing abilities of UWB TR systems were experimentally investigated, showing promising potentials of the systems in reducing the complexity of UWB receivers and for secure communications.

In [277, 278], the effect of dispersion and losses on the performance of the TR operator were investigated for inhomogeneous scatter media. In [176], two types of UWB MIMO TR system for spatial multiplexing were proposed and two corresponding equalizers were designed to mitigate MUI. To further improve the system performance, an antenna selection scheme for the UWB MIMO TR systems proposed in [176] was presented in [177].

Some TR applications in industry can be found in [43, 259], for example.

This chapter will present our recently obtained results on UWB TR systems. We will extensively use the convolution $\ast$ of two matrix-valued functions. Similar to the convolution of two scalar functions, we define

$$M_1(t) \ast M_2(t) = \int_{-\infty}^{\infty} M_1(\tau)M_2(t - \tau)d\tau,$$

where $M_1$ and $M_2$, as functions of time $t$, are two matrices with compatible dimensions and $\ast$ denotes the convolution.

### 6.2 Motivation for the Time-Reversal Approach in UWB Systems

In this section we discuss why a TR pre-filter instead of other kinds of pre-filters should be used at the transmitter side if the channel state information is available at the transmitter. Basically, a TR filter can be considered as a preprocessing filter in the sense that the filter is placed at the transmitter side. The basic purpose of the preprocessing filter for wideband or UWB communication systems, similar to that of the post-processing filter (in the sense that the filter is placed at the receiver side), is to capture or focus the signal energy dispersed by the channel. Therefore, there are two basic ways to design the preprocessing filters.

The first way is to consider the problem in the time domain. This leads to the TR filter, by which all the dispersed channel energy is collected coherently at a specific tap of the receiver, while the energies are summed noncoherently at other taps of the composite channel. In this way, the equivalent composite channel behaves like a Dirac $\delta$ function in the time domain if the original channel disperses sufficiently long and randomly.

The second way is to consider the problem in the frequency domain. This leads to the inverse channel (IC) filter (also called zero-forcing (ZF) filter in the literature), by which the composite channel is whitened and thus the equivalent composite channel is a Dirac $\delta$ function in the ideal case, i.e., when the inverse channel transfer function can be realized
at the transmitter. Generally, the IC filter yields a time-unlimited CIR (CIR). Therefore, it is difficult to realize the ideal IC filter.

For the case where the CIR of the preprocessing filter is of finite length in time, Joham et al. [113] presented a systematic design and performance comparison for general MIMO wideband communication systems. In the continuous-time domain, it has been mathematically proved [37] (see also [288]) that the TR pre-filter is optimum in the sense that it maximizes the amplitude of the receiver output at a specific time instant.

For the sake of completeness, we will first show that the pre-filter of a time-limited CIR should be like that of TR. Then we present some performance comparison results between the TR- and IC-based pre-filters. The comparison is focused on the robustness of the two kinds of filters, showing the limitation of the IC-based pre-filter and superiority of the TR-based pre-filter.

6.2.1 Theoretical Basis

A basic problem to motivate the TR approach is how to design a modulation scheme or a weighting filter at the transmitter to maximize the received SNR at the receiver under the condition that the channel state information \( \{h_0, h_1, \ldots, h_{L-1}\} \) is available at the transmitter. Mathematically, we need to find an optimal weighting filter \( g_0, g_1, \ldots, g_{L-1} \), as shown in Figure 6.2, such that

\[
\frac{\mathbb{E}[(y(t))^2|t=(L-1)\Delta T]}{\mathbb{E}(n^2)}
\]

is maximized, where \( \Delta T \) is the sampling interval and the coefficients \( g_0, g_1, \ldots, g_{L-1} \) should satisfy

\[
\sum_{l=0}^{L-1} g_l^2 = 1,
\]

i.e., no additional power gain will be provided by the weighting filter.

Since the weighting filter does not affect the power of the noise, maximizing the output SNR is equivalent to maximizing \( \mathbb{E}[(y(t))^2|t=(L-1)\Delta T] \) under constraint (6.2). Suppose that

\[
G(z) = \sum_{l=0}^{L-1} g_l z^{-l}
\]

and

\[
H(z) = \sum_{l=0}^{L-1} h_l z^{-l}.
\]

![Figure 6.2](image_url) Block diagram of the TR and IC filters at the transmitter side, where \( G(z) = \sum_{l=0}^{L-1} g_l z^{-l} \) and \( H(z) = \sum_{l=0}^{L-1} h_l z^{-l} \).
the transmitted symbol is $S$. The transmitted signal after passing through the weighting filter is

$$x(t) = S \sum_{l=0}^{L-1} g_l w(t - l \Delta T),$$

where $w(t)$ is the monopulse waveform of the UWB transmission system. The received signal is the convolution of $x(t)$ with the CIR $h(t)$. After sampling, the received signal at time instant $t = (L - 1) \Delta T$ reads as follows

$$y(L - 1) = S \sum_{l=0}^{L-1} g_l h_{L-1-l} + n,$$

where $n$ is the receiver noise. Let

$$\mathbf{g} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{L-1} \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_{L-1} \\ h_{L-2} \\ \vdots \\ h_0 \end{bmatrix}.$$

Then we have

$$y(L - 1) = S \mathbf{g}^T \mathbf{h} + n.$$

Therefore:

$$\mathbb{E}[(y(L - 1))^2] = S^2 (\mathbf{g}^T \mathbf{h})^2 + \sigma_n^2 \leq S^2 (\mathbf{g}^T \mathbf{g}) (\mathbf{h}^T \mathbf{h}) + \sigma_n^2,$$

where $\sigma_n^2$ is the variance of the noise $n$. In inequality (6.4), we have used the Cauchy–Schwarz inequality, and the equality holds true if and only if

$$\mathbf{g} = c \mathbf{h},$$

where $c$ is a scalar constant. The scalar form of Equation (6.5) reads as follows

$$g_l = c h_{L-1-l}, \quad l = 0, 1, \ldots, L - 1.$$

Equation (6.6) shows that the optimal filter for achieving a maximum output SNR is a TR filter. The transmitted symbol can be decoded by sampling the output signal at the time instant $t = (L - 1) \Delta T$.

Substituting Equation (6.5) into constraint (6.2) gives

$$c = \pm \frac{1}{\sqrt{\sum_{l=0}^{L-1} h_l^2}}.$$

As mentioned above, another way for recovering the transmitted symbol is to use the IC filter at the transmitter side. From Equation (6.3) we can see that

$$Y(z) = S G(z) H(z) + N(z),$$
where $Y$, $G$, $H$, and $N$ are the $z$-transforms of the sequence $y$, $g$, $h$, and $n$ respectively. Clearly, if $G(z)$ is so designed

$$G(z) = \frac{1}{H(z)},$$

(6.7)

then the transmitted symbol $S$ can be easily decoded. Notice that the noise power is not changed by the IC filter at the transmitter side. This is quite different from the IC filter at the receiver side.

Generally, the ideal IC filter (6.7) cannot be implemented with a finite-tap filter. Instead, it should be approximated by a finite-tap filter. Let

$$\frac{1}{H(z)} \approx \sum_{l=-\bar{L}}^{\bar{L}} g_l z^{-l} = G(z).$$

There are many ways and criteria to select the coefficients $\{g_l\}$. If the transfer function $\frac{1}{H(z)}$ is expanded as

$$\frac{1}{H(z)} = \sum_{l=-\infty}^{\infty} \tilde{h}_l z^{-l},$$

then one way to select $\{g_l\}$ is as follows:

$$g_l = \tilde{h}_l, \quad l = -\bar{L}, -\bar{L} + 1, \ldots, \bar{L} - 1, \bar{L}.$$  

Such a choice will minimize

$$\left| \frac{1}{H(z)} - \sum_{l=-\bar{L}}^{\bar{L}} g_l z^{-l} \right|^2$$

at $z = e^{j\omega}$ [90].

One notorious problem for the IC filter is that the channel transfer function $H(z)$ should be of minimum phase, i.e., all the zeros of $H(z)$ should be located inside the unit circle. If this is not the case, we should partition $H(z)$ into two parts

$$H(z) = H_+(z)H_-(z),$$

where all the zeros of $H(z)$ that are outside or on the unit circle are included in $H_+(z)$ and all the zeros of $H(z)$ that are inside the unit circle are included in $H_-(z)$. Based on this partition of $H(z)$, Equation (6.7) should be changed to

$$G(z) = \frac{1}{H_-(z)}.$$  

(6.8)

The BER performance of the composite system will be degraded by the nonideal IC filter (6.8).

### 6.2.2 Simulation Results

Several examples are provided to show the performance difference between the TR and IC approaches.
6.2.2.1 Minimum Phase Channel

This is a designed case. In practical UWB channels, it is almost certain that the channel is of nonminimum phase!

In this case, all the zeros of $H(z)$ are randomly chosen and uniformly distributed inside the unit circle, as shown in Figure 6.3a. Therefore, the composite channel of the IC-based approach behaves like a Dirac delta function, producing an AWGN channel, as shown in Figure 6.3c, whereas for the TR approach there are some other side taps, except the peak tap, in the impulse response of the composite channel. This will cause a certain level of ISI if the symbol rate is high. In all the simulations in this subsection, the symbol rate is $1/(4\Delta T)$.

![Figure 6.3](image)

**Figure 6.3** The zero distribution of the channel transfer function, impulse responses of the channel and composite channel, and BER performance for the composite channel: the case of minimum phase (ideal case). The symbol rate is $1/(4\Delta T)$. (a) zero distribution of $H(z)$; (b) CIR of propagation channel itself; (c) CIR of composite channel; (d) BER performance for the composite channel.
6.2.2.2 Simulated Practical Channel

In this case, the channel taps \( h_l, l = 0, 1, \ldots, 99 \), are generated based on the UWB channel model with Nakagami fading as described by Equations (2.11) and (2.13) with \( \kappa = 6 \) and \( \varrho = 0.9 \). The generated channel is shown in Figure 6.4b and the zeros of the channel transfer function are shown in Figure 6.4a. It can be seen that the channel is of nonminimum phase. Therefore, filter (6.8), instead of (6.7), should be used for the IC approach. If filter (6.7) is used, it can be shown that the weighting coefficient \( \{g_l\} \) will be divergent rapidly, causing a great difficulty in implementing the filter.

The BER performance comparison is shown in Figure 6.3d and 6.4e for two different cases. To make the comparison fair, the peak values of impulse responses of the composite channels are normalized to unity, as shown in Figures 6.3c and 6.4c and d respectively. For the TR and IC approaches, different normalization coefficients are needed.

6.2.3 Robustness of TR- and IC-Based Systems

To obtain the CIR at the transmitter, basically we can use two approaches: (i) the receiver estimates the CIR using a training sequence sent by the transmitter and then sends the CIR back to the transmitter; (ii) based on the property of channel reciprocity, the transmitter estimates the CIR using a training sequence sent by the receiver. By either approach, errors will inevitably appear in the estimated CIR. The error sources include

- the typical error appeared in the process of parameter estimation, which is normally AWGNs;
- the error appeared in the feedback channel (such as channel fading) if the forward channel parameters are identified at the receiver;
- the channel reciprocity may not be strictly valid due to changing scattering environments.

Now we investigate what happens for the TR and IC approaches if errors appear in the estimate of the CIR. We consider the simplest error model for the CIR estimation, i.e., the estimated CIR \( \hat{h}_l \) is described by

\[
\hat{h}_l = h_l + \tilde{n}_l, \quad l = 0, 1, \ldots, L - 1,
\]

where \( \tilde{n}_l \) is an AWGN with zero mean and variance of \( \sigma^2_{\tilde{n}} \). The corresponding simulation results are depicted in Figures 6.5 and 6.6 respectively for the minimum phase channel and nonminimum phase channel, where \( \sigma^2_{\tilde{n}} = -15 \text{ dB} \).

It can be seen from Figure 6.5 that, in the case of minimum phase, the CIR of the composite channel for the IC approach is severely changed under moderately noise-corrupted CIR estimate, while the peaky property of the CIR of the composite channel for the TR approach is well kept. Thus, the TR approach outperforms the IC approach in the BER performance when the channel estimation is subjected to errors, as shown in Figure 6.5b; contrast with Figure 6.3d. The similar phenomenon can be observed in Figure 6.6 for the case of nonminimum phase yet practical channels. In this case, the IC approach fails to work properly and the TR approach outperforms greatly the IC approach in the BER performance, as seen in Figure 6.6b.
Figure 6.4  The zero distribution of the channel transfer function, impulse responses of the channel and composite channel, and BER performance for the composite channel: the case of non-minimum phase (practical case). The symbol rate is $1/(4\Delta T)$. (a) zero distribution of $H(z)$; (b) CIR of propagation channel itself; (c) CIR of composite channel for TR; (d) CIR of composite channel for IC; (e) BER performance for the composite channel.
Figure 6.5 The effect of CIR estimation errors on the system performance for the case of minimum phase (ideal case): (a) Estimated CIR with errors; (b) BER performance for the composite channel; (c) CIR of composite channel for TR; (d) CIR of composite channel for IC.

From the aforementioned analysis and simulation results, we can conclude that the TR approach outperforms the IC approach in

- **Output SNR.** Theoretically, the TR filter will produce a maximal output SNR at a specific time instant over all the constrained weighting filters.
- **Performance for practical UWB channels.** In the practical UWB channels, which is generally of nonminimum phase, the TR approach can yield better BER performance than the IC approach due to the fact that the frequency selectivity of the channel cannot be completely compensated by the IC filter.
- **Robustness.** The TR approach is quite robust to CIR estimation errors in keeping the peaky property, while the IC approach is very sensitive to the CIR estimation errors.
- **Complexity.** To implement the IC filter, one needs to factorize the channel transfer functions in real time, which is computationally expensive for a moderate-scale system.
Figure 6.6  The effect of CIR estimation errors on the system performance for the case of nonminimum phase (practical case): (a) Estimated CIR with errors; (b) BER performance for the composite channel; (c) CIR of composite channel for TR; (d) CIR of composite channel for IC.

For a system with multiple antennas, this factorization is prohibitive in computation. However, for the TR approach there is no such a computational burden for a system with either a single antenna or multiple antennas.

6.3 Two Schemes of UWB MIMO TR Systems

In a general sense, the multiple antennas of a MIMO system can be distributed in two ways: centralized and distributed. In the centralized way, all the transmit antennas and receive antennas are located in a somewhat packed form like the case of the beamforming problem, while in the distributed way all the transmit antennas and receive antennas are located in a somehow ad hoc form like the case of opportunistic MIMO communications. Corresponding to these two distribution ways, there are two TR schemes: the full TR scheme and the diagonal TR scheme for UWB MIMO TR communication systems. The two schemes will be described in the following.
6.3.1 Full TR Scheme

Consider the UWB MIMO system illustrated in Figure 6.7, where $N_T$ transmit antennas and $N_R$ receive antennas are deployed at the transmitter and receiver respectively.

Let $h_{i_1 i_2}(t)$ be the CIR of the channel from the $i_2$th transmit antenna to the $i_1$th receive antenna, which can be typically modelled as

$$h_{i_1 i_2}(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{k,l}^{(i_1 i_2)} \delta(t - T_k^{(i_1 i_2)} - \tau_{k,l}^{(i_1 i_2)}),$$

where $\alpha_{k,l}^{(i_1 i_2)}$, $T_k^{(i_1 i_2)}$ and $\tau_{k,l}^{(i_1 i_2)}$ are the amplitude fading, cluster delay and ray delay respectively. For the details about the above model, readers are referred to Chapter 2.

The MIMO channel is characterized by the CIR matrix $G(t)$:

$$G(t) = \begin{bmatrix}
    h_{11}(t) & h_{12}(t) & \cdots & h_{1N_T}(t) \\
    h_{21}(t) & h_{22}(t) & \cdots & h_{2N_T}(t) \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{N_R 1}(t) & h_{N_R 2}(t) & \cdots & h_{N_R N_T}(t)
\end{bmatrix}.$$  \hspace{1cm} (6.9)

In the full TR scheme, the TR filter matrix, denoted as $\tilde{G}(t)$, is

$$\tilde{G}(t) = [G(-t)]^T = \begin{bmatrix}
    h_{11}(-t) & h_{12}(-t) & \cdots & h_{1N_T}(-t) \\
    \|h_{11}\| & \|h_{21}\| & \cdots & \|h_{N_R 1}\| \\
    h_{12}(-t) & h_{22}(-t) & \cdots & h_{2N_T}(-t) \\
    \|h_{12}\| & \|h_{22}\| & \cdots & \|h_{N_R 2}\| \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{1N_T}(-t) & h_{2N_T}(-t) & \cdots & h_{N_R N_T}(-t) \\
    \|h_{1N_T}\| & \|h_{2N_T}\| & \cdots & \|h_{N_R N_T}\|
\end{bmatrix},$$  \hspace{1cm} (6.10)

where $\| \cdot \|$ denotes the 2-norm in the $L^2$ space defined by

$$\|h\| = \sqrt{\int_{-\infty}^{\infty} h^2(t) \, dt}.$$
The TR filter matrix $\mathbf{G}(t)$ in (6.10) is based on the original CIR matrix reversed in the time domain and transposed in the spatial domain.

The CIR matrix of the equivalent composite channel is

$$\hat{\mathbf{G}}(t) = \mathbf{G}(t) \ast \mathbf{G}(t). \quad (6.11)$$

Denote by $\mathbf{x}(t)$ and $\mathbf{y}(t)$ (both are of dimensions $N_R$) the transmit and receive signals of the TR system at time $t$ respectively. Then we have

$$y_i(t) = \sum_{k=1}^{N_T} h_{ik}(t) \ast \frac{h_{ik}(-t)}{\|h_{ik}\|} \ast x_i(t) + \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \sum_{j \neq i}^{N_T} h_{jk}(t) \ast \frac{h_{jk}(-t)}{\|h_{jk}\|} \ast x_j(t) + n_i(t), \quad (6.12)$$

$$i = 1, \ldots, N_R,$$

where $n$ is the receiver noise and $x_i, y_i$ and $n_i$ denote the $i$th entry of $\mathbf{x}, \mathbf{y}$ and $\mathbf{n}$ respectively.

The data symbol contained in $x_i$ can be decoded from the receive signal $y_i$. Thus, the first item in the right-hand side of Equation (6.12) is the desired data stream. The equivalent composite channel for this data stream is the coherent summation of the auto-correlation functions of the CIRs of the $N_T$ channels, which typically produces a peak at a specific instant. The second item in the right-hand side of Equation (6.12) is undesired interference, called multistream interference (MSI). The equivalent composite channel for the MSI is the summation of the cross-correlation functions of the CIRs of different channels, which is typically small.

If all the CIRs of the original channels are orthogonal, then it can be seen from Equation (6.12) that the SNR of the UWB MIMO TR system will be increased by $N_R^2$-fold.

Generally, the CIRs of the original channels are not orthogonal. Let us assume that the cross-correlation coefficients of the CIRs of different channels are uniformly bounded by a small constant $\varepsilon$, i.e.:

$$\left| \int_{-\infty}^{\infty} h_{i_1i_3}(t)h_{i_3i_4}(t) \, dt \right| \leq \varepsilon,$$

$$\sqrt{\int_{-\infty}^{\infty} h_{i_1i_2}^2(t) \, dt} \sqrt{\int_{-\infty}^{\infty} h_{i_3i_4}^2(t) \, dt} \quad \forall i_1, i_3 \in \{1, \ldots, N_R\}, \forall i_2, i_4 \in \{1, \ldots, N_T\}, \ |i_1 - i_3| + |i_2 - i_4| \neq 0.$$

Suppose that the transmitted signal takes the form $x_i(t) = S_i \delta(t)$ with $\mathbb{E}(S_i^2) = E_{TX}, \ i = 1, \ldots, N_R$, where $S_i$ is the data symbol. It is easy to see from Equation (6.12) that the peak instant of the received signal happens at $t = 0$. Therefore, we use the received signal at time instant $t = 0$ to decode the transmitted symbols $S_i$. Applying the Cauchy–Schwarz inequality to the second item of the right-hand side of Equation (6.12) yields a bound of

---

1 If the different data streams are considered as different users, the MSI is also called MUI.

2 It is interesting to compare the SNR increasing rates between the MISO TR system and MISO MRC or SIMO MRC system. The rate for the latter system is $N_T$-fold or $N_R$-fold respectively. This is due to the fact that the TR filter for the MISO TR system does not change the receiver noise.
the power of the MSI (denoted as $P_{\text{MSI},i}$) as follows:

$$P_{\text{MSI},i} = \left| \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \left[ h_{ik}(t) * \frac{h_{jk}(-t)}{\|h_{jk}\|} \right] \right|_{t=0}^2 E_{\text{TX}}$$

$$\leq E_{\text{TX}} \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \frac{1}{\|h_{jk}\|^2} \left| \left[ h_{ik}(t) * h_{jk}(-t) \right] \right|_{t=0}^2$$

$$\leq E_{\text{TX}} \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \frac{1}{\|h_{jk}\|^2} \varepsilon^2 \|h_{ik}\|^2 \|h_{jk}\|^2$$

$$= (N_R - 1) E_{\text{TX}} \varepsilon^2 \sum_{k=1}^{N_T} \|h_{ik}\|^2. \quad (6.13)$$

The received signal power for the desired information symbol (denoted as $P_{S,i}$) is

$$P_{S,i} = \left| \sum_{k=1}^{N_T} \left[ h_{ik}(t) * \frac{h_{ik}(-t)}{\|h_{ik}\|} \right] \right|_{t=0}^2 E_{\text{TX}}$$

$$= \left| \sum_{k=1}^{N_T} \frac{1}{\|h_{ik}\|} \varepsilon E_{\text{TX}} \left[ \sum_{k=1}^{N_T} \|h_{ik}\| \right]^2.$$

Therefore, the signal to interference-plus-noise power ratio (SINR) of the full TR system for the $i$th data stream (denoted $\rho_i$) is

$$\rho_i = \frac{P_{S,i}}{P_{\text{MSI},i} + \sigma_n^2} \geq \frac{E_{\text{TX}} \left[ \sum_{k=1}^{N_T} \|h_{ik}\| \right]^2}{(N_R - 1) E_{\text{TX}} \varepsilon^2 \sum_{k=1}^{N_T} \|h_{ik}\|^2 + \sigma_n^2},$$

where $\sigma_n^2$ is the variance of the receiver noise $\mathbf{n}_i$. From Equation (6.13) we can see that the power of the MSI increases proportionally with the number of data streams and also with the number of the transmit antennas through the different channels, while the signal power increases proportionally with the square of the number of the transmit antennas through the different channels. If the MSI power is much larger than the noise power, i.e., when

$$E_{\text{TX}}(N_R - 1) \varepsilon^2 \sum_{k=1}^{N_T} \|h_{ik}\|^2 \gg \sigma_n^2,$$

we have

$$\rho_i \approx \frac{\left[ \sum_{k=1}^{N_T} \|h_{ik}\| \right]^2}{(N_R - 1) \varepsilon^2 \sum_{k=1}^{N_T} \|h_{ik}\|^2}. \quad (6.14)$$
Further, if all the channels $h_{ik}$, $k = 1, \ldots, N_T$, have the same power for each $i$, then we have

$$\rho_i \approx \frac{N_T}{(N_R - 1)\varepsilon^2}.$$  \hspace{1cm} (6.15)

Equations (6.14) and (6.15) show that the SINR of the full TR system will be saturated when the transmit power is very high: the SINR depends only on the numbers of transmit and receive antennas, the power of the channels, and the cross-correlation coefficients of the CIRs of the channels, but it is not affected by the transmitted power. Thus, a BER floor of the system will be produced at the high SNR range. This phenomenon was verified by the simulations in [176]. Two kinds of equalizers were thus proposed there to mitigate the MSI, with which the BER floor was broken up.

### 6.3.2 Diagonal TR Scheme

In the full TR scheme, all the channels among the transmit and receive antennas should be available at the transmitter side. This requirement increases the complexity of the transmitter on one hand. On the other hand, it is difficult to obtain the channel state information for those channels which are not communication partners in many cases. For example, in multiuser communications, it is relatively easy to learn the channel between two communicating users, but it is difficult to obtain other channels. In these cases, the diagonal TR scheme provides a solution.

In the diagonal TR scheme, it is assumed that $N_T = N_R$. The TR filter matrix is

$$\tilde{G}_d(t) = \begin{bmatrix} \frac{h_{11}(-t)}{\|h_{11}\|} & 0 & \cdots & 0 \\ 0 & \frac{h_{22}(-t)}{\|h_{22}\|} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{h_{N_TN_T}(-t)}{\|h_{N_TN_T}\|} \end{bmatrix}.$$  \hspace{1cm}

Therefore, the TR pre-filter only applies to the channel between the communication partners. The CIR matrix of the equivalent composite channel is

$$\tilde{G}_d(t) = G(t) * \tilde{G}_d(t).$$

The output of the system reads as follows

$$y_i(t) = h_{ii}(t) \ast \frac{h_{ii}(-t)}{\|h_{ii}\|} \ast x_i(t) + \sum_{j=1}^{N_R} \sum_{j \neq i}^{N_R} h_{ij}(t) \ast \frac{h_{jj}(-t)}{\|h_{jj}\|} \ast x_j(t) + n_i(t),$$

$$i = 1, \ldots, N_R.$$  \hspace{1cm} (6.16)

If all the CIRs of the relevant channels are orthogonal, the peak signal produced by the autocorrelation of the $(i, i)$th channel makes it very easy to decode the transmitted symbol at the $i$th transmit antenna. However, as can be seen from Equation (6.16), there is no diversity gain in the diagonal TR scheme.
If the CIRs of the relevant channels are not orthogonal, the SINR of the system will be decreased. Similar to the case of the full TR scheme, assume that
\[
\left| \int_{-\infty}^{\infty} h_{ij}(t) h_{jj}(t) \, dt \right| \leq \varepsilon, \quad \forall i, j \in \{1, \ldots, N_R\}.
\]
Using the same notation as in the case of the full TR scheme, we have
\[
P_{MSI,i} = \left| \sum_{j=1, j \neq i}^{N_R} \left[ h_{ij}(t) \ast h_{jj}(-t) / \|h_{jj}\| \right] \right|_{t=0}^2 E_{TX}
\]
\[
\leq E_{TX} \sum_{j=1, j \neq i}^{N_R} \frac{1}{\|h_{jj}\|^2} \left| \left[ h_{ij}(t) \ast h_{jj}(-t) \right] \right|_{t=0}^2
\]
\[
\leq E_{TX} \sum_{j=1, j \neq i}^{N_R} \frac{1}{\|h_{jj}\|^2} \varepsilon^2 \|h_{ij}\|^2 \|h_{jj}\|^2 = E_{TX} \varepsilon^2 \sum_{j=1, j \neq i}^{N_R} \|h_{ij}\|^2. \quad (6.17)
\]
The received signal power for the desired information symbol is
\[
P_{S,i} = E_{TX} \|h_{ii}\|^2.
\]
Therefore, the SINR of the diagonal TR system for the \(i\)th data stream is
\[
\rho_i = \frac{P_{S,i}}{P_{MSI,i} + \sigma_n^2} \geq \frac{E_{TX} \|h_{ii}\|^2}{E_{TX} \varepsilon^2 \sum_{j=1, j \neq i}^{N_R} \|h_{ij}\|^2 + \sigma_n^2}
\]
From Equation (6.17) we can also see that the power of the MSI increases proportionally with the number of data streams. If the MSI power is much larger than the noise power, i.e., when
\[
E_{TX} \varepsilon^2 \sum_{j=1, j \neq i}^{N_R} \|h_{ij}\|^2 \gg \sigma_n^2,
\]
then we have
\[
\rho_i \approx \frac{\|h_{ii}\|^2}{\varepsilon^2 \sum_{j=1, j \neq i}^{N_R} \|h_{ij}\|^2}. \quad (6.18)
\]
Further, if all the channels \(h_{ik}, k = 1, \ldots, N_T\), have the same power for each \(i\), then we have
\[
\rho_i \approx \frac{1}{(N_R - 1) \varepsilon^2}. \quad (6.19)
\]
Equations (6.18) and (6.19) again show that the SINR of the diagonal TR system will be saturated when the transmit power is very high: the SINR depends only on the numbers of transmit and receive antennas, the power of the channels, and the cross-correlation coefficients of the CIRs of the channels, and it is not affected by the transmit power. Thus, a BER floor of the system will be produced for the system at the high SNR range.

6.4 Pre-Equalizer Design for Spatial Multiplexing
UWB SIMO TR Systems

As analysed in the preceding section and illustrated by both simulation results and indoor measurement trials [96, 176], an SDMA multiuser system based solely on TR can suffer from strong MSI because of cross-correlations between the radio links, and increasing the number of transmit antennas cannot fully solve the problem due to the resultant BER floor.

In this section, two kinds of pre-equalizers, which are applied before the TR filter at the transmitter, are proposed to deal with both MSI and ISI problems in MIMO TR systems. Since the channel information is already available at the transmitter in a TR system, the pre-equalizer only adds a little computational complexity to the transmitter. On the other hand, since the equivalent composite CIR of the TR systems is well focused in the time domain, only a few taps of the equivalent CIR are needed to design the pre-equalizer. Therefore, shortened equivalent channels are used to design the pre-equalizer.

With the help of the TR filter and the pre-equalizer, several data streams can be transmitted over only a single antenna at the same time. These kinds of system, namely TR UWB SM-SIMO, work like the system using virtual transmit antennas to convey the data streams. The simulation results will show that the cascade of the TR filter and the pre-equalizer is an efficient method to deal with the MSI and ISI for TR UWB SM-SIMO systems.

6.4.1 System Description

Consider an SM-MIMO system with $N_T$ transmit and $N_R$ receive antennas, as shown in Figure 6.7. The CIR matrix of the original MIMO channel of the system is characterized by Equation (6.9). The TR filter of the system is described by Equation (6.10). The CIR matrix of the equivalent composite channel is given by Equation (6.11).

Two remarks can be drawn from the matrix of the equivalent channel. First, the maximum number of independent data streams the system can achieve is $N_R$, which is the number of receive instead of transmit antennas. Second, the TR technique in MIMO-UWB can exploit $N_T$-order transmit diversity, and the diversity gain depends on the number of transmit antennas instead of the number of receive antennas.

However, both MSI and ISI are introduced in the received signal due to the fact that the equivalent composite MIMO channel is neither an ideal diagonal matrix in the spatial domain nor a $\delta$-like function in the temporal domain. In some cases, the MSI will cause a BER floor in an SM-MIMO system. Therefore, using a pre-equalizer before the TR filter at the transmitter is helpful to combat the ISI and MSI.
Because of the compressed CIR in the temporal domain for a TR system, the pre-equalizer can be simply implemented. There is no need to use full CSI of the equivalent channel $\hat{G}$. Table 6.1 shows how the power of the equivalent composite channel of a SISO TR system distributes across the channel taps. It is obtained based on the average of 20 realizations of the simulated standard UWB channel models CM1–CM4 [77]. The parameters of the channel models used in the simulations are shown in Table 2.1, except that here $\gamma_C$ is chosen to be 22.61 ns, 26.27 ns, 14.6 ns and 19.8 ns respectively for CM1, CM2, CM3 and CM4. The sampling interval is chosen to be 0.125 ns. It is demonstrated in Table 6.1 that most energy of the equivalent channel falls into taps near to the peak instant of the autocorrelation. For example, 50% of energy falls into a single tap (the peak tap of the equivalent CIR) for the cases of CM1, CM3 and CM4. The same amount of energy is occupied by three taps for the equivalent channel of CM2. From Table 6.1, we can see that channel CM3 has the best compression property in the TR technique.

### 6.4.2 Pre-Equalizer Design

The block diagram of the MIMO TR system with a pre-equalizer is shown in Figure 6.8. Let us discretize the CIR of the equivalent composite channel of the system. Denote the CIR matrix as

$$\hat{G}(t) = \sum_{l=0}^{L_m} \hat{G}_l \delta(t - l\Delta T),$$

**Table 6.1** Power distribution of the equivalent composite channel of SISO UWB systems with a TR pre-filter. (Note that in this table the tails of the simulated CIRs are not cut; hence, many insignificant taps are kept to give a panoramic view for the power distribution profile.)

<table>
<thead>
<tr>
<th>Percentage power</th>
<th>CM1</th>
<th>CM2</th>
<th>CM3</th>
<th>CM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>75%</td>
<td>71</td>
<td>106</td>
<td>15</td>
<td>41</td>
</tr>
<tr>
<td>90%</td>
<td>230</td>
<td>356</td>
<td>44</td>
<td>98</td>
</tr>
<tr>
<td>95%</td>
<td>363</td>
<td>569</td>
<td>69</td>
<td>140</td>
</tr>
<tr>
<td>100%</td>
<td>6407</td>
<td>5043</td>
<td>7249</td>
<td>1105</td>
</tr>
</tbody>
</table>

**Figure 6.8** Block diagram of a MIMO TR system with a pre-equalizer.
where $\Delta T$ is the sampling time, $L_m$ is the maximum number of taps in all the subchannels and

$$
\hat{G}_l = \begin{bmatrix}
\hat{g}_{l,11} & \hat{g}_{l,12} & \cdots & \hat{g}_{l,1N_R} \\
\hat{g}_{l,21} & \hat{g}_{l,22} & \cdots & \hat{g}_{l,2N_R} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{g}_{l,N_R1} & \hat{g}_{l,N_R2} & \cdots & \hat{g}_{l,N_R,N_R}
\end{bmatrix},
$$

which characterizes the relationship between the input symbol (before the TR filter!) $x(k) = [x_1(k) \ x_2(k) \ \cdots \ x_{N_R}(k)]^T$ and the received signal $y(k) = [y_1(k) \ y_2(k) \ \cdots \ y_{N_R}(k)]^T$ corresponding to the $l$th tap component.

Consider a block of the received signals at time instants $k, k+1, \ldots, k+K-1$, which can be represented as

$$
\vec{y} = \hat{G}_B \vec{x} + \vec{n}, \quad (6.20)
$$

where

$$
\vec{x} = \frac{1}{\sqrt{N_T}} \begin{bmatrix}
x(k-L_m+1) \\
x(k-L_m+2) \\
\vdots \\
x(k) \\
x(k+K-1)
\end{bmatrix}, \quad
\vec{y} = \begin{bmatrix}
y(k) \\
y(k+1) \\
\vdots \\
y(k+K-1)
\end{bmatrix}, \quad
\vec{n} = \begin{bmatrix}
n(k) \\
n(k+1) \\
\vdots \\
n(k+K-1)
\end{bmatrix},
$$

and $\vec{n}$ is the receiver noise (assumed to be Gaussian). The scale factor $1/\sqrt{N_T}$ in $\vec{x}$ keeps the total transmit power the same as that of the SISO case for fair comparison.

Owing to the temporal focusing property of the TR channel, the effective length of the equivalent composite TR channel can be greatly reduced to a small number centring around the peak tap. For a given threshold of the collected energy level (in per cent such as shown in Table 6.1), choose $L_s$ taps ($L_s \ll L_m$) of the equivalent channel to approximate the full equivalent channel $\hat{G}$. The new CIR of the equivalent channel is called the shortened CIR, denoted as $\hat{G}_S$. The pre-equalizer will be designed and implemented based on the shortened channel $\hat{G}_S$. Suppose $L_s$ is an odd number. Let the tap index of the peak tap of $\hat{G}$ (in the sense of $\|\hat{G}\|$) be $l_p$, and define $l_s = (L_s - 1)/2$. Then, $\{\hat{G}_{l_p-l_s}, \hat{G}_{l_p-l_s+1}, \ldots, \hat{G}_{l_p}, \ldots, \hat{G}_{l_p+l_s-1}, \hat{G}_{l_p+l_s}\}$ will be selected as the CIR matrix
of the shortened equivalent channel $\hat{G}_S$. Based on this idea, Equation (6.20) can be approximated by
\[ \tilde{y} \approx \hat{G}_S \tilde{x}_S + \hat{n}, \]
where
\[
\hat{G}_S = \begin{bmatrix}
\hat{G}_{l_p+l_s} & \hat{G}_{l_p+l_s-1} & \cdots & \hat{G}_{l_p-1} & \hat{G}_{l_p-l_s} & 0 & \cdots & 0 \\
0 & \hat{G}_{l_p+l_s} & \cdots & \hat{G}_{l_p-2} & \hat{G}_{l_p-l_s} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \hat{G}_{l_p+l_s} & \hat{G}_{l_p+l_s-1} & \hat{G}_{l_p+l_s-2} & \cdots & \hat{G}_{l_p-l_s}
\end{bmatrix},
\]
\[
\tilde{x}_S = \frac{1}{\sqrt{N_T}} \begin{bmatrix}
x(k - L_s + 1) \\
x(k - L_s + 2) \\
\vdots \\
x(k) \\
\vdots \\
x(k + K - 1)
\end{bmatrix},
\]
where the time index for the input symbols has been shifted correspondingly.

Denote the input symbols for each data stream as $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{N_R}$. Define
\[
\tilde{x}(k) = [\tilde{x}_1(k) \ \tilde{x}_2(k) \ \cdots \ \tilde{x}_{N_R}(k)]^T,
\]
\[
\tilde{x}_B = [\tilde{x}_1^T(k) \ \tilde{x}_2^T(k + 1) \ \cdots \ x^T(k + K - 1)]^T.
\]
The pre-equalizer is applied to the block data stream $\tilde{x}_B$. Therefore, the pre-equalizer is characterized by a matrix $F$ of dimension $N_R(K + L_s - 1) \times N_R K$. The system model can be rewritten as
\[ \tilde{y} = \hat{G}_S F \tilde{x}_B + \tilde{n}, \quad (6.21) \]
where the vector $\tilde{y} = [y^T(k) \ y^T(k + 1) \ \cdots \ y^T(k + K - 1)]^T$ is the approximation of the received signal with the application of both the pre-equalizer and TR filter. The covariance matrices of the transmitted signal and noise are characterized by $\mathbb{E}[\tilde{x}_B \tilde{x}_B^T] = \sigma_x^2 I$ and $\mathbb{E}[\tilde{n}\tilde{n}^T] = \sigma_n^2 I$, respectively.

The general principle for the design of the pre-equalizer $F$ is to make $\tilde{y}$ approach $\tilde{x}_B$ as near as possible. The criterion for the near can be either the ZF or MMSE. Both of them are analysed in the following.

### 6.4.2.1 Zero-Forcing Pre-Equalization

The pre-equalization with the ZF criterion guarantees that the received data is identical to the transmitted data in the absence of receiver noise. This simple equalizer, however,
causes some difficulty in its implementation when the channel suffers deep fading. In this case, the values of some entries of the equalizer matrix, which is derived from the inverse of the CIR matrix, might be very large, causing the relevant power amplifier to work inefficiently. To deal with this problem, the power constraint for the transmit signal (after the pre-equalizer and TR filter) is introduced below.

It is easy to find the pre-equalizer matrix $F$ in Equation (6.21) as follows (denoted as $F_{ZF}$ for the ZF case):

$$F_{ZF} = \alpha_{ZF} \tilde{G}_S^+ = \alpha_{ZF} (\tilde{G}_S^T \tilde{G}_S)^{-1} \tilde{G}_S^T,$$

where the superscript $^+$ denotes the Moore–Penrose pseudo-inverse of a matrix. The coefficient $\alpha_{ZF}$ is introduced for the power constraint of the transmit signal, which is determined by

$$E \left[ \| \bar{G}_B F_{ZF} \tilde{x}_B \|^2_2 \right] = E \left[ \| \alpha_{ZF} \bar{G}_B \tilde{G}_S^+ \tilde{x}_B \|^2_2 \right] = E_{tx}, \quad (6.22)$$

where $\| \cdot \|_2$ denotes the Euclidean norm of a vector. Equation (6.22) says that the total power of the transmitted signal at the transmit antennas for all the processing schemes is fixed to be $E_{tx}$. The matrix $\bar{G}_B$ represents the TR filter in the format of block-data processing, which is an $N_T K \times N_R (K + L_s - 1)$ block Toeplitz matrix defined by

$$\bar{G}_B = \begin{bmatrix} \tilde{G}_0 & \ldots & \tilde{G}_{L_s-1} & 0 & \ldots & 0 \\ 0 & \tilde{G}_0 & \ldots & \tilde{G}_{L_s-1} & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \tilde{G}_0 & \ldots & \tilde{G}_{L_s-1} \end{bmatrix}, \quad (6.23)$$

where

$$\tilde{G}_l = \begin{bmatrix} h_{11}[l] & h_{21}[l] & \ldots & h_{N_R 1}[l] \\ h_{12}[l] & h_{22}[l] & \ldots & h_{N_R 2}[l] \\ \vdots & \vdots & \ddots & \vdots \\ h_{1N_T}[l] & h_{2N_T}[l] & \ldots & h_{N_R N_T}[l] \end{bmatrix}. \quad (6.24)$$

In Equation (6.24), $h_{ij}[l]$ represents the value of the CIR $h_{ij}(t)$ at the sampling instant $t = l \Delta T$. Equations (6.23) and (6.24) jointly represent the TR filter (6.10) in the format of block-data processing.

The power constraint (6.22) reduces to

$$\alpha_{ZF} \sigma_s^2 \text{tr}[ (\bar{G}_B \tilde{G}_S^+)^T \bar{G}_B \tilde{G}_S^+ ] = E_{tx}, \quad (6.25)$$

where $\text{tr}(\cdot)$ denotes the trace of a square matrix. The solution to the power constraint (6.25) is

$$\alpha_{ZF} = \sqrt{\frac{E_{tx}}{\sigma_s^2 \text{tr}[ (\bar{G}_B \tilde{G}_S^+)^T \bar{G}_B \tilde{G}_S^+]}}.$$
6.4.2.2 Minimum Mean-Square Error Pre-Equalization

The MMSE pre-equalizer is designed to minimize the mean-square error (MSE) between the receive and transmit data. The equalizer matrix is the solution of the optimization problem below:

\[
\text{arg min}_F \mathbb{E}[\| \tilde{\mathbf{x}}_B - \tilde{\mathbf{y}} \|^2_2]
\]

such that \( \mathbb{E}[\| \mathbf{G}_B F \tilde{\mathbf{x}}_B \|^2_2] = E_{tx} \).

The MSE equation can be rewritten as

\[
\mathbb{E}[\| \tilde{\mathbf{x}}_B - \tilde{\mathbf{y}} \|^2_2] = \mathbb{E}[\| \tilde{\mathbf{x}}_B - \hat{G}_S F \tilde{\mathbf{x}}_B - \bar{n} \|^2_2]
\]

\[
= \sigma_x^2 \text{tr}(\mathbf{I} - \hat{G}_S F)^T (\mathbf{I} - \hat{G}_S F) + \sigma_n^2.
\]

The power constraint equation gives

\[
\sigma_x^2 \text{tr}(\mathbf{G}_B F)^T \mathbf{G}_B F - E_{tx} = 0.
\]

Construct the Lagrangian function

\[
\mathcal{L} = \mathbb{E}[\| \tilde{\mathbf{x}}_B - \tilde{\mathbf{y}} \|^2_2] + \lambda (\mathbb{E}[\| \mathbf{G}_B F \tilde{\mathbf{x}}_B \|^2_2] - E_{tx})
\]

\[
= \sigma_x^2 \text{tr}(\mathbf{I} - \hat{G}_S F)^T (\mathbf{I} - \hat{G}_S F) + \sigma_n^2 + \lambda \{\sigma_x^2 \text{tr}(\mathbf{G}_B F)^T \mathbf{G}_B F - E_{tx}\},
\]

where the positive number \( \lambda \) is a Lagrangian multiplier. Setting the first derivative of the Lagrangian function to zero, we obtain the solution for the MMSE pre-equalizer as follows (denoted as \( F_{\text{MMSE}} \) for the MMSE case):

\[
F_{\text{MMSE}} = (\hat{G}_S^T \hat{G}_S + \lambda \mathbf{G}_B^T \hat{G}_B)^{-1} \hat{G}_S^T,
\]

where \( \lambda \) is the solution of the equation

\[
\text{tr}[\mathbf{G}_B (\hat{G}_S^T \hat{G}_S + \lambda \mathbf{G}_B^T \mathbf{G}_B)^{-1} \hat{G}_S^T \hat{G}_S (\hat{G}_S^T \hat{G}_S + \lambda \mathbf{G}_B^T \mathbf{G}_B)^{-1} \mathbf{G}_B] = \frac{E_{tx}}{\sigma_x^2}.
\]

When the transmit power is large, the solution \( \lambda \) to Equation (6.26) will be very small. In this case, the MMSE pre-equalizer matrix is close to the ZF pre-equalizer matrix.

6.4.3 Numerical Results

Simulations are conducted to verify the performance of the TR SM-MIMO-UWB systems with the proposed pre-equalizers. In the simulations, the data symbols are modulated in the binary pulse amplitude modulation (BPAM) format. The monocycle UWB waveform is

\[
p(t) = \left[1 - 4\pi \left(\frac{t - t_c}{\tau_p}\right)^2\right] \exp\left[-2\pi \left(\frac{t - t_c}{\tau_p}\right)^2\right],
\]

where \( \tau_p \) is a parameter corresponding to the pulse width and \( t_c \) is a time shift of the pulse. In the simulations, \( \tau_p = 1 \) ns and \( t_c = \tau_p/2 \). The symbol duration is 1.5 ns,
corresponding to a data rate of 667 Mb/s per data stream. It is assumed that the signal is perfectly synchronized at the receiver. In the simulations, the IEEE 802.15.3a channel models CM1–CM4 [77] are used. The average BER is evaluated over 100 channel realizations for each simulation.

Figure 6.9 illustrates the transmit diversity capability of the TR SM-MIMO-UWB system for different channel models. At the same SNR = 12 dB, when the number of transmit antennas increases, the average BER decreases. It can be seen that transmit diversity has been achieved. However, the increasing rate becomes small when the number of transmit antennas is high. For high-speed data transmission and a small number of transmit antennas, mitigation is necessary to deploy the equalizer for ISI and MSI.

The BER performances of the TR UWB SM-SIMO systems with and without the pre-equalizer for the CM1–CM4 channels are shown in Figures 6.10–6.13 respectively. The ZF pre-equalizer without power constraint is designed based on the shortened equivalent channels. The length of the shortened channels is chosen from Table 6.1 according to the criterion that the power threshold is 90% of the total power. In the simulation, \( N_R (2, 3 \text{ and } 4 \text{ respectively}) \) independent data streams are transmitted over a single transmit antenna. The performance of the SM-SIMO system severely degrades if the equalizer is not used. The greater the number of data streams, the poorer the BER performance of the system. This is because the MSI involved increases with the number of data streams. From Figures 6.10–6.13, it can be seen that the proposed pre-equalizer can significantly improve the system performance. The error floor of the average BER is reduced by almost one order. For example, when the number of data streams \( N_R = 4 \), the BER performance for the CM3 channel decreases from \( \text{BER} = 3 \times 10^{-2} \) for the case of no equalizer to \( \text{BER} = 9 \times 10^{-4} \) for the case of using the ZF equalizer. If the number of data streams is small, for example, \( N_R = 2 \) and 3, then the BER error floor of the system with the ZF pre-equalizer is broken up for the CM1, CM2 and CM3 channels.

![Figure 6.9](image_url)

**Figure 6.9** The BER performance of an SM-MIMO-UWB system versus the number of transmit antennas. Here, SNR = 12 dB and \( N_R = 4 \).
Figure 6.10  The BER performance of the UWB SM-SIMO with and without a ZF equalizer for the CM1 channel.

Figure 6.11  The BER performance of the UWB SM-SIMO with and without a ZF equalizer for the CM2 channel.

Figure 6.14 shows the dependence of the average BER on the length of the shortened channel used in the pre-equalizer. The BER performance is plotted with respect to the ratio between the length of the shortened channel \(L_s\) and the full channel length \(L_m\). The SNR is fixed at 12 dB. When the length of the shortened channel increases, the BER decreases
drastically first and then approaches to some saturation points. At the saturation point, if the shortened channel length increases further, the BER decreases slightly. Simulation results show that the equivalent channel CM3 possesses a better compression property than other channels CM1, CM2, and CM4. These results agree with the fact as illustrated in Table 6.1.
When there is a power constraint at the transmitter side, the performance of two pre-equalizers, i.e., the power-constraint ZF and MMSE pre-equalizers, is compared. The result is illustrated in Figure 6.15. As can be seen, the performance of the MMSE pre-equalizer is slightly better than that of the ZF pre-equalizer.
Notice that, in the simulations, the spatial correlation among the different channels is not considered. Therefore, the results are generally valid when the receive antennas are separated sufficiently far. In the TR SM-UWB-SIMO systems considered in this section, the multiple receive antennas can be antennas from different users. Therefore, a separation of several tens of centimetres among the antennas is quite normal. On the other hand, it is shown in [118, 152] that the system performance of UWB-MIMO systems will not suffer too much degradation if the antennas are separated from each other by more than 10 cm.

6.5 Antenna Selection for Time-Reversal UWB MIMO Systems

In general, the greater the number of transmit antennas in a communication system, the more spatial diversity the system possesses and, hence, the better the system performance. However, the complexity and cost of the system increase considerably with the number of transmit antennas. In narrowband MIMO systems, antenna selection has been widely exploited to compromise the complexity/cost and performance of the system [91, 102, 188]. For UWB TR systems, antenna selection can be naturally applied since the channel state information has already been available at the transmitter side. In this section, some performance simulation results will be presented to show the benefit of antenna selection in UWB TR systems.

We consider the UWB MIMO TR system as addressed in the preceding section. Assume that \( N_R \) data streams are transmitted over some \( \tilde{N}_T \) antennas instead of all \( N_T \) antennas. There are \( \binom{N_T}{\tilde{N}_T} \) possible subsets of the selected transmit antennas. The criterion for antenna subset selection is to maximize the signal-to-interference power ratio (SIR).

Let the CIR between the \( j \)th transmit antenna and the \( i \)th receive antenna be

\[
g_{ij}(t) = \sum_{l=1}^{L} h_{ij}(l) \delta(t - \Delta T).\]

Define vector \( \mathbf{h}_{ij} \) and \( L \times (2L - 1) \)-dimensional Toeplitz matrix \( \mathbf{H}_{ij} \) as follows:

\[
\mathbf{h}_{ij} = \begin{bmatrix} h_{ij}(0) & \cdots & h_{ij}(L - 1) \\ 0 & \cdots & h_{ij}(L - 1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_{ij}(0) & \cdots & h_{ij}(L - 1) \end{bmatrix},
\]

\[
\mathbf{H}_{ij} = \begin{bmatrix} h_{ij}(0) & \cdots & h_{ij}(L - 1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_{ij}(0) & \cdots & h_{ij}(L - 1) \end{bmatrix}.
\]

Then the SIR at the \( i \)th receiver reads as follows [177]

\[
\text{SIR}_i = \frac{\left\| \sum_{j=1}^{\tilde{N}_T} \mathbf{h}_{ij} \mathbf{H}_{ij} \right\|_F^2}{\left\| \sum_{j=1}^{\tilde{N}_T} \sum_{k=1, k \neq i}^{N_R} \mathbf{h}_{ij} \mathbf{H}_{kj} \right\|_F^2}, \quad i = 1, \ldots, N_R, \quad (6.28)
\]
where $\| \cdot \|_F$ denotes the Frobenius norm. This ratio is calculated from the channel state information obtained at the transmitter side.

Notice that it is not necessary for the number of transmit antennas $N_T$ or $\bar{N}_T$ to be greater than the number of data streams $N_R$ because of the capability of the TR approach in the spatial multiplexing.

Simulations are conducted to examine the performance of the proposed antenna selection scheme for UWB MIMO TR systems. Binary data, taking values from $\{\pm 1\}$, is modulated to the BPAM format. The UWB monopulse (6.27) with $\tau_p = 1$ ns and $t_c = \tau_p/2$ is used in the simulation. The symbol duration is 5 ns, giving a data rate of 200 Mb/s per data stream. Assume that the received signal is perfectly synchronized at the receiver. In the simulation, the IEEE 802.15.3a CM4 channel model [165] is used for each channel.

The BER performance of UWB MIMO TR systems using the antenna selection scheme is shown in Figures 6.16 and 6.17. Figure 6.16 is for the case of the systems with two, three and four data streams transmitted over a single transmit antenna, where the antenna selection algorithm chooses one transmit antenna from two available transmit antennas. Figure 6.17 is for the case of the systems selecting two antennas from four available transmit antennas, where the numbers of data streams to be transmitted are three and four. It can be seen that, by exploiting the antenna selection scheme, the BER performance of these systems is considerably improved compared with the system without antenna selection. For example, the $2 \times 3$ MIMO system with four available transmit antennas (hence selecting two out of four transmit antennas) yields an SNR gain of about 5 dB at $BER = 10^{-3}$ compared with the $2 \times 3$ MIMO system without antenna selection.

![Figure 6.16](image-url)  
**Figure 6.16** BER performance of UWB MIMO TR systems $1 \times N_R$: $N_R (=2, 3$ and $4)$ data streams are transmitted over a single transmit antenna. One antenna is selected from two available transmit antennas. (From [177]. Reproduced by permission of © IEEE 2009.)
6.6 Impact of Channel Imperfection on UWB TR Systems

In the previous discussions, it is assumed that the channel information can be exactly obtained. In practice, this assumption rarely holds true due to the facts mentioned in Section 6.2.3. This raises an important problem regarding how to evaluate (and, hence, to improve if possible) the performance of a wireless system employing the TR technique when the channel information is not exactly known. In this section, we will study the first part of this problem, i.e., the evaluation of the effect of inexact channel parameters on the UWB TR system performance.

The rest of the section is organized as follows. The channel model that is used in this section will be briefly reviewed in Section 6.6.1. Then the BER and SNR or SINR analysis for SISO TR systems and MIMO TR systems will be presented in Sections 6.6.2 and 6.6.3 respectively. In Section 6.6.4, a multiuser communication scheme using the TR technique will be proposed and the SNR performance will be discussed. Numerical results are given in Section 6.6.5.

6.6.1 Channel Model

The channel model described in Chapter 2 will be used in Section 6.6. For reasons of notational clarity, the model will be briefly summarized here. First, consider the UWB SISO channel. We will use the simplified version of general UWB channel models, which
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is described by the following equation:

\[
g(t) = \sum_{l=0}^{L_m-1} h(l) \delta(t - l \Delta T), \tag{6.29}
\]

where \( g \) is the CIR, \( \delta \) is the Dirac delta function, \( L_m \) is the number of multipath components, \( \Delta T \) is the sampling period and \( h(l) \) is the channel fading in the \( l \)th delay bin, which describes the small-scale fading of the channel. The channel fading is characterized by the Nakagami distribution.

Denote \( h(l) \) as \( h(l) = \nu_l \xi_l \). The statistics of \( \xi_l \) are described by the following pdf:

\[
p_{\xi_l}(x) = \frac{2^{\kappa/2}}{\Gamma(\kappa/2)} x^{(\kappa/2) - 1} e^{-\beta_l x^2} U(x), \quad \kappa \geq 1,
\]

where \( \Gamma(\cdot) \) denotes the Gamma function, \( \kappa = 2[\mathbb{E}(\xi_l^2)]^2/\text{Var}[\xi_l^2], \beta_l = \kappa/(2\Omega_l), \Omega_l = \mathbb{E}(\xi_l^2) \) and the unit step function \( U(\cdot) \) is defined as

\[
U(x) = \begin{cases} 
1 & \text{when } x \geq 0, \\
0 & \text{otherwise}. \end{cases}
\] (6.31)

Define \( \eta_l = h^2(l) = \xi_l^2 \). Then \( \eta_l \) is of the Gamma distribution, whose pdf is

\[
p_{\eta_l}(x) = \frac{\beta_l^{\kappa/2}}{\Gamma(\kappa/2)} x^{(\kappa/2) - 1} e^{-\beta_l x} U(x), \quad \kappa \geq 1.
\] (6.32)

The variable \( \nu_l \) takes the signs \(+1\) and \(-1\) with equal probability. The power of the amplitude fading is exponentially decreasing with the excess delay. Thus, we have

\[
\Omega_l = \varrho \Omega_{l-1},
\] (6.33)

where \( \varrho < 1 \) is a constant, referred to as the channel decay exponent.

Throughout this section, we assume that (i) \( \kappa \) is an even number and (ii) \( \kappa \) is of the same value for all the delay bins. The first assumption on \( \kappa \) is necessary for the theoretical BER analysis in Section 6.6.2.1, while the second one is just for the convenience of the analysis and simulations.

The \( \ell \)th moment of the variable \( \xi_l \) is given by [192, p. 47]

\[
\mathbb{E}[\xi_l^\ell] = \frac{\Gamma((\kappa + \ell)/2)}{\Gamma(\kappa/2)} \left( \frac{2\Omega_l}{\kappa} \right)^{\ell/2}.
\]

It is assumed that \( \nu_l \) and \( \xi_l \), \( \xi_{l_1} \) and \( \xi_{l_2} \), and \( \nu_{l_1} \) and \( \nu_{l_2} \) are independent of each other respectively for all \( l, l_1, \) and \( l_2 \) such that \( l_1 \neq l_2 \).

Next, consider the UWB MIMO channel. Let \( N_T \) and \( N_R \) denote the numbers of transmit antennas and receive antennas respectively. The MIMO channel model is described by the following Equation [118]:

\[
G(t) = \sum_{l=0}^{L_m-1} H(l) \delta(t - l \Delta T), \tag{6.34}
\]
where $H(l)$ is the channel fading matrix in the $l$th delay bin and $G$ is the CIR matrix with $G_{ij}$ being the CIR from the $j$th transmit antenna to the $i$th receive antenna. To make further analysis tractable, we need the following assumption.

**Assumption 6.1** The amplitude fading matrices $H(l)$, $l = 0, \ldots, L_m - 1$, are assumed to be independent, and all the entries of $H(l)$, $l = 0, \ldots, L_m - 1$, are also assumed to be independent. Suppose $H(l) = [h(l)_{k_1,k_2}]$ with $h(l)_{k_1,k_2} = \nu_{l,k_1,k_2} \zeta_{l,k_1,k_2}$, $l = 0, \ldots, L_m - 1$, $k_1 = 1, \ldots, N_R$, $k_2 = 1, \ldots, N_T$. Then $\nu_{l,k_1,k_2}$ takes values $\pm 1$ with equal probability and $\zeta_{l,k_1,k_2}$ assumes distribution (6.30), with $\Omega_l$ being governed by model (6.33).

### 6.6.2 Analysis of UWB SISO TR Systems

Denote the estimate of the CIR as $\hat{h}(l)$, $l = 0, 1, 2, \ldots, L - 1$ with $L \leq L_m$, which means that only the first $L$ taps of the estimated CIR will be used in the TR pre-filter at the transmitter. At the transmitter, each data symbol, denoted as $s$, will be modulated by the time-reversed CIR estimate $\hat{h}(L - 1 - l)$, i.e., the actual transmitted signal $x_s(t)$ for the symbol $s$ is

$$x_s(t) = s \sum_{l=0}^{L-1} \hat{h}(L - 1 - l) w(t - l \Delta T).$$

(6.35)

The equivalent CIR for the system with the TR pre-filter is

$$\hat{h}(l) = h(l) * \hat{h}(L - 1 - l), \ l = 0, 1, 2, \ldots, L + L_m - 2.$$  

(6.36)

Expanding Equation (6.36) yields

$$\hat{h}(l) = \begin{cases} 0 & \text{if } l < 0 \text{ or } l \geq L_m + L - 1, \\ \sum_{k=0}^{L-1} h(k) \hat{h}(L - 1 - l + k) & \text{if } 0 \leq l \leq L - 1, \\ \sum_{k=l-L+1}^{L_m-1} h(k) \hat{h}(L - 1 - l + k) & \text{if } L \leq l \leq L_m - 1, \\ \sum_{k=l-L+1}^{L_m-1} h(k) \hat{h}(L - 1 - l + k) & \text{if } L_m \leq l \leq L_m + L - 2. \end{cases}$$

(6.37)

Generally, channel estimation errors can be modelled as AWGNs if a linear channel estimator is used. Hence, we can write

$$\hat{h}(l) = h(l) + \tilde{n}(l), \ l = 0, 1, 2, \ldots, L - 1,$$

(6.38)

where $\hat{h}$ is the estimate of $h$ and $\tilde{n}$ is the channel estimation error, which is assumed to be white Gaussian with zero mean and variance $\sigma_n^2$. For the convenience of later exposition, we define the *noise level* of the channel estimation as $10 \log_{10}(\Omega_0/\sigma_n^2)$ (in decibels), i.e., the ratio between the power of the strongest channel tap and the noise power contained in the channel estimator.
The equivalent composite CIR $\hat{h}(l)$ statistically achieves its peak at $l = L - 1$:

$$\hat{h}(L - 1) = \sum_{l=0}^{L-1} h(l)\hat{h}(l) = \sum_{l=0}^{L-1} [h^2(l) + h(l)\tilde{n}(l)].$$

Equation (6.35) describes how a single symbol is modulated in a TR system. For a symbol sequence $s(i)$, $i = 0, \pm 1, \ldots$, the general modulation structure is as follows:

$$x_s(t) = \sum_{i=-\infty}^{+\infty} s(i) \sum_{l=0}^{L-1} \hat{h}(L - 1 - l)w(t - l\Delta T - iL_s\Delta T),$$

where $L_s$ is a parameter controlling the symbol rate. The received signal, after passing through the matching filter (matched to the waveform of the UWB monocycle) and sampling, can be expressed as

$$r(l) = \sum_{i=-\infty}^{+\infty} s(i)\hat{h}(l - iL_s) + n(l),$$

where $n$ is the receiver noise, which is white Gaussian with zero mean and variance $\sigma_n^2$.

It is clear that the CIR $h(l)$ at the $l$th tap possesses relatively less power when $l$ becomes larger due to the exponentially decayed power delay profile. Therefore, the estimated CIR $\hat{h}(l)$ at the $l$th tap will contain more erroneous information about the original channel when $l$ becomes larger. Hence, the length of adopted CIR in the TR $L$ and the channel estimation accuracy, characterized by the parameter $\sigma_n^2$, have a critical influence on the system performance. The purpose of this section is to analyse the effect of the two parameters $L$ and $\sigma_n^2$ on the performance of UWB TR systems in the sense of BER and SINR.

In the following, we separate the problem into two cases: low data rate and high data rate.

### 6.6.2.1 The Case of Low Data Rate

It is assumed that $L_s \geq L_m$ in the case of low data rate. To decode the transmitted symbols $\{s(i)\}$, the received signals $r$ are sampled at the time instants $t = (iL_s + L - 1)T$, $i = 0, \pm 1, \ldots$. Therefore, no ISI is contained in the received signal. Thus, it is sufficient to investigate the system performance for one symbol transmission. Let us define

$$r_0 := r(L - 1) = s(0) \sum_{l=0}^{L-1} [h^2(l) + h(l)\tilde{n}(l)] + n(L - 1).$$

In the following, the BER will be calculated. Assume that binary data bits are transmitted. Throughout this section, it is assumed that the transmitted bits at different time instants are independent of each other and each bit takes the value $\pm 1$ with equal probability. Suppose $s(0) = +1$. Then the probability that the decision device outputs an erroneous bit $-1$ is given by

$$P_b = \Pr \left\{ \sum_{l=0}^{L-1} [h^2(l) + h(l)\tilde{n}(l)] + n(L - 1) < 0 \right\}.$$
Define the event \( A_1 = \{ \sum_{j=0}^{L-1} [h^2(l) + h(l)\bar{n}(l)] + n(L-1) < 0 \} \). To calculate the probability \( P_b \), we first derive the conditional probability \( \Pr(A_1 \mid \bar{h}) \) for a given channel realization \( \bar{h} := [h(0), \ldots, h(L_m - 1)] \). This reads as follows

\[
\Pr(A_1 \mid \bar{h}) = \Pr \left\{ \sum_{l=0}^{L-1} h(l)\bar{n}(l) + n(L-1) < -\sum_{l=0}^{L-1} h^2(l) \right\} = 1 - Q \left( \frac{-\sum_{l=0}^{L-1} h^2(l)}{\sqrt{\sigma_n^2 + \sigma_n^2 \sum_{l=0}^{L-1} h^2(l)}} \right),
\]

where \( Q(\cdot) \) denotes the \( Q \)-function [192, p. 60]. Similarly, if a data bit \(-1\) is transmitted, then the same error probability is produced. Therefore, the conditional bit error probability \( P_b \) is given by \( \Pr(A_1 \mid \bar{h}) \) for a given \( \bar{h} \).

Let

\[
\eta = \sum_{l=0}^{L-1} h^2(l).
\]

Then \( \eta \) is the summation of \( L \) independent Gamma-distributed random variables. The pdf of \( \eta \) reads as follows [52]:

\[
p_{\eta}(\eta) = \left\{ \begin{array}{l}
(-1)^{(L-1)k/2} \left( \prod_{l=0}^{L-1} \beta_l^{k/2} \right) \sum_{j_1=1}^{\beta_l} \sum_{j_2=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \sum_{j_{L-2}=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \\
\times \sum_{j_{L-3}=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \sum_{j_{L-4}=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \\
\times \cdots \times \sum_{j_2=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \sum_{j_1=1}^{\beta_l} \frac{\beta_l^k}{\Gamma(j_L-2)^k} \\
\{ e^{-\beta \eta}, \ \text{if} \ \beta_l \neq \beta_l', \ l, l' \in \{0, 1, \ldots, L-1\}, \ l \neq l'; \\
\beta_l^{\frac{k}{L}} \eta^{\frac{k}{L}} - 1 e^{-\beta \eta}, \ \text{if} \ \beta_0 = \beta_1 = \cdots = \beta_{L-1} = \beta, \\
\end{array} \right.
\]

where \( C(j_1, j_2) \) is defined as

\[
C(j_1, j_2) = \binom{j_1 + j_2 - 2}{j_1 - 1} = \binom{j_1 + j_2 - 2}{j_2 - 1},
\]

\[
C(j_1, 1) = C(1, j_1) = 1, \ \text{for all} \ j_1 \geq 1,
\]
and $\beta(j; l)$ is the $j$th element of the ordered set $\{\beta_0, \beta_1, \ldots, \beta_{L-1}\} \setminus \{\beta_l\}$ (where ‘\setminus’ denotes the set difference), i.e.,

$$
\beta(j; l) = \begin{cases} 
\beta_{j-1} & \text{if } l > j - 1, \\
\beta_j & \text{if } l \leq j - 1.
\end{cases}
$$

The BER $P_b$ can be obtained based on Equations (6.39) and (6.40) as follows:

$$
P_b = \int_0^\infty Q\left(\frac{\eta}{\sqrt{\sigma_n^2 + \sigma_n'^2 \eta}}\right) p_\eta(\eta) \, d\eta. \tag{6.41}
$$

It is difficult to find the closed-form expression. However, the numerical solution for $P_b$ can be easily obtained from Equations (6.40) and (6.41).

The average SNR for the signal $r_0$ is (noting that $\mathbb{E}[\kappa^2(0)] = 1$):

$$
\rho_{r_0} = \frac{\mathbb{E}\left\{\sum_{l=0}^{L-1} h^2(l_1) \sum_{l_2=0}^{L-1} h^2(l_2)\right\}}{\mathbb{E}\left\{\sum_{l_1=0}^{L-1} h(l_1) \tilde{n}(l_1) \sum_{l_2=0}^{L-1} h(l_2) \tilde{n}(l_2)\right\} + \sigma_n^2}.
$$

By using the statistical property of $h(l)$, it is found that

$$
\mathbb{E}\left\{\sum_{l_1=0}^{L-1} h^2(l_1) \sum_{l_2=0}^{L-1} h^2(l_2)\right\} = \mathbb{E}\left\{\sum_{l_1=0}^{L-1} h^2(l)\right\} + \mathbb{E}\left\{\sum_{l_1=0}^{L-1} \sum_{l_2=0, l_2 \neq l_1}^{L-1} h^2(l_1) h^2(l_2)\right\}
$$

$$
= \sum_{l=0}^{L-1} \frac{\Gamma[(\kappa/2) + 2]}{\Gamma(\kappa/2)} \left(\frac{2\Omega_l}{\kappa}\right)^2 + \sum_{l_1=0}^{L-1} \sum_{l_2=0, l_2 \neq l_1}^{L-1} \Omega_{l_1} \Omega_{l_2}
$$

$$
= \sum_{l=0}^{L-1} \frac{2}{k} \Omega_l^2 + \left(\sum_{l=0}^{L-1} \Omega_l \right)^2 = \left[\frac{2(1 - \rho^{2L})}{\kappa (1 - \rho^2)} + \frac{(1 - \rho^L)^2}{(1 - \rho)^2}\right] \Omega_0^2, \tag{6.42}
$$

$$
\mathbb{E}\left\{\sum_{l_1=0}^{L-1} h(l_1) \tilde{n}(l_1) \sum_{l_2=0}^{L-1} h(l_2) \tilde{n}(l_2)\right\} = \mathbb{E}\left\{\sum_{l=0}^{L-1} h^2(l)\right\} \sigma_n^2
$$

$$
+ \mathbb{E}\left\{\sum_{l_1=0}^{L-1} \sum_{l_2=0, l_2 \neq l_1}^{L-1} h(l_1) \tilde{n}(l_1) h(l_2) \tilde{n}(l_2)\right\}
$$

$$
= \sum_{l=0}^{L-1} \Omega_l \sigma_n^2 = \frac{1 - \rho^L}{1 - \rho} \Omega_0 \sigma_n^2.
$$

Thus, we have

$$
\rho_{r_0} = \frac{\left[\frac{2(1 - \rho^{2L})}{\kappa (1 - \rho^2)} + \frac{(1 - \rho^L)^2}{(1 - \rho)^2}\right] \Omega_0^2}{\frac{1 - \rho^L}{1 - \rho} \Omega_0 \sigma_n^2 + \sigma_n^2}.
$$
6.6.2.2 The Case of High Data Rate

It is assumed that $L_s < L_m$ in the case of high data rate. In this case, both the previous and later transmitted symbols will cause interference to the presently transmitted symbol. This is due to the fact that the symbol is modulated via the TR approach across a time interval centred around the time instant of interest. Let $M_a = \lfloor (L_m - 1)/L_s \rfloor$ and $M_p = \lfloor (L - 1)/L_s \rfloor$, where $\lfloor x \rfloor$ means to round $x$ to the nearest integer towards minus infinity.

Consider the $i$th transmitted symbol $s(i)$. The received signal $r$ sampled at the time instant $t = (iL_s + L - 1) T$ will be fed to the receiver to decode the symbol $s(i)$. Therefore, the sampled signal will contain the ISI from the a priori symbols $s(i - 1), \ldots, s(i - M_a)$ and a posteriori symbols $s(i + 1), \ldots, s(i + M_p)$ (see Figure 6.18 for an illustration).

![ISI structure for the high data rate TR system.](image)
Let \( r_i \) denote the received signal \( r \) sampled at time instants \( t = (iL_s + L - 1)T, i = 0, \pm 1, \ldots, \) and let \( M_d = \lceil (L_m - L)/L_s \rceil \), where \( \lceil x \rceil \) means to round \( x \) to the nearest integer towards plus infinity. Then

\[
 r_i = \sum_{k=-M_d}^{M_p} \hat{h}(L - 1 - kL_s)s(i - k) + n(iL_s + L - 1)
\]

\[
= \sum_{k=0}^{M_p} \sum_{l=0}^{L-1-kL_s} h(l)\hat{h}(kL_s + l)s(i - k) + \sum_{k=-M_d}^{-1} \sum_{l=-kL_s}^{L-1-kL_s} h(l)\hat{h}(kL_s + l)s(i - k)
\]

\[
+ \sum_{k=-M_d}^{-M_d-1} \sum_{l=-kL_s}^{L_m-1} h(l)\hat{h}(kL_s + l)s(i - k) + n(iL_s + L - 1)
\]

\[
= \sum_{l=0}^{L-1} [h^2(l) + h(l)\hat{n}(l)]s(i) + \sum_{k=1}^{M_p} \sum_{l=0}^{L-1-kL_s} h(l)[h(l + kL_s) + \hat{n}(l + kL_s)]s(i - k)
\]

\[
+ \sum_{k=1}^{M_d} \sum_{l=kL_s}^{L-1+kL_s} h(l)[h(l - kL_s) + \hat{n}(l - kL_s)]s(i + k)
\]

\[
+ \sum_{k=M_d+1}^{M_a} \sum_{l=kL_s}^{L_m-1} h(l)[h(l - kL_s) + \hat{n}(l - kL_s)]s(i + k) + n(iL_s + L - 1).
\]

In the following, we calculate the BER. As in the case of low data rate, we assume that binary data bits are transmitted. Suppose \( s(i) = +1 \) for the time instant \( t = iT \) considered. Then the probability that the decision device outputs an erroneous bit \(-1\) is given by

\[
P_b = \Pr \left\{ \sum_{l=0}^{L-1} [h^2(l) + h(l)\hat{n}(l)] + \xi_1 + n(iL_s + L - 1) < 0 \right\},
\]

where \( \xi_1 \) denotes the ISI, which is equal to

\[
\xi_1 = \sum_{k=1}^{M_p} \sum_{l=0}^{L-1-kL_s} h(l)[h(l + kL_s) + \hat{n}(l + kL_s)]s(i - k)
\]

\[
+ \sum_{k=1}^{M_d} \sum_{l=kL_s}^{L-1+kL_s} h(l)[h(l - kL_s) + \hat{n}(l - kL_s)]s(i + k)
\]

\[
+ \sum_{k=M_d+1}^{M_a} \sum_{l=kL_s}^{L_m-1} h(l)[h(l - kL_s) + \hat{n}(l - kL_s)]s(i + k)
\]

\[
= \sum_{k=1}^{M_p} \sum_{l=kL_s}^{L-1} h(l - kL_s)[h(l) + \hat{n}(l)]s(i - k)
\]
\[
+ \sum_{k=1}^{M_d} \sum_{l=0}^{L-1} h(l + kL_s)\[h(l) + \tilde{n}(l)\]s(i + k) \\
+ \sum_{k=M_d+1}^{M_a} \sum_{l=0}^{L_m-1-kL_s} h(l + kL_s)\[h(l) + \tilde{n}(l)\]s(i + k).
\]

(6.43)

To make the expression of the relevant noise variance compact, we assume that (i) both integers \(L_m - 1\) and \(L - 1\) can be divided by integer \(L_s\) and (ii) \(L \geq 2L_s\). The second assumption guarantees that \(L_1 := L_m - 1 - (M_d + 1)L_s \geq 0\). Based on these assumptions, the ISI \(\xi_1\) can be rewritten as

\[
\xi_1 = \sum_{l=0}^{L-1} \sum_{k=1}^{\lfloor l/L_s \rfloor} h(l - kL_s)\[h(l) + \tilde{n}(l)\]s(i - k) \\
+ \sum_{l=0}^{L-1} \sum_{k=1}^{M_d} h(l + kL_s)\[h(l) + \tilde{n}(l)\]s(i + k) \\
+ \sum_{l=0}^{L_1} \sum_{k=M_d+1}^{\lfloor (L_m-1-l)/L_s \rfloor} h(l + kL_s)\[h(l) + \tilde{n}(l)\]s(i + k).
\]

For the purpose of BER calculation, the transmitted symbols \(s(k), k \neq i\), in the ISI \(\xi_1\) can be considered to be absorbed into the sign of the channel fading, i.e., \(\xi_1\) has the same distribution as that of \(\xi_2\), where

\[
\xi_2 = \xi_3 + \xi_4,
\]

\[
\xi_3 = \sum_{l=0}^{L-1} \sum_{k=1}^{\lfloor l/L_s \rfloor} h(l - kL_s)h(l) + \sum_{l=0}^{L-1} \sum_{k=1}^{M_d} h(l + kL_s)h(l) \\
+ \sum_{l=0}^{L_1} \sum_{k=M_d+1}^{\lfloor (L_m-1-l)/L_s \rfloor} h(l + kL_s)h(l),
\]

\[
\xi_4 = \sum_{l=0}^{L-1} \sum_{k=1}^{\lfloor l/L_s \rfloor} h(l - kL_s)\tilde{n}(l) + \sum_{l=0}^{L-1} \sum_{k=1}^{M_d} h(l + kL_s)\tilde{n}(l) \\
+ \sum_{l=0}^{L_1} \sum_{k=M_d+1}^{\lfloor (L_m-1-l)/L_s \rfloor} h(l + kL_s)\tilde{n}(l).
\]

Let

\[
\xi_5 = \sum_{l=0}^{L-1} h(l)\tilde{n}(l) + \xi_4.
\]
In the following, we need to calculate the variance of the noise $\xi_5$, denoted $\sigma_{\xi_5}^2$. Since $\tilde{n}$ is white, it is easy to get

$$\sigma_{\xi_5}^2 = \sum_{l=0}^{L_s-1} \left[ h(l) + \sum_{k=1}^{M_d} h(l + kL_s) + \sum_{k=M_d+1}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$+ \sum_{l=L_s}^{L_1} \left[ h(l) + \sum_{k=1}^{M_d} h(l + kL_s) + \sum_{k=M_d+1}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$+ \sum_{l=L_1+1}^{L-1} \left[ h(l) + \sum_{k=1}^{M_d} h(l + kL_s) + \sum_{k=M_d+1}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$= \sum_{l=0}^{L_s-1} \left[ \sum_{k=0}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2 + \sum_{l=L_s}^{L_1} \left[ \sum_{k=0}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$+ \sum_{l=L_1+1}^{L-1} \left[ \sum_{k=0}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

when $L_1 \geq L_s$ and

$$\sigma_{\xi_5}^2 = \sum_{l=0}^{L_1} \left[ h(l) + \sum_{k=1}^{M_d} h(l + kL_s) + \sum_{k=M_d+1}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$+ \sum_{l=L_1+1}^{L_s-1} \left[ h(l) + \sum_{k=1}^{M_d} h(l + kL_s) \right]^2 \sigma_n^2 U(L_s - L_1 - 2)$$

$$+ \sum_{l=L_s}^{L-1} \left[ h(l) + \sum_{k=1}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

$$= \sum_{l=0}^{L_1} \left[ \sum_{k=0}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2 + \sum_{l=L_1+1}^{L_s-1} \left[ \sum_{k=0}^{M_d} h(l + kL_s) \right]^2 \sigma_n^2 U(L_s - L_1 - 2)$$

$$+ \sum_{l=L_s}^{L-1} \left[ \sum_{k=0}^{(L_m-1)/L_s} h(l + kL_s) \right]^2 \sigma_n^2$$

when $L_1 < L_s$. Define the event

$$\mathcal{A}_2 = \left\{ \sum_{l=0}^{L-1} \left[ h^2(l) + h(l)\tilde{n}(l) \right] + \xi_1 + n(iL_s + L - 1) < 0 \right\}.$$
To calculate the probability $P_b$, we first derive the conditional probability \( \Pr(A_2 \mid \vec{h}) \) for a given channel realization \( \vec{h} \). It is easy to obtain the following:

\[
\Pr(A_2 \mid \vec{h}) = \Pr \left\{ \xi_5 + n(iL_s + L - 1) < - \sum_{l=0}^{L-1} h^2(l) - \xi_3 \right\} = 1 - Q \left( \frac{- \sum_{l=0}^{L-1} h^2(l) + \xi_3}{\sqrt{\sigma_n^2 + \sigma_{\xi_5}^2}} \right) = Q \left( \frac{\sum_{l=0}^{L-1} h^2(l) + \xi_3}{\sqrt{\sigma_n^2 + \sigma_{\xi_5}^2}} \right).
\]

Similarly, if a data bit \(-1\) is transmitted, the same error probability is produced. Therefore, the conditional bit error probability is given by \( \Pr(A_2 \mid \vec{h}) \) for a given \( \vec{h} \). Finally, the average BER is given by

\[
P_b = \int Q \left( \frac{\sum_{l=0}^{L-1} h^2(l) + \xi_3}{\sqrt{\sigma_n^2 + \sigma_{\xi_5}^2}} \right) p_{\vec{h}}(\vec{h}) d\vec{h}, \quad (6.44)
\]

where we have abused the notation of random vector \( \vec{h} \) and its realization. Since \( h(0), \ldots, h(L_m - 1) \) are assumed to be independent of each other, the multivariate distribution of \( \vec{h} \) can be easily found. However, it is difficult to find the closed-form expression of Equation (6.44).

The average SINR for the signal \( r_i \) is

\[
\rho_{r_i} = \frac{\mathbb{E} \left\{ \left[ \sum_{l=0}^{L-1} h^2(l) \right]^2 \right\}}{\mathbb{E} \left\{ \left[ \sum_{l=0}^{L-1} h(l)\tilde{n}(l)s(i) + \xi_1 \right]^2 \right\} + \sigma_n^2}. \quad (6.45)
\]

Using the fact that \( \mathbb{E}[s(i)s(j)] = \delta(i - j) \), \( \mathbb{E}[\tilde{n}(i)\tilde{n}(j)] = \sigma_{\tilde{n}}^2 \delta(i - j) \), the statistical property of \( h(l) \) and Equation (6.43), we can obtain

\[
\sigma_{ISI}^2 := \mathbb{E} \left\{ \left[ \sum_{l=0}^{L-1} h(l)\tilde{n}(l)s(i) + \xi_1 \right]^2 \right\} = \mathbb{E} \left\{ \left[ \sum_{l=0}^{L-1} h(l)\tilde{n}(l) \right]^2 \right\} + \sum_{k=1}^{M_p} \left[ \sum_{l=kL_s}^{L-1} h(l - kL_s)[h(l) + \tilde{n}(l)] \right]^2
\]

\[
+ \sum_{k=1}^{M_d} \left[ \sum_{l=0}^{L-1} h(l + kL_s)[h(l) + \tilde{n}(l)] \right]^2
\]

\[
+ \sum_{k=M_d+1}^{M_d+M_a} \left[ \sum_{l=0}^{L_m-1-kL_s} h(l + kL_s)[h(l) + \tilde{n}(l)] \right]^2
\]
In the above derivation, we have implicitly used the independence property among \( s(i) \), \( h(l) \) and \( n(k) \) for all relevant indices \( i, l \) and \( k \) respectively. Substituting Equations (6.42)
and (6.46) into Equation (6.45) yields

$$\rho_{ri} = \frac{2(1-\varrho^2L)}{\kappa(1-\varrho^2)} + \frac{(1-\varrho L)^2}{(1-\varrho)^2} \Omega_0^2 \sigma_{ISI}^2 + \sigma_n^2.$$  

### 6.6.3 Analysis of UWB MIMO TR Systems

Denote the estimate of the channel matrix as $\hat{H}(l)$, $l = 0, \ldots, L - 1$. At the transmitter side, each time $N_R$ data symbol, denoted by an $N_R$-dimensional vector $s$, will be first modulated by the time-reversed CIR matrix estimate $\hat{H}(L - 1 - l)$ and then multiplexed to the transmitters, i.e., the actual transmitted signal $x(t)$ for the symbol $s$ is

$$x_s(t) = \sum_{l=0}^{L-1} \hat{H}^T(L - 1 - l)s w(t - l\Delta T).$$

Therefore, the equivalent CIR matrix for a TR channel is

$$\hat{H}(l) = H(l) * \hat{H}^T(L - 1 - l), \quad l = 0, 1, 2, \ldots, L + L_m - 2.$$  

Expanding Equation (6.47) yields

$$\hat{H}(l) = \begin{cases} 0 & \text{if } l < 0 \text{ or } l \geq L_m + L - 1, \\ \sum_{k=0}^{l} H(k)\hat{H}^T(L - 1 - l + k) & \text{if } 0 \leq l \leq L - 1, \\ \sum_{k=l-L+1}^{L_m-1} H(k)\hat{H}^T(L - 1 - l + k) & \text{if } L \leq l \leq L_m - 1, \\ \sum_{k=l-L+1}^{L_m-L+1} H(k)\hat{H}^T(L - 1 - l + k) & \text{if } L_m \leq l \leq L_m + L - 2. \end{cases}$$

Similar to the SISO case, the estimate of the CIR matrix can be modelled as

$$\hat{H}(l) = H(l) + \tilde{n}(l), \quad l = 0, 1, 2, \ldots, L - 1,$$

where $\tilde{n}$ is a white Gaussian noise matrix of the same size as that of $H$, with each entry of $\tilde{n}$ being independent of all other entries and having zero mean and variance $\sigma_{\tilde{n}}^2$.

The diagonal entries of $\hat{H}(l)$ statistically achieve their peak at $l = L - 1$.

$$\hat{H}(L - 1) = \sum_{l=0}^{L_m-1} H(l)\hat{H}^T(l),$$

$$[\hat{H}(L - 1)]_{ij} = \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} [H(l)]_{ik}([H(l)]_{jk} + [\tilde{n}(l)]_{jk}), \quad i, j = 1, \ldots, N_R.$$  

\footnote{In the rest of this section we will use subscripts to denote the entry of a vector or matrix. For example, $r_i$ (or $[r]_i$) and $A_{ij}$ (or $[A]_{ij}$) stand for the $i$th entry and $ij$th entry of $r$ and $A$ respectively.}
From Equation (6.48) we can see that the power of the diagonal entries of the composite TR CIR matrix $\hat{\mathbf{H}}$ can be strengthened due to the coherent superposition of the convolutions of the forward CIRs and the time-reversed CIRs, while the power of the off-diagonal entries of $\hat{\mathbf{H}}$ is relatively and statistically small, which contributes to the MUI.

For a symbol sequence $s(i), i = 0, \pm 1, \ldots$, the general modulation structure and receiver signal (after passing through the matching filter and sampling) are respectively described by the following equations:

$$
x_s(t) = \sum_{i=-\infty}^{+\infty} \sum_{l=0}^{L-1} \hat{\mathbf{H}}^T(L-1-l)s(i)w(t-l\Delta T - iL_s\Delta T),
$$

$$
r(l) = \sum_{i=-\infty}^{+\infty} \hat{\mathbf{H}}(l-iL_s)s(i) + n(l), \quad (6.48)
$$

where $n$ is the receiver noise vector, which is white Gaussian with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}$. From the structure of $\hat{\mathbf{H}}$ and Equation (6.48) we can see that the received signal suffers from two kinds of interference. The first kind is the ISI, which is caused by prior and posterior transmitted symbols if $L_s$ is too short; the second kind is the MUI, which is caused by symbols other than the symbol of interest.

### 6.6.3.1 The Case of Low Data Rate

In this case it is assumed that $L_s \geq L_m$. To decode the transmitted symbols $\{s(i)\}$, the received signal $r$ is sampled at time instants $t = (iL_s + L - 1)T, i = 0, \pm 1, \ldots$, at each receive antenna. Therefore, no ISI is contained in the received signal, but MUI exists at each receive antenna. In this case, it is sufficient to investigate the system performance for one symbol transmission. Without causing confusion, we omit the time indices in the transmitted symbol and received signal and denote them as $s$ and $r$ respectively. Then the $i$th entry of $r$, denoted as $r_i$, will be used to decode the $i$th symbol of $s$. We can write

$$
r := r(L-1) = \hat{\mathbf{H}}(L-1)s + n(L-1),
$$

$$
r_i = \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} [\mathbf{H}(l)]_{ik} \left\{ [\mathbf{H}(l)]_{ik} + [\tilde{n}(l)]_{ik} \right\} s_i + \sum_{j=1, j \neq i}^{N_R} \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} [\mathbf{H}(l)]_{ik} \left\{ [\mathbf{H}(l)]_{jk} + [\tilde{n}(l)]_{jk} \right\} s_j + n_i(L-1).$$

To calculate the BER, assume that binary data bits are transmitted. Consider the $i$th symbol of $s$. Suppose $s_i = +1$. Let us denote the event

$$
\mathcal{A}_3 = \left\{ \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} [\mathbf{H}(l)]_{ik}^2 + [\mathbf{H}(l)]_{ik} [\tilde{n}(l)]_{ik} \right\} + \sum_{j=1, j \neq i}^{N_R} \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} [\mathbf{H}(l)]_{ik} \left\{ [\mathbf{H}(l)]_{jk} + [\tilde{n}(l)]_{jk} \right\} s_j + n_i(L-1) < 0 \right\}.
$$
Then the probability that the decision device outputs an erroneous bit $-1$ is given by

$$P_b = \Pr(A_3).$$

Since $\mathbf{s}$ is independent of both $\mathbf{H}$ and $\tilde{\mathbf{n}}$, it can be absorbed into the sign of either $\mathbf{H}$ or $\tilde{\mathbf{n}}$. Thus:

$$P_b = \Pr(A_3) = \Pr(A_4),$$

where

$$A_4 = \left\{ \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik}^2 + \left[ \mathbf{H}(l) \right]_{ik} \left[ \tilde{\mathbf{n}}(l) \right]_{ik} \right. $$

$$ + \sum_{j=1, j \neq i}^{N_R} \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik} \left[ \mathbf{H}(l) \right]_{jk} + \left[ \tilde{\mathbf{n}}(l) \right]_{jk} + \mathbf{n}_s(L - 1) < 0 \right\}. $$

To calculate the probability $P_b$, we first derive the conditional probability $\Pr(A_4 \mid \tilde{\mathbf{H}})$ for a given channel realization $\tilde{\mathbf{H}} = [\mathbf{H}(0), \ldots, \mathbf{H}(L_m - 1)]$:

$$\Pr(A_4 \mid \tilde{\mathbf{H}}) = \Pr \left\{ \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik} \left[ \tilde{\mathbf{n}}(l) \right]_{ik} + \sum_{j=1, j \neq i}^{N_R} \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik} \left[ \mathbf{H}(l) \right]_{jk} + \mathbf{n}_s(L - 1) < 0 \right\} $$

$$= 1 - Q \left( \frac{\sum_{l=0}^{L_m-1} \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik} \left[ \mathbf{H}(l) \right]_{jk}}{\sqrt{\sigma_n^2 + \sigma_\tilde{n}^2 N_R \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik}^2}} \right) $$

$$= Q \left( \frac{\sqrt{\sigma_n^2 + \sigma_\tilde{n}^2 N_R \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik}^2}}{\sqrt{\sigma_n^2 + \sigma_\tilde{n}^2 N_R \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik}^2}} \right).$$

(6.49)

Similarly, if a data bit $-1$ is transmitted for the $i$th symbol of $\mathbf{s}$, then the same error probability is produced. Therefore, the conditional bit error probability is given by $\Pr(A_4 \mid \mathbf{H})$ for a given $\mathbf{H}$.

Finally, the average BER is given by

$$P_b = \int Q \left( \frac{\sum_{l=0}^{L_m-1} \sum_{j=1}^{N_R} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik} \left[ \mathbf{H}(l) \right]_{jk}}{\sqrt{\sigma_n^2 + \sigma_\tilde{n}^2 N_R \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \left[ \mathbf{H}(l) \right]_{ik}^2}} \right) p_{\tilde{\mathbf{H}}}(\tilde{\mathbf{H}}) d\tilde{\mathbf{H}}. $$

(6.50)

In Equation (6.50), we again abused the notation of random matrix $\tilde{\mathbf{H}}$ and its realization. Since $\mathbf{H}(0), \ldots, \mathbf{H}(L_m - 1)$ are assumed to be independent of each other, the multivariate distribution of $\tilde{\mathbf{H}}$ can be easily found. However, it is difficult to find the closed-form expression of Equation (6.50).
Noting that $\mathbb{E}(s_i^2) = 1$ and $\mathbb{E}(s_i s_j) = 0$ for $i \neq j$, we can obtain the average SNR for the signal $r_i$ as the following:

$$\rho_{r_i} = \frac{\gamma_1}{N_R \gamma_3 + \gamma_2 + \sigma_n^2},$$

where

$$\gamma_1 = \mathbb{E} \left\{ \sum_{l_1=0}^{L_m-1} \sum_{k_1=1}^{N_T} [H(l_1)]_{ik_1}^2 \sum_{l_2=0}^{L_m-1} \sum_{k_2=1}^{N_T} [H(l_2)]_{ik_2}^2 \right\}$$

$$= \mathbb{E} \left\{ \sum_{l_1=0}^{L_m-1} \sum_{k_1=1}^{N_T} [H(l_1)]_{ik_1}^4 + \sum_{l_1=0}^{L_m-1} \sum_{k_1=1}^{N_T} \sum_{l_2=1, k_2 \neq k_1}^{N_T} [H(l_1)]_{ik_1}^2 [H(l_2)]_{ik_2}^2 \right\}$$

$$= \sum_{l_1=0}^{L_m-1} \frac{\Gamma[(\kappa/2) + 2]}{\Gamma(\kappa/2)} \left( \frac{2\Omega_{l_1}}{\kappa} \right)^2 + \sum_{l_1=0}^{L_m-1} \sum_{k_1=1}^{N_T} \sum_{l_2=1, k_2 \neq k_1}^{N_T} \Omega_{l_1}^2$$

$$+ \sum_{l_1=0}^{L_m-1} \sum_{l_2=0, l_2 \neq l_1}^{L_m-1} \sum_{k_1=1}^{N_T} \sum_{k_2=1}^{N_T} \Omega_{l_1} \Omega_{l_2}$$

$$= \frac{\kappa + 2}{\kappa} N_T \sum_{l=0}^{L_m-1} \Omega_l^2 + N_T(N_T - 1) \sum_{l=0}^{L_m-1} \sum_{k=1}^{N_T} \Omega_l^2$$

$$= \left[ 2(1 - \frac{2L_m}{\kappa(1 - \varrho^2)}) \left( \frac{1 - \varrho^{L_m}}{(1 - \varrho)^2} \right) N_T \right] \Omega_0^2.$$
Combining the above equations gives

\[
\rho_{ri} = \frac{2(1-\varrho^{2L_m})N_T + (1-\varrho^{2L_m})\varrho^2}{N_{R}N_{T}1-\varrho^{L_m}\Omega_0\sigma_r^2 + (N_R - 1)N_{T}1-\varrho^{2L_m}\Omega_0^2 + \sigma_n^2} \Omega_1^2 
\]

Since the BER and average SNR analysis for the case of high data rate MIMO TR is extremely complicated, we omit the analysis for this case.

### 6.6.4 MISO TR Multiuser Communications

In a MISO TR system, there are two cases to be considered. The first case is that all the transmit antennas transmit the same data symbol at the same time. In this case, the multiple transmit antennas are used to exploit the diversity of the channels. The data modulation at the transmitter side is relatively simple. The \(i\)th transmit antenna modulates data symbols using the time-reversed CIR from the \(i\)th transmit antenna to the receive antenna. Each transmitted signal will propagate to the receiver and produces a peaky signal at the receiver. All the peaky signals will be coherently superpositioned at the peaky instant if the channel tap length from all transmit antennas to the receiver takes the same value. Hence, the data symbol can be easily decoded. The second case is that different transmitters want to communicate with the same receiver separately. In this case, if the peaks of the signals received from different transmitters coincide in time at the receiver, they will be catastrophically superpositioned and, hence, it is difficult to decode the symbols. Therefore, some kind of processing at the transmitters should be performed.

A simple processing scheme is similar to the time-division accessing. The basic idea is to use an attractive characteristic of the TR technique, namely its temporal focal point at the receiver can be adjusted by a change in the channel length used at the pre-filter. The analysis for the first case is essentially the same as the MIMO TR problem studied in the preceding subsection. Now we study the second case.

Let \(N_T\) denote the number of transmit antennas. It is supposed that each transmit antenna has the estimate of the channel from this transmit antenna to the receive antenna only. Similar to the SISO and MIMO cases, the channel model from the \(j\)th transmit antenna to the receive antenna can be described by the following equation:

\[
g_j(t) = \sum_{l=0}^{L_m-1} h_j(l)\delta(t - l\Delta T).
\]

The following simplifying assumption is made.

**Assumption 6.2** The amplitude fading taps \(h_j(l), l = 0, \ldots, L_m - 1, j = 1, \ldots, N_T\), are assumed to be independent of each other. Suppose \(h_j(l) = v_{jl}\xi_{jl}, j = 1, \ldots, N_T, l = 0, 1, \ldots, L_m - 1\). Then \(v_{jl}\) takes values \(\pm 1\) with equal probability and \(\xi_{jl}\) assumes distribution (6.30), with \(\Omega_l\) being governed by model (6.33).

Denote the estimate of the channel matrix as \(\hat{h}_j(l), l = 0, \ldots, L_j - 1, j = 1, \ldots, N_T\), where \(L_j\) is the length of the time-reversed channel used for the \(j\)th transmit antenna.
Notice that a different transmitter (user) will use a different length of the channel estimate. From the measured CIRs of TR systems, it can be observed that, neighbouring and in both sides of the peaky tap, there exist a number of taps whose power is considerably higher than that of other taps far from the peaky tap. Denote by $L_c$ the single-side length of these taps with conspicuous stronger power. Then $L_j$ can be chosen as follows:

$$L_j = L_0 + (j - 1)L_c \quad \text{with} \quad L_j \leq L_m, \quad j = 1, \ldots, N_T,$$

where $L_0$ is a given integer. When $L_j$, $j = 1, \ldots, N_T$, are so chosen, the peaky signals from different users will be separated sufficiently far from each other.

At the $j$th transmitter, the information symbols $s_j(i)$, $i = 0, \pm 1, \ldots$, will be modulated by the time-reversed CIR estimate $\hat{h}_j(L_j - 1 - l)$, i.e., the actual transmitted signal $x_j(t)$ is

$$x_j(t) = \sum_{i=\infty}^{+\infty} \sum_{k=0}^{L_j-1} \hat{h}_j(L_j - 1 - l)s_j(i)w(t - k\Delta T - iL_s\Delta T).$$

The received signal, after passing through the matching filter and sampling, can be expressed as

$$r(l) = \sum_{i=\infty}^{+\infty} \sum_{j=1}^{N_T} \hat{h}_j(l - iL_s)s_j(i) + n(l), \quad (6.52)$$

where $\hat{h}_j$ is the equivalent CIR of the TR channel from the $j$th transmit antenna to the receive antenna:

$$\hat{h}_j(l) = h_j(l) \ast \hat{h}_j(L_j - 1 - l), \quad l = 0, 1, 2, \ldots, L_j + L_m - 2. \quad (6.53)$$

Equation (6.53) is of a similar expansion as Equation (6.37), which we will not repeat here.

As in the preceding subsections, the channel estimate is modelled as

$$\hat{h}_j(l) = h_j(l) + \tilde{n}_j(l), \quad l = 0, 1, \ldots, L_j - 1,$$

where $\tilde{n}_j$ is a white Gaussian noise with zero mean and variance $\sigma^2_{\tilde{n}}$. All $\tilde{n}_j$, $j = 1, \ldots, N_T$, are assumed to be independent of each other.

From the structure of $\hat{h}_j$ and Equation (6.52) we can see that the received signal also suffers from two kinds of interference: ISI and MUI.

We omit the BER and average SNR analysis for the case of high data rate MISO TR systems since it is extremely complicated. In the following, we will analyse the case of low data rate. In this case, it is assumed that $L_s \geq L_m$. To decode the transmitted symbols $\{s_j(i)\}$, we need to synchronize the received signal $r$ with the time instants $t = (iL_s + L_j - 1)T$, $i = 0, \pm 1, \ldots$. Without loss of generality, we need only to consider the time instant for $i = 0$. For this reason, we omit the time index $i$ hereafter. Let us define

$$r_{j_0} := r(L_{j_0} - 1) = \sum_{j=1}^{N_T} \hat{h}_j(L_{j_0} - 1)s_j + n(L_{j_0} - 1).$$
The signal $r_{j_0}$ is used to decode the symbol transmitted at the $j_0$th transmit antenna. By using Equation (6.37) we can obtain

$$r_{j_0} = \sum_{j=1}^{j_0-1} \sum_{k=L_{j_0}-L_j}^{L_{j_0}-1} h_j(k) \hat{h}_j(L_j - L_{j_0} + k)s_j$$

$$+ \sum_{j=j_0}^{N_T} \sum_{k=0}^{L_{j_0}-1} h_j(k) \hat{h}_j(L_j - L_{j_0} + k)s_j + n(L_{j_0} - 1)$$

$$= \sum_{k=0}^{L_{j_0}-1} h_{j_0}(k)[h_{j_0}(k) + \tilde{n}_{j_0}(k)]s_{j_0}$$

$$+ \sum_{j=1}^{j_0-1} \sum_{k=L_{j_0}-L_j}^{L_{j_0}-1} h_j(k)[h_j(L_j - L_{j_0} + k) + \tilde{n}_j(L_j - L_{j_0} + k)]s_j$$

$$+ \sum_{j=j_0+1}^{N_T} \sum_{k=0}^{L_{j_0}-1} h_j(k)[h_j(L_j - L_{j_0} + k) + \tilde{n}_j(L_j - L_{j_0} + k)]s_j + n(L_{j_0} - 1).$$

The BER can be calculated by using the same way as shown in the preceding subsections, which will not be repeated here.

The average SNR for the signal $r_{j_0}$ is (noting again that $\mathbb{E}(s_i^2) = 1$ and $\mathbb{E}(s_i s_j) = 0$ for $i \neq j$)

$$\rho_{j_0} = \frac{\Upsilon_4}{\Upsilon_5 + \sigma_n^2},$$

where

$$\Upsilon_4 = \mathbb{E} \left\{ \sum_{k_1=0}^{L_{j_0}-1} h_{j_0}^2(k_1) \sum_{k_2=0}^{L_{j_0}-1} \hat{h}_{j_0}^2(k_2) \right\} = \left[ \frac{2(1 - \varrho^{2L_{j_0}})}{k(1 - \varrho^2)} + \frac{(1 - \varrho^{L_{j_0}})^2}{(1 - \varrho)^2} \right] \Omega_0^2,$$

$$\Upsilon_5 = \mathbb{E} \left\{ \left[ \sum_{k=0}^{L_{j_0}-1} h_{j_0}(k) \tilde{n}_{j_0}(k)s_{j_0} + \sum_{j=1}^{j_0-1} \sum_{k=L_{j_0}-L_j}^{L_{j_0}-1} h_j(k)[h_j(L_j - L_{j_0} + k)$$

$$+ \tilde{n}_j(L_j - L_{j_0} + k)]s_j$$

$$+ \sum_{j=j_0+1}^{N_T} \sum_{k=0}^{L_{j_0}-1} h_j(k)[h_j(L_j - L_{j_0} + k) + \tilde{n}_j(L_j - L_{j_0} + k)]s_j \right]^2 \right\}$$

$$= \mathbb{E} \left\{ \sum_{k=0}^{L_{j_0}-1} h_{j_0}^2(k)\sigma_n^2 + \sum_{j=1}^{j_0-1} \sum_{k=L_{j_0}-L_j}^{L_{j_0}-1} [h_j^2(k)\sigma_n^2 + h_j^2(k)h_j^2(L_j - L_{j_0} + k)] \right\}$$
In the calculation of $\Upsilon$ is assumed that $\varrho$ is normalized to unity, i.e., $\frac{\sum_j \varrho L_j}{\Omega_1 \sigma^2}$.

In this subsection, some simulation results are presented to show how the channel imperfection affects the system performance. In all the simulations, unless stated otherwise, it is assumed that $\varrho = 0.8$, $\kappa = 6$ and $L_m = 50$. The power possessed by the $L_m$ multipaths is normalized to unity, i.e., $\sum_{i=0}^{L_m-1} \Omega_i = 1$, which renders $\Omega_0 \approx 0.2$.

Three representative cases for the variance of channel estimation errors are considered: $\sigma_n^2 = 0$ (perfect estimation), $\sigma_n^2 = 0.1$ (typical error case) and $\sigma_n^2 = 1$ (worst error case).

Except when stated otherwise, other system parameters are set as follows:

- in the SISO high data rate case, we choose $L_s = 7$ and $L = 22$;
- in the MIMO case, we fix $N_T = 3$ but vary $N_R$ from 1 to 3;
- in the MISO multiuser case, we fix $L_0 = 20$ but vary $L_c$ from 1 to 15.

Figures 6.19 and 6.20 show the BER performance and output average SNR respectively of the SISO TR system for the low data rate case. The channel is perfectly estimated in Figure 6.19a, whereas channel estimation errors are considered in Figure 6.19b. It can be seen from Figures 6.19 and 6.20 that the BER of the TR system decreases monotonously with the parameter $L$, the channel length used in the TR pre-filter; on the other hand, the output average SNR of the TR system increases monotonously with $L$, no matter whether channel estimation errors exist. However, when $L > 20$, the system performance improvement is marginal. Therefore, an optimal $L$, denoted $L_{\text{opt}}$, for the channel length used in the TR pre-filter exists, considering the complexity of the transmitter. Of course, $L_{\text{opt}}$ depends on the real communication scenarios. As shown in Figures 6.19 and 6.20,
Figure 6.19 The BER performance of the SISO TR system for the low data rate case: (a) the channel is perfectly estimated; (b) channel estimation errors exist.
the TR system performance is quite good if we set $L_{\text{opt}} = 20$. Our other simulations show that this observation holds true even when $\varphi$ is as high as 0.95.

From Figure 6.19 it can also be observed that the BER performance of the TR system is quite satisfactory when the variance of the channel estimation error is $\sigma_n^2 = 0.1$, while when $\sigma_n^2 = 1$ the BER is so high that reliable communications cannot be performed and a BER floor is produced. Note that $\Omega_0 \approx 0.2$. Hence, the fact that $\sigma_n^2 = 0.1$ means that the noise level in the channel estimation for the strongest tap is approximately $-3$ dB. For the weaker channel taps, the noise level in the estimates of those taps is even lower than $-3$ dB. This estimation error ($\sigma_n^2 = 0.1$) is quite large in the channel estimation. Therefore, it can be concluded that the performance of the TR system is quite robust to the channel estimation error.

Figure 6.21 shows the BER performance of the SISO TR system for the high data rate case, compared with the corresponding scenario for the low data rate case. It can be seen that when the channel is perfectly obtained or estimated with a reasonable accuracy ($\sigma_n^2 \leq 0.1$), using the TR approach to conduct high data rate communications is viable. When the channel is poorly estimated, for example, when $\sigma_n^2 \geq 1$ (which is too pessimistic to happen in practice), it is impossible to conduct reliable communications using the TR approach at either low or high data rates. Therefore, to estimate the channel with a reasonable accuracy is critical for successfully using the TR technique.

To show the effect of $L_s$ on the output average SNR for the high data rate case, we set $L_m = 61$ and $L = 31$. The results are presented in Figure 6.22, from which it can be seen that, for a fixed input SNR, the output average SNR of the system increases with the symbol interval $L_s$, i.e., decreases with the symbol rate. When $L_s = 5$, a strong ISI will be introduced, causing serious deterioration in the output average SNR compared with the cases of $L_s = 10$ and $L_s = 15$. 

Figure 6.20 The output average SNR of the SISO TR system for the low data rate case.
Figure 6.21  The BER performance of the SISO TR system for the high data rate case.

Figure 6.22  The output average SNR of the SISO TR system for the high data rate case.
Figure 6.23 The BER performance of the MIMO TR system for the low data rate case: (a) the channel is perfectly estimated; (b) channel estimation errors exist ($\sigma_n^2 = 0.1$).
Figures 6.23 and 6.24 show the BER performance and output average SNR respectively of the MIMO TR system for the low data rate case. The channel is perfectly estimated in Figure 6.23a, whereas channel estimation errors are considered in Figure 6.23b. From Figures 6.23 and 6.24 we can again see that the BER of the TR system decreases monotonously with $L$, and the output average SNR of the TR system increases monotonously with $L$, no matter whether channel estimation errors exist. However, when $L > 20$, the system performance improvement is marginal. Therefore, the optimal channel length $L_{opt}$ can be taken as 20. Similar to the SISO case, the BER performance of the MIMO TR system is quite satisfactory when the variance of the channel estimation error is $\sigma^2 = 0.1$, as shown in Figure 6.23b.

As demonstrated in Figure 6.23, the MUI is introduced when using multiple receiver antennas ($N_R > 1$). This MUI causes a BER floor for the TR system, which greatly degrades the system performance, especially when the channel estimate is not perfect.

A special case in Figure 6.23 is when $N_R = 1$. In this case, the MIMO system becomes a MISO system, but this MISO system is different from the case illustrated in Figure 6.25. Here (in Figure 6.23), only one user’s communications are deployed and the multiple transmit antennas are actually used to obtain diversity. From Figure 6.23 we can see that very good BER performance can be achieved by this MISO system even when only five taps of the channel are used in the TR pre-filter.

Figure 6.25 shows the BER performance and output average SNR of the MISO multiuser communications for the low data rate case. As illustrated in Figure 6.25a, it is
Figure 6.25 The BER performance and output average SNR of the MISO multiuser communications for the low data rate case: (a) BER performance; (b) output average SNR, where the input SNR for all the three users is 10 dB.
possible to conduct multiuser communications by using the proposed processing scheme at the transmitter, especially when the channel can be perfectly estimated. In this case, the system achieves almost the same BER performance and output average SNR for different users even though different users use different resources (different channel lengths). However, when channel estimation errors exist, considerable imbalance in the BER and output average SNR among the users appears: the user using a longer channel length can obtain a better BER and output average SNR. A better processing scheme might overcome this problem.

6.7 Summary

The TR technique combined with wideband/UWB communications provides another promising possibility for multiuser communications. In a rich multipath environment, a CIR is unique for any pair of communication points. This unique property can be used to differentiate users in different directions and even in the same direction but in different distances. This offers another kind of multiple access technique, quite different from the popularly used code-division multiple access (CDMA) technique. But some fundamental issues exist in this technique. For example, in the CDMA technique, the SINR can always be controlled in some way if the complexity of the system is not an issue, while the channel response is mainly determined by the scattering environment instead of by the system design in the TR technique. This raises two problems:

(i) Is it possible to adjust or design the system parameters so that the system performance satisfies given requirements?
(ii) What is the effect of imperfect channels on the system performance?

This chapter provides an initial analysis from both analytical and numerical aspects for the aforementioned problems in the context of UWB communications. The closed-form expressions for the BER and output average SNR of relevant systems for several cases have been presented. The numerical results reveal that the BER performance of the UWB TR systems, whether in the forms of SISO, MISO or MIMO, are quite robust to the imperfection of CIRs of the systems. It is shown that:

- For typical UWB channels, it is sufficient to use 20 channel taps, instead of the full channel, for the TR pre-filter to achieve satisfactory BER performance when the channel decay exponent is in the range 0.8–0.95. A further increase in the number of channel taps beyond 20 helps improve the system performance, but only marginally.
- The TR system can tolerate inaccurate channel estimates, at least at a noise level of $-3$ dB ($\sigma^2_{\tilde{n}} = 0.1$) for each channel tap. But when the noise level in the channel estimate is beyond 7 dB ($\sigma^2_{\tilde{n}} \geq 1$), using the TR technique cannot yield satisfactory performance.

A simple multiple accessing scheme for using the UWB MISO TR system to conduct multiuser communications has been proposed. Simulation results show that the multiple accessing scheme works well when the channel estimate is of reasonable accuracy. For a UWB MIMO TR system, since the equivalent composite MIMO channel is neither an ideal diagonal matrix in the spatial domain nor a $\delta$-like function in the temporal domain,
both MSI and ISI exist. In this chapter, ZF and MMSE pre-equalizers are proposed to reduce the effects of ISI and MSI in the SM MIMO UWB system. The pre-equalizer is designed based on shortened equivalent channels to reduce the complexity of systems. Simulation results show that the performance of the system can be considerably improved when the pre-equalizer is applied. The cascade of the proposed pre-equalizer and TR filter makes the TR UWB SM-SIMO system with only one transmit antenna capable of conveying several independent data streams. The TR UWB SM-SIMO system works like a system using virtual multi-transmit antennas for communications.

In this chapter, we have also discussed the advantages of the TR approach compared with the IC approach. It is shown that the TR approach outperforms the IC approach in output SNR, BER performance for UWB channels of nonminimum phase, robustness against CIR estimation errors, and complexity.

From the discussions in this chapter, we can see that, using the TR technique, multiuser communications can be performed via MIMO, MISO or SIMO, combined with the UWB radio.
7

UWB Relay Systems

Kiattisak Maichalernnukul,1 Thomas Kaiser and Feng Zheng

7.1 Introduction

Since a UWB system overlays its spectrum with many licensed narrower bandwidth radios, the permitted power spectral density of a UWB signal is rather limited due to the regulations imposed by relevant authorities, such as the FCC in the USA [63] and the EC document in Europe [105]. For example, it is typically restricted that the spectral density (measured with EIRP, equivalent isotropically radiated power) should not exceed \(-41.3\ \text{dBm/MHz}\) across the 3.1–10.6 GHz frequency band. This regulation puts up a strong barrier for the data rates and communication distances that the UWB systems can achieve. One way to solve this problem is to use a relay (or repeater in some references, for example, [186]) scheme.

The theoretical study on relay channels from the information theoretic point of view was started in the late 1960s by van der Meulen [253, 254], where a model for discrete memoryless multi-terminal networks was introduced. The later proposed two-hop relay channel can be considered as a special form of a three-terminal network in [253]. Substantial advances in the theory were made by Cover and El Gamal [55], who developed two fundamental coding strategies for single-relay systems and proved an upper bound for the highest achievable rate for single-relay channels. The first strategy achieving the rates in [55, Theorem 1] was found to be one of a class of relaying schemes now commonly called decode-and-forward (DF) [136, 181, 186]. Many other researchers developed capacity-achieving codes for other classes of channels by combining the strategies in [55], for example, [19, 126, 128, 271, 281]. An extensive survey can be found in [128]. Notice that most of the aforementioned studies, except [128, 136, 181, 186], are for nonfading AWGN channels, while [128] extended some results for non-fading narrowband channels to narrowband fading channels. Recently, the research in this field has been focused on the

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capacity scaling laws for large-scale narrowband wireless relay networks [29, 60, 83, 194]. Here, the term ‘large-scale’ means that there are a large number of relay nodes between the (single) source node and (single) destination node. On the other hand, the performance investigation in terms of BERs or SNR/capacity outage probabilities for various narrowband single-relay wireless systems has been well studied [18, 100, 137, 172, 249]. According to the forwarding strategies, these systems can be mainly classified as: (i) DF relaying, in which the relay node fully decodes and again encodes the received signal for retransmission; and (ii) amplify-and-forward (AF) relaying, in which the relay node simply amplifies and retransmits the received signal.

The first result for addressing the wideband wireless relay networks appeared in [257], where two-hop large-scale wireless relay networks were studied. The achievable rates and scaling laws of this kind of relay network were presented for both narrowband and wideband cases under the AF scheme.

However, there have been only very few research studies on IR-based UWB relay systems until now. The first UWB relaying scheme was proposed by Cho et al. in 2004 [45] and further developed in 2007 [46] by the same authors. As Cho et al. [45, 46] pointed out, the traditional AF relay for narrowband systems does not work for UWB systems owing to the backward coupling between the transmit and receive antennas at the relay. This coupling comes from the leakage of the amplified version of the already received UWB signal which is to be transmitted at the transmit antenna, causing severe interference for the incoming UWB signal (which is typically very weak) at the receive antenna. In [46], it was found that a detect-and-forward (DTF, which can be viewed as uncoded DF) relay works well for UWB systems. In the DTF relay, the detected data bits are modulated again and transmitted to the destination node after a delay, say $\tau_d$. If $\tau_d$ is properly designed, the interference between the received and transmitted signals can be neglected.

A typical DTF relay for UWB systems includes three parts, namely a receiver, a transmitter and a relay controller [45, 46]. The receiver and transmitter at the relay may share the same antenna, since the reception and transmission of UWB signals happen at different times. The relay controller is equipped with a switch unit, which controls the transmission/reception, and a delay unit, which provides a proper delay for the retransmitted signal.

In [45, 46], a two-hop relay combined with simple pulse position modulation (PPM) was used to demonstrate the basic idea of relaying in UWB systems, and the system performance improvement was shown via simulations. In [279], the upper bounds on the ergodic and outage capacities of two-hop UWB relay networks were given based on the approach developed in [55], where the frequency selectivity of the considered UWB channels was treated by equivalent multiple parallel channels, similar to the orthogonal frequency-division multiplexing (OFDM) approach. In [9], UWB nodes which are neither the source node nor the destination node are treated as relay nodes opportunistically. Then several STC schemes based on cyclic division algebras are proposed to improve the performance of the overall system. Note that each node considered in [9] is equipped with only a single antenna, while the ‘space’ refers to the multiple antennas across multiple nodes, including both the source and relay nodes. In [147, 149], a multi-antenna relaying scheme is proposed, where multiple antennas are mounted at the relay node to increase the reliability of the relay by making use of the spatial diversity of the channels. Note that, in [149], the multiple antennas are used for the purposes of both reception and
transmission at the relay node, and the gain thus obtained will be doubled compared with the cases in which these antennas are deployed either at the source node or at the destination node. Since UWB systems typically use Rake receivers to collect the energy spread over multipaths and the number of Rake fingers is an important factor which affects the complexity/cost of UWB systems, the compromise between the number of antennas at the relay node and the number of Rake fingers was investigated in [147, 148]. The results show that generally increasing the number of antennas at the relay node yields better BER performance than increasing the number of Rake fingers for a fixed product of the two numbers. In [150], the results of [147, 148] are extended to the case where multiple antennas are also equipped at the source and destination nodes. In [148], a differential coded transmitted-reference (DCTR) relay system is proposed, by which the traditional synchronizer and Rake receiver can be removed. As is well known, the design of synchronizers for UWB receivers is a critical and not yet fully solved issue, and the Rake receiver with a reasonable number of fingers considerably increases the complexity/cost of UWB systems. In [151], the performance comparison between an antenna-selection scheme in multi-antenna single-relay systems and a relay-selection scheme in single-antenna multi-relay systems is investigated.

In this chapter, we will extend some results on narrowband relay systems to UWB relay systems, combined with some advanced modulation and coding schemes. In Section 7.2, we first investigate simple UWB relay systems: the source and destination are equipped with one transmit and receive antenna respectively, while the relay may be equipped with one or two transmit/receive antennas. Through this basic setup, the benefits of relaying in UWB systems are examined, and the DCTR scheme is illustrated for how to remove the synchronizer in UWB relay systems. Then, in Section 7.3, we study general UWB relay systems: all the source, relay and destination are equipped with multiple antennas. The STC and pre-Rake techniques are introduced into the UWB relay systems and the system performance is comprehensively investigated. Some basic guidelines for the compromise between the system performance and the number of transmit/receive antennas are illustrated through numerical results.

Throughout this chapter, perfect synchronization (if necessary for the schemes to be discussed) at both the relay and destination is assumed. Here, we emphasize some acronyms which could be easily mixed up: DF for decode-and-forward, DCF for decouple-and-forward, and DTF for detect-and-forward. In this chapter, we adopt the following notational convention. \( \mathbf{1}_N \) is the all-ones column vector of length \( N \). \([ \cdot ]_{ij}\) and \([ \cdot ]_k\) denote the \((i, j)\)th entry and the \(k\)th row of a matrix respectively. \( \| \cdot \|_F \) denotes the Frobenius norm, \( \otimes \) is the Kronecker product, and \( \text{sign}(\cdot) \) is the sign operator. The notations \( p_X(x; a, b) \) and \( P_X(x; a, b) \) refer to the pdf and cdf respectively of a random variable \( X \) possibly with parameters \( a \) and \( b \). \( M_X(s) = \mathbb{E}[\exp(sX)] \) denotes the moment-generating function (MGF) of \( X \). The subscripts S, R and D denote the source, relay and destination in a relay system respectively. The superscripts \((t)\) and \((r)\) denote the transmitter side and the receiver side respectively. \( \Gamma(\cdot, \cdot) \), \( \Gamma(\cdot) \), \( K_n(\cdot) \) and \( 2F_1(\cdot, \cdot; \cdot; \cdot) \) respectively denote the upper incomplete Gamma function, the ordinary Gamma function, the \( n \)th-order modified Bessel function of the second kind and the Gauss hypergeometric function, as defined by Equations (8.350.2), (8.310.1), (8.432) and (9.100) respectively of [93].
7.2 UWB Relay Systems with SISO at Source and Destination

The general structure of a two-hop relay system is illustrated in Figure 7.1, where the source and destination are equipped with $M_S$ transmit antennas and $M_D$ receive antennas, respectively, and the relay is equipped with max($M_R^{(t)}$, $M_R^{(r)}$) antennas, of which $M_R^{(r)}$ antennas and $M_R^{(t)}$ antennas are used for reception and transmission, respectively. The switching between the reception and transmission is controlled by the relay controller. Note that $M_R^{(t)}$ is not necessarily equal to $M_R^{(r)}$. In this section, we consider a special case of Figure 7.1: $M_S = M_D = 1$ and $M_R^{(t)} = M_R^{(r)} = 1$ or $M_R^{(r)} = M_R^{(t)} = 2$. The antenna(s) at the relay can be used for both reception and transmission. Antipodal modulation signaling is used at the source and relay, and coherent and noncoherent detection schemes will be discussed.

The channel impulse response (CIR) for a UWB transmission link is described by

$$h(t) = \sum_{l=0}^{L_t-1} \alpha_l \delta(t - \tau_l) = \sqrt{G} \sum_{l=0}^{L_t-1} \varphi_l \delta(t - \tau_l), \quad (7.1)$$

where $L_t$ is the number of multipath components, $l$ the path index, $\alpha_l$ the path coefficient, $\tau_l$ the path delay, $\varphi_l$ the energy-normalized path coefficient with $\mathbb{E}[\sum_{l=0}^{L_t-1} \varphi_l^2] = 1$, and $G = \sum_{l=0}^{L_t-1} \alpha_l^2$ the total multipath gain. We consider the resolvable multipath channel with $\tau_l = lT_w$, where $T_w$ is the width of the UWB monocycle $w(t)$ [84, 146]. The coefficients $\varphi_l$, $l = 0, 1, \ldots, L_t - 1$, are assumed to be quasi-static over several bit durations.

7.2.1 Coherent Detection Systems

In the following systems, a selective-Rake reception scheme based on $L$ Rake fingers [36] with pilot-aided channel estimation is employed. In this reception scheme, only a subset of the resolvable paths is exploited. The reason for using a selective-Rake receiver is that it is simpler than the all-Rake receiver [36] while yields sufficiently good performance if $L$ is reasonably large. In this subsection, it is assumed that the channel estimation is corrupted by an AWGN. As shown in [51, 214] and Chapter 6, channel estimation errors have a great impact on the performance of relevant UWB systems. However, imperfect channel estimation has not yet been considered in the study of UWB relay systems. Therefore, the major difference between the result presented in this subsection and the

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existing result in the literature [46] lies in that the former applies to a more practical situation: selective-Rake and imperfect channel estimation.

7.2.1.1 Nonrelay Systems

The UWB transmitted signal at the source can be modeled as

\[ s(t) = \sum_{i=0}^{N_b-1} b_i w(t - iT_b), \]  

(7.2)

where \( N_b \) is the number of data-modulated pulses, \( b_i \in \{ \pm 1 \} \) the binary data bit with equal probability, \( w(t) \) the monocycle waveform with the width \( T_w \) and energy \( E_b \), and \( T_b \) the bit duration. Note that \( E_b \) is equal to the transmit energy per bit owing to the nonrepetitive transmission of the pulses here. Since the transmitted signal propagates through a UWB multipath channel, the duration of the received waveform for each transmitted pulse is \( T_r = T_{mds} + T_w \) where \( T_{mds} \) is the maximum excess delay of the channel in Equation (7.1). The bit duration is chosen such that \( T_b \geq T_r \) to preclude intersymbol interference (ISI). To facilitate channel estimation, the sequence of \( N_p \) unmodulated pulses, i.e., the pilot signal

\[ p(t) = \sum_{i=-N_p}^{-1} w(t - iT_b) \]

is transmitted before the data signal \( s(t) \). This requires that the channel is quasi-static during the time interval \([-N_p T_b, N_b T_b)\). The received UWB signal at the destination can be generally expressed as

\[ r(t) = h(t) * s(t) + n(t) = \sum_{l=0}^{L-1} \alpha_l s(t - lT_w) + n(t), \]  

(7.3)

where \( n(t) \) is the AWGN with zero mean and double-sided PSD \( N_0/2 \). Following [51, 214], the channel coefficient \( \alpha_l \) is estimated by maximizing the log-likelihood function of the received signal conditioned on \( \alpha_l \). As a result, the maximum-likelihood estimate of the channel coefficient can be expressed as

\[ \hat{\alpha}_l = \alpha_l + \varepsilon_l, \]

where \( \varepsilon_l \) represents the channel estimation error, which is an independent and identically distributed (i.i.d.) zero-mean Gaussian random variable with the variance \( N_0/(2N_p E_b) \) for all \( l \). See [51, 214] for more details.

In the \( L \)-finger selective-Rake reception, the correlation template can be constructed as follows:

\[ \hat{h}(t) = \sum_{l \in \mathcal{P}_L} \hat{\alpha}_l w(t - lT_w), \]
where $P_L$ is the ordered set of the indices of the $L$ strongest paths. Using this template, the decision variable for the data bit $b_i$ can be obtained as

$$z = \int_{iT_b}^{(i+1)T_b} r(t) \hat{h}(t - iT_b) \, dt = d + u,$$

where $d$ is the desired signal term, while $u$ is the noise term, given respectively by

$$d = b_i E_b \sum_{l \in P_L} \alpha_l^2,$$

$$u = b_i \int_{iT_b}^{(i+1)T_b} \sum_{l=0}^{L-1} \sum_{l' \in P_L} \alpha_l \varepsilon_{l'} w(t - lT_w) w(t - l'T_w) \, dt$$

$$+ \int_{iT_b}^{(i+1)T_b} \sum_{l' \in P_L} \alpha_{l'} w(t - l'T_w) n(t) \, dt + \int_{iT_b}^{(i+1)T_b} \sum_{l' \in P_L} \varepsilon_{l'} w(t - l'T_w) n(t) \, dt.$$  

(7.4b)

The first two terms of $u$ are Gaussian distributed, whereas the last term is not. According to the central limit theorem, the last term can be approximated as Gaussian if $L$ is large enough. It can be shown that all the three terms are uncorrelated and of zero mean. The variance of $u$ is given by

$$\sigma_u^2 = E_b^2 \sum_{l \in P_L} \alpha_l^2 \sigma_{\varepsilon_l}^2 + \frac{E_b N_0}{2} \sum_{l \in P_L} \alpha_l^2 + \frac{E_b N_0}{2} \sum_{l \in P_L} \sigma_{\varepsilon_l}^2.$$  

(7.5)

The SNR of the decision variable $z$, i.e., the received SNR conditioned upon the channel coefficients, is defined as

$$\gamma = \frac{d^2}{\sigma_u^2}.$$  

(7.6)

Using Equations (7.4) and (7.5), we get

$$\gamma = 2N_p E_a \frac{E_b}{N_0} \left[ N_p + 1 + \frac{L}{2E_a} \left( \frac{E_b}{N_0} \right)^{-1} \right]^{-1},$$  

(7.7)

where $E_a = \sum_{l \in P_L} \alpha_l^2$ can be interpreted as the portion of the total multipath energy captured by the selective-Rake receiver with the diversity level $L$. The ratio $E_b/N_0$ can be referred to as the transmitted SNR per bit. The average BER can be calculated from Equation (7.7) as

$$P_c = \int_0^\infty Q(\sqrt{x}) p_{\gamma}(x) \, dx,$$

(7.8)

where $p_{\gamma}(x)$ is the pdf of $\gamma$ and $Q(\cdot)$ denotes the $Q$-function [192, p. 60].
7.2.1.2 Single-Antenna Relay Systems

In this kind of system, a DTF UWB relay equipped with one antenna is employed to relay the signal received from the source to the destination. This relay not only can alleviate the path loss (if appropriately placed), but also exploit multipath diversity in part by using the selective-Rake scheme. Due to practical limitations as a conventional relay, such a kind of relay cannot transmit and receive the signal simultaneously. The transmitted signal at the source is then given by

$$s(t) = \sum_{i=0}^{N_b-1} b_i w_{T1}(t - i2T_b),$$

where $w_{T1}(t)$ is similar to $w(t)$ except that its energy is $E_{b,T1}$. Similarly, the pilot signal used to estimate the UWB channel is

$$p(t) = \sum_{i=-N_p}^{-1} w_{T1}(t - i2T_b).$$

This means that two time slots, each of duration $T_b$, are needed to relay either a pilot pulse or a data-modulated pulse. Therefore, it is required that the channel is quasi-static over the interval $[-2N_pT_b, 2N_bT_b)$. Note that the transmission rate of this system is half of that of the nonrelay system, i.e., $1/(2T_b)$. In the following, we describe only the strategy for relaying the data signal. The strategy for relaying the pilot signal is similar.

In the first time slot, only the relay receives the signal sent from the source. The received signal can be written as

$$r_R(t) = h_{T1}(t) * s(t) + n_R(t) = \sum_{l=0}^{L_t-1} \alpha_{T1,l} s(t - lT_w) + n_R(t), \quad (7.9)$$

where $h_{T1}(t)$ is the CIR from the source to the relay, whose path coefficients are $\alpha_{T1,l}$, $l = 0, 1, \ldots, L_t - 1$, and $n_R(t)$ is the AWGN with zero mean and the same PSD as $n(t)$ in Equation (7.3). On the other hand, the receiver of the destination neglects the signal sent directly from the source, for example, it is idle, because this signal suffers from more path loss than the received signal at the relay which is closer to the source. The relay employs the same selective-Rake scheme as discussed before. The decision variable for $b_i$ at the relay is given by

$$z_R = \int_{2iT_b}^{(2i+1)T_b} r_R(t) \hat{h}_{T1}(t - 2iT_b) dt = d_R + u_R, \quad (7.10)$$

where $d_R$ and $u_R$ are similar to Equation (7.4) except that the integration interval is changed and some subscripts are added. The corresponding received SNR $\gamma_R$ can be obtained from Equation (7.7) with $E_\alpha$ and $E_b$ being replaced by $E_{a,T1} = \sum_{l\in P_{T1,L}} \alpha_{T1,l}^2$ and $E_{b,T1}$, respectively, and the average BER $P_{e1}$ is obtained from Equation (7.8) with $\gamma$ being replaced by $\gamma_R$. Here $P_{T1,L}$ is the ordered set of the indices of the $L$ strongest paths in the link between the source and relay.
In the second time slot, the relay transmits the detected data bit \( \hat{b}_i = \text{sign}(z_R) \) to the destination as

\[
s_R(t) = \sum_{i=0}^{N_b-1} \hat{b}_i w_{T2}(t - (2i + 1)T_b)
\]

after the pilot signal

\[
p_R(t) = \sum_{i=-1}^{-N_p} w_{T2}(t - (2i + 1)T_b),
\]

where \( w_{T2}(t) \) is similar to \( w(t) \) except that its energy is \( E_{b,T2} \). Hence, the received signal at the destination is

\[
r(t) = h_{T2}(t) * s_R(t) + n(t) = \sum_{l=0}^{L_t-1} \alpha_{T2,l} s_R(t - lT_w) + n(t),
\]

where \( h_{T2}(t) \) is the CIR from the relay to the destination, whose path coefficients are \( \alpha_{T2,l}, l = 0, 1, \ldots, L_t - 1 \). Similar to Equation (7.10), the decision variable for \( \hat{b}_i \) at the destination is given by

\[
z = \int_{(2i+1)T_b}^{(2i+2)T_b} r(t) \hat{h}_{T2}(t - (2i + 1)T_b) dt = d + u,
\]

where \( d \) and \( u \) are similar to Equation (7.11) except that the integration interval is changed and some subscripts are added. The SNR of \( z \) (i.e., \( \gamma \)) can thus be computed from Equation (7.7) with \( E_b \) and \( E_\alpha \) being replaced by \( E_{b,T2} \) and \( E_{\alpha,T2} = \sum_{l \in \mathcal{P}_{T2,L}} \alpha_{T2,l}^2 \), respectively, and the average BER of \( \hat{b}_i \) (i.e., \( P_{e2} \)) is given by Equation (7.8). Here \( \mathcal{P}_{T2,L} \) is the ordered set of the indices of the \( L \) strongest paths in the link between the relay and destination.

Using \( P_{e1} \) and \( P_{e2} \), the average BER for the single-antenna relay system can be obtained as

\[
P_e = P_{e1}(1 - P_{e2}) + P_{e2}(1 - P_{e1}). \tag{7.11}
\]

Notice that in deriving Equation (7.11), we have used the fact that erroneous data bit detection at both the relay and destination yields correct data bit detection finally due to the binary bit transmission. Therefore, the term \( P_{e1} P_{e2} \) corresponding to the probability of the event of erroneous data bit detection at both the relay and destination is not included in \( P_e \).

### 7.2.1.3 Two-Antenna Relay Systems

In this kind of system, a DTF relay equipped with two antennas is used to relay the signals received from the source to the destination. These antennas are spatially spaced in such a way that the received/transmitted signals undergo statistically independent fading.
In addition to multipath diversity, the proposed relay can exploit spatial diversity by performing spatial diversity reception [239] for those signals, and spatial transmit diversity [273] for the regenerated signals from the detected data. The transmitted signal at the source for this system is given by

\[
s(t) = \sum_{i=0}^{(N_b/2)-1} \{b_{2i}w_{T1}(t - 4iT_b) + b_{2i+1}w_{T1}(t - (4i + 1)T_b)\},
\]

where \(N_b\) is an even number. Similarly, the pilot signal is

\[
p(t) = \sum_{i=-N_p/2}^{-1} \{w_{T1}(t - 4iT_b) + w_{T1}(t - (4i + 1)T_b)\},
\]

where \(N_p\) is an even number. This means that two time slots, each of duration \(2T_b\), are needed to relay either two pilot pulses or two data-modulated pulses. As in the single-antenna relay system, it is required that the channel is quasi-static over \([-2N_pT_b, 2N_bT_b]\).

Note that the transmission rate of this system is equal to that of the single-antenna relay system. In the following, we describe only the relaying protocol for the data signal. The relaying protocol for the pilot signal is similar to that for the data signal.

In the first time slot, the received signal at the \(j\)th antenna of the relay can be written as

\[
r_{R,j}(t) = h_{T1,j}(t) * s(t) + n_{R,j}(t) = \sum_{l=0}^{L_t-1} \alpha_{T1,l,j}s(t - lT_w) + n_{R,j}(t),
\]

where \(h_{T1,j}(t)\) is the CIR from the source to the \(j\)th antenna of the relay, whose path coefficients are \(\alpha_{T1,l,j}, l = 0, 1, \ldots, L_t - 1, j = 1, 2\), and \(n_{R,j}(t)\) is the AWGN at the \(j\)th antenna of the relay with zero mean and the same PSD as \(n_R(t)\) in Equation (7.9). Employing the \(L\)-finger selective-Rake scheme at each antenna and the maximum ratio combining (MRC) algorithm for the signals from both antennas, the decision variables for \(b_{2i}\) and \(b_{2i+1}\) at the relay can be expressed as

\[
z_R = \sum_{j=1}^{2} \int_{(4i+1)T_b}^{(4i+1)T_b} r_{R,j}(t) \hat{h}_{T1,j}(t - 4iT_b) dt = \sum_{j=1}^{2} (d_{R,j} + u_{R,j}) = d_R + u_R
\]

and

\[
\tilde{z}_R = \sum_{j=1}^{2} \int_{(4i+1)T_b}^{(4i+1)T_b} r_{R,j}(t) \hat{h}_{T1,j}(t - (4i + 1)T_b) dt = \sum_{j=1}^{2} (\tilde{d}_{R,j} + \tilde{u}_{R,j}) = \tilde{d}_R + \tilde{u}_R
\]

respectively. After some algebraic manipulations, we obtain the SNRs of \(z_R\) and \(\tilde{z}_R\) (denoted as \(\gamma_R\) and \(\tilde{\gamma}_R\), respectively) as follows:

\[
\gamma_R = \tilde{\gamma}_R = 2N_p\left(E_{T1,1} + E_{T1,2}\right)\frac{E_{b,T1}}{N_0} N_p + 1 + \frac{L}{E_{T1,1} + E_{T1,2}} \left(\frac{E_{b,T1}}{N_0}\right)^{-1},
\]
where \( E_{T1,1} = \sum_{l \in \mathcal{P}_{T1,L}} \alpha_{T1,l,1}^2 \), \( E_{T1,2} = \sum_{l \in \mathcal{Q}_{T1,L}} \alpha_{T1,l,2}^2 \), and \( \mathcal{P}_{T1,L} \) and \( \mathcal{Q}_{T1,L} \) are the ordered sets of the indices of the \( L \) strongest paths of \( h_{T1,1}(t) \) and \( h_{T1,2}(t) \), respectively. The corresponding average BER, i.e., \( P_{e1} \), can then be calculated from Equation (7.8) with \( \gamma \) being replaced by \( \gamma_R \) or \( \gamma_{R} \).

Using the spatial transmit diversity in the second time slot, the relay transmits the detected data bits, i.e., \( \hat{b}_{2i} = \text{sign}(\hat{z}_R) \) and \( \hat{b}_{2i+1} = \text{sign}(\hat{z}_R) \), from the first and second antennas to the destination, respectively, as

\[
s_{R,1}(t) = \frac{1}{\sqrt{2}} \sum_{i=0}^{(N_b/2)-1} [\hat{b}_{2i}w_{T2}(t - (4i + 2)T_b) - \hat{b}_{2i+1}w_{T2}(t - (4i + 3)T_b)],
\]

\[
s_{R,2}(t) = \frac{1}{\sqrt{2}} \sum_{i=0}^{(N_b/2)-1} [\hat{b}_{2i+1}w_{T2}(t - (4i + 2)T_b) + \hat{b}_{2i}w_{T2}(t - (4i + 3)T_b)]
\]

after the pilot signal

\[
p_{R,1}(t) = \frac{1}{\sqrt{2}} \sum_{i=-N_p/2}^{-1} [w_{T2}(t - (4i + 2)T_b) - w_{T2}(t - (4i + 3)T_b)],
\]

\[
p_{R,2}(t) = \frac{1}{\sqrt{2}} \sum_{i=-N_p/2}^{-1} [w_{T2}(t - (4i + 2)T_b) + w_{T2}(t - (4i + 3)T_b)].
\]

The factor \( 1/\sqrt{2} \) is introduced to ensure that the total transmit energy from the two-antenna relay is identical to that from the single-antenna relay. The received signal at the destination is

\[
r(t) = \sum_{j=1}^{2} h_{T2,j}(t) * s_{R,j}(t) + n(t) = \sum_{j=1}^{2} \sum_{l=0}^{L_l-1} \alpha_{T2,l,j} s_{R,j}(t - lT_w) + n(t),
\]

where \( h_{T2,j}(t) \) are the CIR from the \( j \)th antenna of the relay to the destination, whose path coefficients are \( \alpha_{T2,l,j}, l = 0, 1, \ldots, L_l - 1, j = 1, 2 \). Employing the MRC with the selective-Rake templates \( \hat{h}_{T2,1}(t) \) and \( \hat{h}_{T2,2}(t) \), the decision variables for \( \hat{b}_{2i} \) and \( \hat{b}_{2i+1} \) at the destination can be expressed, respectively, as

\[
z = \int_{(4i+2)T_b}^{(4i+3)T_b} r(t)\hat{h}_{T2,1}(t - (4i + 2)T_b) \, dt + \int_{(4i+3)T_b}^{(4i+4)T_b} r(t)\hat{h}_{T2,2}(t - (4i + 3)T_b) \, dt = d + u.
\]

\[
\bar{z} = \int_{(4i+2)T_b}^{(4i+3)T_b} r(t)\hat{h}_{T2,2}(t - (4i + 2)T_b) \, dt - \int_{(4i+3)T_b}^{(4i+4)T_b} r(t)\hat{h}_{T2,1}(t - (4i + 3)T_b) \, dt = \bar{d} + \bar{u}.
\]

After some straightforward calculations, and neglecting the term

\[
\sum_{l \in \mathcal{P}_{T2,L}} \alpha_{T2,l,1} \alpha_{T2,l,2} - \sum_{l \in \mathcal{Q}_{T2,L}} \alpha_{T2,l,1} \alpha_{T2,l,2}
\]
(since this term is much smaller than other relevant terms), the SNRs of $z$ and $\tilde{z}$ (denoted as $\gamma$ and $\tilde{\gamma}$, respectively) can be simplified to

$$
\gamma = \tilde{\gamma} = \frac{2N_p (E_{T2,1} + E_{T2,2}) E_{b,T2}}{N_0} \left[ 2N_p + 1 + \frac{\tilde{E}_{T2,1} + \tilde{E}_{T2,2}}{E_{T2,1} + E_{T2,2}} + \frac{2L}{E_{T2,1} + E_{T2,2}} \left( \frac{E_{b,T2}}{N_0} \right)^{-1} \right]^{-1}, \quad (7.12)
$$

where

$$
E_{T2,1} = \sum_{l \in P_{T2,L}} \alpha_{T2,l,1}^2, \quad E_{T2,2} = \sum_{l \in Q_{T2,L}} \alpha_{T2,l,2}^2,
$$

$$
\tilde{E}_{T2,1} = \sum_{l \in P_{T2,L}} \tilde{\alpha}_{T2,l,2}^2, \quad \tilde{E}_{T2,2} = \sum_{l \in Q_{T2,L}} \tilde{\alpha}_{T2,l,1}^2.
$$

Note that if $E_{T1,1} + E_{T1,2} = E_{T2,1} + E_{T2,2}$, $E_{b,T1} = E_{b,T2}$, and $N_p = \infty$ (which implies perfect channel estimation), $\gamma = \gamma_R / 2$, and $\tilde{\gamma} = \tilde{\gamma}_R / 2$. The average BER of $\hat{b}_{2i}$ or $\hat{b}_{2i+1}$ (i.e., $P_{e2}$) can then be obtained from Equation (7.8) with $\gamma$ or $\tilde{\gamma}$ given by Equation (7.12). Lastly, the average BER of this system is obtained by substituting $P_{e1}$ and $P_{e2}$ into Equation (7.11).

### 7.2.2 Noncoherent Detection Systems

A noncoherent detection (specifically differential transmitted-reference, DTR) scheme composed of differential encoding and autocorrelation demodulation is used for noncoherent detection. In the Rake scheme considered in the preceding subsection, the received signal $r(t)$ correlates with the locally generated template, denoted by $\hat{h}(t)$, at the receiver side. This correlation operation reduces the noise bandwidth to the signal bandwidth. In the DTR scheme, the received signal correlates with a delayed replica of the received data signal itself [104]. Hence, a filtering operation is essential for the DTR scheme to limit the noise bandwidth. For analytical simplicity, an ideal bandpass filter with single-sided bandwidth $W$ is deployed at the receiver front end. The filter bandwidth is chosen to be wide enough, i.e., larger than the 10 dB bandwidth of the transmitted UWB signal [283], to avoid filter-induced ISI. Notice that the CIRs $h(t)$, $h_{T_1}(t)$, and $h_{T_1,j}(t)$ ($i, j = 1, 2$) are not distorted by the bandpass filter.

### 7.2.2.1 Nonrelay Systems

The transmitted signal at the source is

$$
s(t) = \sum_{i=-1}^{N_b-1} a_i w(t - iT_b), \quad (7.13)
$$

where $a_i = a_{i-1} b_i$ is the differentially encoded data with a given initial binary bit $a_{-1}$, and other parameters are the same as defined in Equation (7.2). To preclude the ISI as in
the coherent detection nonrelay system, the bit duration $T_b$ satisfies $T_b \geq T_{mds} + T_w$. The received signal at the destination can be written in the same form as Equation (7.3) with $s(t)$ given by Equation (7.13). After bandpass filtering, the filtered received signal can be expressed as

$$\tilde{r}(t) = h(t) * s(t) + \tilde{n}(t) = \sum_{l=0}^{L_t-1} \alpha_l s(t - lT_w) + \tilde{n}(t),$$

where $\tilde{n}(t)$ is the filtered noise with zero mean and single-sided bandwidth $W$.

The decision variable is produced by the output of the correlator. For the data bit $b_i$, the decision variable is given by

$$z = \int_{iT_b}^{iT_b + T} \tilde{r}(t) \tilde{r}(t - T_b) \, dt = d + u$$

with

$$d = b_i E_b \sum_{l=0}^{L_{int}-1} \alpha_i^2,$$  \hspace{1cm} (7.14)

$$u = a_i \int_{iT_b}^{iT_b + T} \sum_{l=0}^{L_{int}-1} \alpha_l w(t - lT_w) \tilde{n}(t - T_b) \, dt$$

$$+ a_{i-1} \int_{iT_b}^{iT_b + T} \sum_{l=0}^{L_{int}-1} \alpha_l w(t - lT_w) \tilde{n}(t) \, dt + \int_{iT_b}^{iT_b + T} \tilde{n}(t) \tilde{n}(t - T_b) \, dt,$$  \hspace{1cm} (7.15)

where $L_{int} = \lfloor T/T_p \rfloor$ if $T < T_{mds}$ and $L_{int} = L_t$ otherwise, and $T$ is the integration interval satisfying $T \leq T_b$. Notice that there exists an optimal value for $T$ which minimizes the average BER [47]. This will be discussed in Section 7.2.3.

It is required that the channel is quasi-static over the duration of $2T_b$. The first two terms of $u$ are Gaussian, whereas the last term is not. However, if the time–bandwidth product $TW$ is large enough, the last term can be approximated as Gaussian distributed [40, 47]. It can be shown that all the three terms of $u$ are uncorrelated and of zero mean. The variance of $u$ is given by

$$\sigma_u^2 = E_bN_0 \sum_{l=0}^{L_{int}-1} \alpha_l^2 + \frac{N_0^2 TW}{2}.$$  \hspace{1cm} (7.16)

Using Equations (7.6), (7.14), (7.15), and (7.16), the SNR of $z$ can be expressed as

$$\gamma = E_b' \frac{E_b}{N_0} \left[ 1 + \frac{TW}{2E_b'} \left( \frac{E_b}{N_0} \right)^{-1} \right]^{-1},$$  \hspace{1cm} (7.17)

where $E_b' = \sum_{l=0}^{L_{int}-1} \alpha_l^2$. The average BER of the system can be computed from Equation (7.8).
7.2.2.2 Single-Antenna Relay Systems

The relay is equipped with one antenna, and the aforementioned ideal bandpass filter is also deployed at the antenna. The transmitted signal at the source is given by

\[ s(t) = \sum_{i=-1}^{N_b-1} a_i w_{T1}(t - i2T_b), \]

which implies that two time slots, each of duration \( T_b \), are needed to relay each data-modulated pulse. The transmission rate is then half of that of the nonrelay system, i.e., \( \frac{1}{2T_b} \). The relaying strategy for this system can be described as follows. In the first time slot, only the relay is used to receive the signal sent from the source. The filtered received signal at the relay can be written as

\[ \tilde{r}_{R}(t) = h_{T1}(t) * s(t) + \tilde{n}_{R}(t) = \sum_{l=0}^{L_1-1} \alpha_{T1,l}s(t - lT_w) + \tilde{n}_{R}(t), \]

where \( \tilde{n}_{R}(t) \) is the filtered noise with zero mean and single-sided bandwidth \( W \). The decision variable for \( b_i \) at the relay is then given by

\[ z_{R} = \int_{2iT_b}^{2iT_b+T} \tilde{r}_{R}(t)\tilde{r}_{R}(t - 2T_b) \, dt = d_{R} + u_{R}, \quad (7.18) \]

where \( d_{R} \) and \( u_{R} \) are similar to Equations (7.14) and (7.15), respectively, except that the integration interval is changed and some subscripts are added. For this system, it is required that the channel is quasi-static during the interval of \( 3T_b \). The corresponding received SNR \( \gamma_{R} \) can be calculated via Equation (7.17) with \( E'_a \) and \( E_b \) being replaced by \( E'_{\alpha,T1} = \sum_{l=0}^{L_{int}-1} \alpha_{T1,l}^2 \) and \( E_{b,T1} \), respectively, and the average BER \( P_{e1} \) is obtained from Equation (7.8) with \( \gamma \) being replaced by \( \gamma_{R} \).

In the second time slot, the relay transmits the detected data bit \( \hat{b}_i = \text{sign}(z_{R}) \) to the destination as

\[ s_{R}(t) = \sum_{i=-1}^{N_b-1} \hat{a}_i w_{T2}(t - (2i + 1)T_b), \]

where

\[ \hat{a}_i = \hat{a}_{i-1}\hat{b}_i \]

is the differentially encoded version of the detected data bit \( \hat{b}_i \), except the initial bit \( \hat{a}_{-1} \). Hence, the filtered received signal at the destination is

\[ \tilde{r}(t) = h_{T2}(t) * s_{R}(t) + \tilde{n}(t) = \sum_{l=0}^{L_1-1} \alpha_{T2,l}s_{R}(t - lT_w) + \tilde{n}(t). \]

Similar to Equation (7.18), the decision variable for \( \hat{b}_i \) at the destination is given by

\[ z = \int_{(2i+1)T_b}^{(2i+1)T_b+T} \tilde{r}(t)\tilde{r}(t - 2T_b) \, dt = d + u, \]
where $d$ and $u$ are similar to Equations (7.14) and (7.15), respectively, except that the integration interval is changed and some subscripts are added. The corresponding received SNR $\gamma$ is given by Equation (7.17) with $E_b'$ and $E_b$ being replaced by $E_b'_{T2} = \sum_{l=0}^{L_{\text{int}}-1} d_{T2,l}$ and $E_{b,T2}$, respectively, and the average BER $P_{e2}$ is given by Equation (7.17). Substituting $P_{e1}$ and $P_{e2}$ into Equation (7.11), the average BER for this system can be obtained.

### 7.2.2.3 Two-Antenna Relay Systems

In this kind of system, the relay is equipped with two antennas, and the aforementioned ideal bandpass filter is also deployed at each antenna. This relay provides a spatial diversity gain by performing spatial diversity reception for the received signals along with the DTR scheme [119], and differential transmit diversity [241] for the regenerated signals from the detected data. The transmitted signal at the source is given by

$$s(t) = \frac{N_b}{2} \sum_{i=-1}^{(N_b/2)-1} \left( a_{2i}w_{T1}(t - 4iT_b) + a_{2i+1}w_{T1}(t - (4i + 1)T_b) \right),$$

where $N_b$ is an even number, and

$$a_{2i} = a_{2i-2}b_{2i}, \quad i \geq 0, \quad \text{with a given } a_{-2};$$

$$a_{2i+1} = a_{2i-1}b_{2i+1}, \quad i \geq 0, \quad \text{with a given } a_{-1}.$$

This means that two time slots, each of duration $2T_b$, are needed to relay two data-modulated pulses. The transmission rate of the two-antenna relay system is then equal to that of the single-antenna relay system.

The relaying protocol for this system is as follows. In the first time slot, the filtered received signal at the $j$th antenna of the relay can be written as

$$\tilde{r}_{R,j}(t) = h_{T1,j}(t) \ast s(t) + \tilde{n}_{R,j}(t) = \sum_{l=0}^{L_{\text{int}}-1} a_{T1,l,j}s(t - lT_w) + \tilde{n}_{R,j}(t).$$

Employing the differential demodulation at each antenna and combining the corresponding output signals of both antennas, the decision variables for $b_{2i}$ and $b_{2i+1}$ at the relay are given, respectively, by

$$z_R = \sum_{j=1}^{2} \int_{4iT_b}^{4iT_b+T} \tilde{r}_{R,j}(t)\tilde{r}_{R,j}(t - 4T_b) \, dt = \sum_{j=1}^{2} (d_{R,j} + u_{R,j}) = d_R + u_R,$$

$$\tilde{z}_R = \sum_{j=1}^{2} \int_{(4i+1)T_b}^{(4i+1)T_b+T} \tilde{r}_{R,j}(t)\tilde{r}_{R,j}(t - 4T_b) \, dt = \sum_{j=1}^{2} (\tilde{d}_{R,j} + \tilde{u}_{R,j}) = \tilde{d}_R + \tilde{u}_R.$$

The above equations are valid under the condition that the channel is quasi-static over the duration of $6T_b$. After some manipulations, we can obtain the SNRs of $z_R$ and $\tilde{z}_R$. 

Using the corresponding differential decoding scheme, the decision variables for \( \hat{E}_1 \) and \( \hat{E}_2 \), respectively, as the following:

\[
\gamma_R = \tilde{\gamma}_R = (E_{T1,1}^\prime + E_{T1,2}^\prime) \frac{E_{b,T1}}{N_0} \left[ 1 + \frac{T W}{E_{T1,1}^\prime + E_{T1,2}^\prime} \left( \frac{E_{b,T1}}{N_0} \right)^{-1} \right]^{-1},
\]

where \( E_{T1,j}^\prime = \sum_{l=0}^{L_{int}-1} a_{T1,l,j}^2 \), \( j = 1, 2 \). Hence, the corresponding average BER \( P_{e1} \) can be computed from Equation (7.8) with \( \gamma \) being replaced by \( \gamma_R \) or \( \tilde{\gamma}_R \).

Employing the differential transmit diversity introduced by [82] in the second time slot, the relay transmits the detected data bits, i.e., \( \tilde{b}_{2j} = \text{sign}(\hat{z}_R) \) and \( \tilde{b}_{2j+1} = \text{sign}(\hat{z}_R) \), from the first and second antennas to the destination, respectively, as

\[
s_{R,1}(t) = \frac{1}{\sqrt{2}} \sum_{i=-1}^{(N_b/2)-1} \{ \hat{a}_{2i} w_{T2}(t - (4i + 2)T_b) - \hat{a}_{2i+1} w_{T2}(t - (4i + 3)T_b) \},
\]

\[
s_{R,2}(t) = \frac{1}{\sqrt{2}} \sum_{i=-1}^{(N_b/2)-1} \{ \hat{a}_{2i+1} w_{T2}(t - (4i + 2)T_b) + \hat{a}_{2i} w_{T2}(t - (4i + 3)T_b) \},
\]

where

\[
\hat{a}_{2i} = \hat{a}_{2i-2} \tilde{b}_{2i} - \hat{a}_{2i-1} \tilde{b}_{2i+1}, \quad \hat{a}_{2i+1} = \hat{a}_{2i-1} \tilde{b}_{2i} + \hat{a}_{2i-2} \tilde{b}_{2i+1},
\]

\( i \geq 0 \) with a given \( \hat{a}_{-2} \) and \( \hat{a}_{-1} \).

Thus the received signal at the destination is

\[
\tilde{r}(t) = \sum_{j=1}^{2} h_{T2,j}(t) * s_{R,j}(t) + \tilde{n}(t) = \sum_{j=1}^{2} \sum_{i=0}^{L_{int}-1} a_{T2,l,j} s_{R,j}(t - iT_w) + \tilde{n}(t).
\]

Using the corresponding differential decoding scheme, the decision variables for \( \hat{b}_{2j} \) and \( \hat{b}_{2j+1} \) at the destination can be expressed, respectively, as

\[
z = \int_{(4i+2)T_b}^{(4i+3)T_b} \tilde{r}(t) \tilde{r}(t - 4T_b) \, dt + \int_{(4i+3)T_b}^{(4i+2)T_b} \tilde{r}(t) \tilde{r}(t - 4T_b) \, dt = d + u,
\]

\[
\tilde{z} = \int_{(4i+2)T_b}^{(4i+3)T_b} \tilde{r}(t) \tilde{r}(t - 3T_b) \, dt - \int_{(4i+3)T_b}^{(4i+2)T_b} \tilde{r}(t) \tilde{r}(t - 5T_b) \, dt = \tilde{d} + \tilde{u}.
\]

After some straightforward calculations, we obtain the SNRs of \( z \) and \( \tilde{z} \) (denoted as \( \gamma \) and \( \tilde{\gamma} \), respectively) as follows:

\[
\gamma = \tilde{\gamma} = \frac{E_{T2,1}^\prime + E_{T2,2}^\prime}{2} \frac{E_{b,T2}}{N_0} \left[ 1 + \frac{T W}{E_{T2,1}^\prime + E_{T2,2}^\prime} \left( \frac{E_{b,T2}}{N_0} \right)^{-1} \right]^{-1}, \tag{7.19}
\]

where \( E_{T2,j}^\prime = \sum_{l=0}^{L_{int}-1} a_{T2,l,j}^2 \), \( j = 1, 2 \). The corresponding average BER \( P_{e2} \) can be calculated from Equation (7.8) with \( \gamma \) given by Equation (7.19). Finally, the average BER of the system is obtained by substituting \( P_{e1} \) and \( P_{e2} \) into Equation (7.11).
7.2.3 Numerical Results and Concluding Remarks

To obtain numerical results, both analysis and simulations are performed using the UWB indoor communication link model as shown in Figure 7.2, where the letters S, R, and D denote the source, relay, and destination, respectively. In Figure 7.2, D at either position A (denoted by D(A)) or B (denoted by D(B)) detects a UWB signal of S via either direct transmission link, i.e., without R, or two-hop relay link, i.e., using R with either one or two antennas. The CIRs for all the links are calculated using Equation (7.1). According to [23], the total multipath gain $G$ in Equation (7.1) decreases with the link distance $D$ as $G = G_0/D^{-\lambda}$, where $G_0$ is the reference value of the power gain evaluated at $D = 1$ m and $\lambda$ is the path loss exponent of the power attenuation law. The value of $G_0$ can be calculated from $G_0 = 10^{-A_0/10}$, where $A_0$ (in dB) represents the path loss at the reference distance $D_0 = 1$ m ($A_0 = 47$ dB and $\lambda = 1.7$ for a LOS environment, and $A_0 = 51$ dB and $\lambda = 3.5$ for an NLOS environment). The coefficients $\varphi_l$, $l = 0, \ldots, L_t - 1$, in Equation (7.1) are generated from the standard UWB channel models CM1–CM4, as shown in Table 2.1. According to this table, the S–R link (LOS, 4 m) and R–D(A) link (LOS, 4 m) belong to the CM1 ($T_{mds} = 40$ ns). The S–D(A) link (NLOS, 5.6 m) and R–D(B) link (NLOS, 7.2 m) belong to the CM3 ($T_{mds} = 120$ ns). The S–D(B) link (NLOS, 10.8 m) belongs to the CM4 ($T_{mds} = 200$ ns). The Gaussian monocycle with $T_w = 0.5$ ns as described in [23] is adopted. To preclude ISI, we choose $T_b = 120.5$ ns when considering D(A), while $T_b = 200.5$ ns when considering D(B). For fair comparison, we set $E_{b,T_1} = E_{b,T_2} = 0.5E_b$, where $E_b$ is the transmit energy per bit for the nonrelay system.

Figure 7.3 shows the BER performance comparison among the coherent detection systems with the single-antenna relay, with the two-antenna relay, and without relays, respectively. The destination is located at the position A. The ratio $E_b/N_0$ is the transmitted SNR per bit.\(^3\) The curves labeled with ‘S-Rake’ refer to the results for the L-finger

\(^3\) Notice that the links in Figure 7.2 are several meters long, which leads to the total multipath gain $G$ in Equation (7.1) on the order of minus several tens dB; see [23] and the aforementioned discussion. This results in a very high transmitted SNR per bit ($E_b/N_0$). For a proper performance comparison between the systems with and without relays, the path loss should be taken into account.
selective-Rake systems. The channel is estimated using the pilot signal with $N_p = 4$. The curves labeled with ‘A-Rake’ refer to the results for the all-Rake systems with perfect channel estimation. The systems with the all-Rake receivers are considered as benchmark systems. It can be seen from Figure 7.3 that the analytical BER results agree quite well with the simulated BER results for all the cases, and that the analytical results of the S-Rake systems with more fingers (comparing $L = 5$ and $L = 1$) are closer to the corresponding simulation results, which indicates the validity of the Gaussian approximation in the central limit theorem discussed in Section 7.2.1. The S-Rake systems with five fingers outperform those with one finger since the former systems can capture more signal energy at the expense of increased complexity. Obviously, the S-Rake systems using the single-antenna relay have much better performance than those without relaying, for example, in the case of $L = 5$, a gain of 17 dB in $E_b/N_0$ at $BER = 10^{-2}$ is obtained by using the single-antenna relay compared to the corresponding nonrelay system. The price for this reward is that the transmission rate of the former systems is half of that of the latter ones. The five-finger S-Rake systems using the two-antenna relay outperform the systems using the single-antenna relay by approximately 2 dB gain in $E_b/N_0$ at $BER = 10^{-2}$. This is a result of the spatial diversity gain obtained using two antennas at the relay. Since the relay systems discussed above provide BER performance gains at the expense of a half
transmission rate, we also consider the case where the relay systems have approximately
the same transmission rate as the nonrelay ones. In this case, we set $T_b = 60$ ns for those
relay systems, resulting in ISI at the receivers of both the relay and destination. The
corresponding simulation results, marked with ‘HR’ (i.e., high-rate), are partly shown in
Figure 7.3. It is noteworthy that this ISI slightly affects the BER performance due to
the fact that the transmission rate is still not too high and the power delay profile of the
considered UWB channel model is exponentially decaying.

The effect of channel estimation errors on the BER performance, and the tradeoff
between the number of selective-Rake fingers, $L$, and the number of antennas at the
relay, $M$, for the selective-Rake relay systems are depicted in Figure 7.4. Only the sim-
ulation results are demonstrated for ease of illustration. The pilot signal with $N_p = 2$
is used to represent the worse case for imperfect channel estimation. In both single-
and multi-antenna relay cases, the difference in BER performance between the systems
with the perfect and imperfect channel estimation increases as the number of fingers
increases. This can be explained as follows. The UWB multipath channel (7.1) has an
exponentially decaying power delay profile. By considering $\hat{h}(t)$, the amplitude of $\alpha_l$
tends to be less while the variance of $\varepsilon_l$ is fixed for all $l$. As a result, the sum $(\alpha_l + \varepsilon_l)$
is more likely to deviate from $\alpha_l$ for larger $l$, leading to performance degradation for
larger $L$.

![Figure 7.4](image)

**Figure 7.4** The effect of channel estimation errors on the BER performance of the coherent
detection Rake systems using (a) the single-antenna relay ($M = 1$) and (b) the two-antenna relay
($M = 2$). The destination is located at position $A$. 
To investigate the tradeoff between \( L \) and \( M \), the BER curves are compared for the same product of \( L \) and \( M \). Under perfect and imperfect channel estimation, the performance of the two-antenna relay system using one finger is very similar to that of the single-antenna relay system using two fingers. The corresponding curves are difficult to differentiate. However, the two-antenna relay system using five fingers outperforms the single-antenna relay system using 10 fingers. Indeed, the superiority of the two-antenna relay system is also found when using more than one finger. One explanation for this is that the relay having two antennas, each of which is equipped with five fingers, can collect more energy than the relay having one antenna, which is equipped with 10 fingers, due to the exponentially decaying power delay profile. These results indicate that, typically, increasing the number of antennas at the relay gains more BER improvement than increasing the number of selective-Rake fingers.

The BER performance of three noncoherent detection DTR systems is illustrated in Figure 7.5, where ‘conv. DTR’ and ‘opt. DTR’ denote, respectively, the conventional DTR using the integration interval \( T = T_{\text{mds}} + T_{\text{w}} \) and the DTR using an optimal integration interval in the sense of minimizing the average BER. An ideal bandpass filter with \( W = 5 \) GHz is adopted. The optimal interval is determined through simulation trials. In fact, it provides an optimal compromise between the amount of captured signal energy and that of noise reduction. The optimal intervals for the \( S-D(A) \) and \( R-D(B) \) links are about 40 ns,

![Figure 7.5](image-url)  

**Figure 7.5** BERs of three noncoherent detection DTR systems: (a) without relaying; (b) using the single-antenna relay; (c) using the two-antenna relay. The destination is located at position \( A \). Analytical and simulation curves are dashed and solid, respectively.
while the optimal intervals for the $S-R$ and $R-D(A)$ links are only 10 ns. For the $S-D(B)$ link, the optimal interval is about 100 ns. It can be seen from Figure 7.5 that on average the opt. DTR systems have a 1.5 dB gain at $BER = 10^{-2}$ compared to the conventional DTR systems. As shown, the analytical results agree well with the simulation results. This is because the time–bandwidth products obtained here are large enough (i.e., $TW \geq 50$) so that the Gaussian approximation is accurate. The conventional DTR system employing the single-antenna relay remarkably outperforms the conventional DTR system without relaying by about 18 dB at $BER = 10^{-2}$. The price for this reward is that the transmission rate of the former system is half of that of the latter one. With the same transmission rate, using the two-antenna relay achieves 2 dB better performance at $BER = 10^{-2}$ than using the single-antenna relay in the conventional DTR system. In Figure 7.5, we also plot the simulation BER results for some HR DTR relay systems ($T = T_b = 60$ ns), i.e., with approximately the same transmission rate as the DTR nonrelay systems. Similar to the case of the HR Rake relay systems, the effect of ISI on the BER performance of the HR DTR relay systems is not conspicuous. By comparing Figures 7.3 and 7.5, the overall performance of the DTR systems is worse than that of the Rake systems. From these figures, we can say that the transmit energy per bit $E_b$ can be greatly reduced by employing the proposed relays even without sacrificing the transmission rate. In other words, the use of these relays can increase the UWB coverage range.

Figure 7.6 shows the BER performance of three Rake systems where the destination is located at the position $B$. The destination is now far away from the relay, resulting in the

![Figure 7.6](image_url)

**Figure 7.6** BERs of three coherent detection Rake systems: (a) without relaying; (b) using the single-antenna relay; (c) using the two-antenna relay. The destination is located at position $B$. 
long NLOS link between them. It is clear from the figure that the S-Rake systems using the single-antenna relay outperform the systems without relaying. However, comparing Figures 7.3 and 7.6, we can see that the performance gains of the single-antenna relay systems over the corresponding nonrelay systems for the scenario in Figure 7.6 are lower than those for the scenario in Figure 7.3. Likewise, the two-antenna relay systems have smaller gains in the BER performance over the single-antenna relay systems. This is due to the severe path loss on the aforementioned NLOS link. Similar trends can also be observed in three DTR systems, as illustrated in Figure 7.7. These results indicate that the distances and LOS versus NLOS conditions of the links among the source, relay, and destination have a significant impact on the performance of the UWB relay systems.

### 7.3 UWB Relay Systems with MIMO at Source and Destination

Consider now the general case as illustrated in Figure 7.1, i.e., all the source, relay and destination are equipped with multiple antennas. We call this case the UWB MIMO relay system. Our focus is the system design and performance analysis under two different assumptions on channel state information (CSI):

(i) partial CSI is only available at the receiver side;
(ii) partial CSI is only available at the transmitter side.
We refer to the UWB MIMO relay systems designed under the first assumption as receiver-CSI-assisted relay systems and to those under the second assumption as transmitter-CSI-assisted relay systems. Two kinds of relaying schemes, DCF and DF, will be discussed for the former systems,\(^4\) while the AF and DTF relaying schemes will be investigated for the latter systems.

The system performance will be analysed in terms of amount of fading (AoF), SNR outage probability and BER. Because spatial correlation in UWB channels is a critical factor affecting the performance of UWB MIMO systems without relays [118, 152], we will also quantify the effect of such correlation on the system performance. Numerical results based on both theoretical analysis and simulations provide some basic guidelines for the design of the UWB MIMO relay systems, for example, about the appropriate use of transmit/receive antennas for different kinds of relay systems.

### 7.3.1 Channel and System Models

The CIR for a UWB transmission link is described by Equation (7.1). In this section (Section 7.3), the Nakagami distribution is used to characterize the fading statistics of \(\phi_l\).

Denote \(\phi_l = \nu_l \zeta_l\), where \(\nu_l = \pm 1, \zeta_l = |\phi_l|\). The variable \(\nu_l\) takes values \(+1\) and \(-1\) with equal probability. The variable \(\zeta_l\) obeys the Nakagami-\(m\) distribution [173] with the pdf given by

\[
p_{\zeta_l}(x; m, \theta_l) = \frac{2x^{2m-1}}{\Gamma(2m)\theta_l^m(m-1)!} \exp\left(-\frac{x^2}{\theta_l}\right) U(x),
\]

where \(U(\cdot)\) is the unit step function as defined in Equation (6.31), \(\theta_l = \Omega_l/m\) and \(\Omega_l = \mathbb{E}[\zeta_l^2]\) is exponentially decreasing with the excess delay, i.e., \(\Omega_l = \varrho \Omega_{l-1}\) where \(\varrho < 1\) is a constant. The value of \(\varrho\) is determined by the communication scenario. Throughout this paper, we assume that \(m\) is an integer and fixed for all the path indices. The first assumption on \(m\) is necessary for the theoretical analysis hereafter, whereas the second one is just for analytical convenience. Define \(\eta_l = \phi_l^2 = \zeta_l^2\). Hence, \(\eta_l\) follows the Gamma distribution whose pdf and cdf are

\[
p_{\eta_l}(x; m, \theta_l) = \frac{x^{m-1}}{\Gamma(m)\theta_l^m(m-1)!} \exp\left(-\frac{x}{\theta_l}\right) U(x),
\]

and

\[
P_{\eta_l}(x; m, \theta_l) = \left[1 - \frac{\Gamma(m, \frac{x}{\theta_l})}{(m-1)!}\right] U(x) = \left[1 - \exp\left(-\frac{x}{\theta_l}\right) \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{x}{\theta_l}\right)^k\right] U(x)
\]

respectively.

We consider the UWB MIMO relay system depicted in Figure 7.1, where the source and destination are equipped with \(M_S\) transmit and \(M_D\) receive antennas respectively.

\(^4\) Note that in the SISO case, i.e., the source, relay and destination have only one antenna, the DCF relaying scheme presented here can be viewed as the AF relaying scheme for the receiver-CSI-assisted relay systems.
and the relay is equipped with $M_R^{(r)}$ receive and $M_R^{(t)}$ transmit antennas. Let $H_{SR,l}$ and $H_{RD,l}$ (whose dimensions are $M_R^{(r)} \times M_S$ and $M_D \times M_R^{(t)}$ respectively) be the channel matrix for the $l$th path between the source-relay link and relay-destination link respectively. Therefore, $[H_{SR,l}]_{ij} = \alpha_{SR,l,ij}$ and $[H_{RD,l}]_{ij} = \alpha_{RD,l,ij}$ are the channel coefficients between the $j$th transmit antenna and the $i$th receive antenna for the first and second hops respectively. These coefficients are assumed to remain constant over the duration of the data transmission at both hops. As mentioned in Equation (7.1), we have that $\alpha_{SR,l,ij} = \sqrt{G_{SR}\phi_{SR,l,ij}}$ and $\alpha_{RD,l,ij} = \sqrt{G_{RD}\phi_{RD,l,ij}}$. To make further analysis tractable, we assume that $\Omega_{SR,l,ij} = \Omega_{SR,l,i'j'}$ and $\Omega_{RD,l,ij} = \Omega_{RD,l,i'j'}$ for all $l, i, j, i', j'$. For this reason and notational simplicity, we omit the subscripts $i$ and $j$ of $\Omega_{SR,l,ij}$, $\Omega_{RD,l,ij}$ and of the related variables in the rest of this section. We ignore the direct link between the source and destination due to the larger distance and additional path loss compared with the source-relay and relay-destination links.

### 7.3.2 Receiver-CSI-Assisted Relay Systems

In these systems, the source has no CSI but the relay and destination have partial knowledge of the source-relay and relay-destination link channels respectively, i.e., the relay knows the channel matrices $H_{SR,l}$, $l = 0, 1, \ldots, L - 1$, and the destination knows the channel matrices $H_{RD,l}$, $l = 0, 1, \ldots, L - 1$, where $L < L_t$. In order to achieve both spatial and multipath diversities, the space-time block coding technique described in Chapter 4 is employed at the transmitter side and Rake reception is performed at the receiver side. In what follows, we describe the relaying schemes for such systems.

#### 7.3.2.1 DCF Relaying

In this scheme, the source with the multiple transmit antennas employs space-time block codes obtained from the ROD [242] as outlined in Section 4.4. Let $B_S$ be the space-time block-coded data matrix for the source designed according to the ROD. Suppose that $B_S$ is of dimension $M_S \times N_c$, where $N_c$ is the block length of the corresponding space-time block code. If binary data bits $\{b_k\}_{k=1}^{N_b}$ are transmitted with this matrix, then the code rate is $N_b/N_c$. In what follows, we concentrate on the space-time block-coded data matrices with the full code rate, i.e., $N_c = N_b$. Define

$$g(t, i) = \begin{bmatrix} w(t - N_b(i - 1)T_i) \\
w(t - N_b(i - 1)T_i - T_i) \\
\vdots \\
w(t - N_b(i - 1)T_i - (N_b - 1)T_i) \end{bmatrix},$$

where $T_i$ is the pulse repetition period and $i$ will be stated in Equation (7.22). In order to preclude ISI, we choose $T_i$ such that $T_i \geq L_t T_w$. In the first hop, the transmitted signal vector at the source can be modelled as

$$x_S(t) = \sqrt{\frac{E_{LS}}{M_S}} \sum_{i=1}^{N_l} B_S g(t, i), \quad (7.22)$$
where $N_f$ is the number of transmitted pulses representing one data bit, $i$ is the index of such pulses, $E_{f,S} = E_{b,S}/N_f$ is the energy of the transmitted pulse and $E_{b,S}$ is the bit energy.

At each receive antenna of the relay, a Rake receiver with $L_f$ fingers, whose correlators use the pulse $w(t)$ as a template, is employed. Corresponding to the $l$th finger, the correlator outputs for all the receive antennas and received data bits at the relay can be expressed in a matrix form as

$$Y_{R,l}(i) = \sqrt{\frac{E_{f,S}}{M_S}}H_{SR,l}B_S + N_{R,l}(i), \ l = 0, \ldots, L - 1, \ i = 1, \ldots, N_f, \ (7.23)$$

where $N_{R,l}(i)$ is an $M_{f} \times N_b$ matrix, each entry of which is the AWGN with zero mean and double-sided PSD $N_0/2$. In order to decouple the received signal matrix $Y_{R,l}(i)$, we define the matrix $V_{R,l}(n)$ which satisfies

$$[H_{SR,l}]_nB_S = b^TV_{R,l}(n), \ (7.24)$$

where $b = [b_1 \ b_2 \ \cdots \ b_{N_b}]^T$ is the vector of the transmitted bits and $n$ is the index of receive antennas. Then $V_{R,l}(n)$ is the companion of the ROD (CROD). Examples of RODs and CRODs are illustrated in Section 4.4. Since the relay was assumed to know the channel matrices $H_{SR,l}, l = 0, 1, \ldots, L - 1$, the matrices $V_{R,l}(n), l = 0, 1, \ldots, L - 1$, in Equation (7.24) are known to the relay. Consequently, the decoupled data bits can be obtained as

$$z_R := [z_{R,1} \ z_{R,2} \ \cdots \ z_{R,N_b}]^T = \sum_{n=1}^{M_{f}} \sum_{i=1}^{N_f} \sum_{l=0}^{L-1} V_{R,l}(n)[(Y_{R,l}(i))_n]^T. \ (7.25)$$

At the relay, the $k$th element of $z_R$ is normalized with respect to

$$\mathbb{E}_H[|z_{R,k}|^2] = N_f \mathcal{J} \left( \frac{E_{b,S} \mathcal{J}}{M_S} + \frac{N_0}{2} \right),$$

where

$$\mathcal{J} = \sum_{l=0}^{L-1} \mathcal{J}_l, \ \mathcal{J}_l = |H_{SR,l}|^2_F.$$

Through the normalization, the transmitted signal vector at the relay as shown in Equation (7.26) has the same average power as that at the source if $E_{f,R}$ in Equation (7.26) is equal to $E_{f,S}$. Those normalized variables, denoted $[z_{R,k}]_{k=1}^{N_b}$, are encoded in a similar way as the data bits $\{b_k\}_{k=1}^{N_b}$ are encoded at the source. Let $\hat{Z}_R$ be the resultant ROD space–time block-coded matrix whose dimension is $M_{f} \times N_c$. The transmitted signal vector at the relay in the second hop is represented by

$$x_R(t) = \sqrt{\frac{E_{f,R}}{M_{f} \sum_{i=1}^{N_f}}} \sum_{k=1}^{N_b} \hat{Z}_Rg(t - N_bN_fT_f, i), \ (7.26)$$
where $E_{f,R} = E_{b,R}/N_f$ is the energy of the transmitted pulse and $E_{b,R}$ is the energy of $\bar{z}_{R,k}$.

A Rake receiver with $L$ fingers is used at each receive antenna of the destination. For the $i$th finger, the $M_D \times N_b$ matrix of the correlator outputs can be written as

$$Y_{D,i}(i) = \sqrt{E_{f,R}/M_R} \mathbf{H}_{RD,i} \bar{Z}_R + N_{D,i}(i), \quad l = 0, \ldots, L - 1, \quad i = 1, \ldots, N_f,$$

(7.27)

where $N_{D,i}(i)$ is the $M_D \times N_b$ matrix of the AWGN with the same statistical properties as the elements of $N_{R,i}(i)$. Exploiting the partial knowledge of the CSI $\mathbf{H}_{RD,i}$, $l = 0, 1, \ldots, L - 1$, the destination generates the matrices $\mathbf{V}_{D,i}(n)$, $l = 0, 1, \ldots, L - 1$, which satisfy

$$[\mathbf{H}_{RD,i}]_n \tilde{\mathbf{Z}}_R = [\bar{z}_{R,1} \bar{z}_{R,2} \cdots \bar{z}_{R,N_b}] \mathbf{V}_{D,i}(n).$$

Finally, the vector of the decision variables can be derived as follows:

$$\mathbf{z}_D = \sum_{n=1}^{M_D} \sum_{i=1}^{N_f} \sum_{l=0}^{L-1} \mathbf{V}_{D,i}(n) ([Y_{D,i}(i)]_n)^T = [z_{D,1} \ z_{D,2} \cdots \ z_{D,N_b}]^T.$$  

(7.28)

For analytical convenience, from now on we assume equal power allocation between the source and relay, i.e., $E_{b,S} = E_{b,R} = E_b/2$, where $E_b$ represents the transmit energy per bit for the whole relay system. Since $E_{f,S} = E_{f,R} = E_b/(2N_f)$, we use $E_{f}$ to denote both $E_{f,S}$ and $E_{f,R}$ in the rest of this section. After some straightforward calculations, the overall end-to-end SNR per bit, i.e., the SNR of $z_{D,k}$, is found to be

$$\gamma = \frac{E_b^2 \mathcal{J} \mathcal{J}'}{M_R^{(t)} E_b N_0 \mathcal{J} + M_S E_b N_0 \mathcal{J}' + M_S M_R^{(t)} N_0^2} = \frac{A_1 \mathcal{J} \mathcal{J}'}{A_2 \mathcal{J} + A_3 \mathcal{J}' + A_4},$$

(7.29)

where

$$\mathcal{J}' = \sum_{l=0}^{L-1} \mathcal{J}'_l,$$

$$\mathcal{J}'_l = \|\mathbf{H}_{RD,i}\|_F^2,$$

and $A_1 = E_b^2$, $A_2 = M_R^{(t)} E_b N_0$, $A_3 = M_S E_b N_0$ and $A_4 = M_S M_R^{(t)} N_0^2$ are introduced for notational convenience. In Appendix 7.A, the pdfs and cdfs of $\mathcal{J}$ and $\mathcal{J}'$ are provided, and it is shown that these functions can be expressed in closed form only for the spatial uncorrelation case. We focus on this case in the following, while the spatial correlation case will be considered in Section 7.3.4.

**SNR Outage Probability Analysis**

A common measure for evaluating the performance of a system in fading channels is the SNR outage probability, which is defined as

$$P_{out} = \Pr[\gamma \leq \gamma_{th}],$$

(7.30)
where $\gamma_{th}$ is a prespecified SNR threshold. Substituting Equation (7.29) into Equation (7.30) yields

$$P_{out} = \Pr \left[ \frac{A_1 JJ' + A_3 JJ' + A_4}{A_2 J + A_3 J' + A_4} \leq \gamma_{th} \right]$$

$$= \int_0^\infty \Pr \left[ \frac{A_1 JJ' x}{A_2 x + A_3 J' + A_4} \leq \gamma_{th} \right] p_J(x) \, dx$$

$$= \int_0^{(A_3/A_1)\gamma_{th}} \Pr [J' (A_1 x - A_3 \gamma_{th}) \leq \gamma_{th} (A_2 x + A_4)] p_J(x) \, dx$$

$$+ \int_{(A_3/A_1)\gamma_{th}}^\infty \Pr [J' (A_1 x - A_3 \gamma_{th}) \leq \gamma_{th} (A_2 x + A_4)] p_J(x) \, dx$$

$$= \int_0^{(A_3/A_1)\gamma_{th}} p_J(x) \, dx + \int_{(A_3/A_1)\gamma_{th}}^\infty P_{J'} \left[ \frac{\gamma_{th} (A_2 x + A_4)}{A_1 x - A_3 \gamma_{th}} \right] p_J(x) \, dx$$

$$:= Q_1 + Q_2,$$

(7.31)

where

$$Q_1 = P_J \left( \frac{A_3}{A_1} \gamma_{th} \right) = \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi \, P_{J_l} \left( \frac{A_3}{A_1} \gamma_{th}; k, \Phi_{SR,l} \right) = 1 - \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi \frac{\Gamma(k, C_1 \gamma_{th}/\gamma_0)}{(k - 1)!},$$

$$Q_2 = \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi' \int_{(A_3/A_1)\gamma_{th}}^\infty P_{J'_l} (x; k, \Phi_{SR,l})$$

$$\times P_{J'_l'} \left[ \frac{\gamma_{th} (A_2 x + A_4)}{A_1 x - A_3 \gamma_{th}}; k', \Phi_{RD,l'} \right] \, dx$$

$$= \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi' \left\{ \left[ 1 - P_{J_l} \left( \frac{A_3}{A_1} \gamma_{th}; k, \Phi_{SR,l} \right) \right] \right.$$

$$- \int_{(A_3/A_1)\gamma_{th}}^\infty \frac{P_{J'_l} (x; k, \Phi_{SR,l})}{(k' - 1)!} \Gamma \left[ k', \frac{\gamma_{th} (A_2 x + A_4)}{\Phi_{RD,l'} (A_1 x - A_3 \gamma_{th})} \right] \, dx \right\}$$

$$= \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi' \left[ \frac{\Gamma(k, C_1 \gamma_{th}/\gamma_0)}{(k - 1)!} - Q_3 \right],$$

(7.32)
\[ Q_3 = \frac{2(C_1 \gamma_0 / \gamma_0)^k}{(k - 1)!} \sqrt{\frac{C_2}{C_1}} \left( 1 + \frac{1}{\gamma_0} \right) \exp \left[ -\frac{\gamma_0}{\gamma_0} (C_1 + C_2) \right] \times \sum_{j_1=0}^{k' - 1} \frac{(C_2 \gamma_0 / \gamma_0)^{j_1}}{j_1!} \sum_{j_2=0}^{j_1} \left( \frac{j_1}{j_2} \right) \left[ \frac{C_1}{C_2} \left( 1 + \frac{1}{\gamma_0} \right) \right]^{j_2/2} \times \sum_{j_3=0}^{k - 1} \left( \frac{k - 1}{j_3} \right) \left[ \frac{C_2}{C_1} \left( 1 + \frac{1}{\gamma_0} \right) \right]^{j_3/2} \frac{2}{\gamma_0} \sqrt{C_1 C_2 \gamma_0 (1 + \gamma_0)} \right], \quad (7.33) \]

\[ C_1 = \frac{M_S}{\Phi_{SR,l}}, \quad C_2 = \frac{M^{(t)}}{\Phi_{RD,l'}}, \]

\( \gamma_0 = E_b / N_0 \) is the transmitted SNR per bit, and \( \mu, \mu', \Phi_{SR,l} \) and \( \Phi_{RD,l'} \) are defined in Appendix 7.A. The term \( \Psi \) is the abbreviation for \( \Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) \) given in Equation (7.60) and the term \( \Psi' \) stands for \( \Psi(l', k', \mu', \{\Phi_{RD,q}\}_{q'=0}^{L-1}) \). The detailed derivation of Equation (7.33) is provided in Appendix 7.B. Combining Equations (7.31)–(7.33) yields

\[ P_{out} = 1 - 2 \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{\mu'} \Psi \left( \frac{(C_1 \gamma_0 / \gamma_0)^k}{(k - 1)!} \right) \sqrt{\frac{C_2}{C_1}} \left( 1 + \frac{1}{\gamma_0} \right) \exp \left[ -\frac{\gamma_0}{\gamma_0} (C_1 + C_2) \right] \times \sum_{j_1=0}^{k' - 1} \frac{(C_2 \gamma_0 / \gamma_0)^{j_1}}{j_1!} \sum_{j_2=0}^{j_1} \left( \frac{j_1}{j_2} \right) \left[ \frac{C_1}{C_2} \left( 1 + \frac{1}{\gamma_0} \right) \right]^{j_2/2} \times \sum_{j_3=0}^{k - 1} \left( \frac{k - 1}{j_3} \right) \left[ \frac{C_2}{C_1} \left( 1 + \frac{1}{\gamma_0} \right) \right]^{j_3/2} \frac{2}{\gamma_0} \sqrt{C_1 C_2 \gamma_0 (1 + \gamma_0)} \right]. \quad (7.34) \]

**AoF Analysis**

We use the AoF to quantify the severity of fading experienced at the output of the DCF relay system. This measure is given by [222, Equation (2.5)]

\[ \text{AoF} = \frac{\mathbb{E}[\gamma^2]}{(\mathbb{E}[\gamma])^2} - 1, \quad (7.35) \]

where

\[ \mathbb{E}[\gamma^n] = \int_0^\infty x^n p_\gamma(x) \, dx \quad (7.36) \]

is the \( n \)-th order moment of \( \gamma \). In order to calculate the AoF, we first need to find the pdf \( p_\gamma(x) \). This pdf is given by Equation (7.67) in Appendix 7.C. To the best of our knowledge, the closed-form solution to Equation (7.36) is not available. One option is to
develop a tight approximation of Equation (7.36). In the following, we assume that the transmitted SNR per bit, $\gamma_0$, is large enough such that

$$\left( \frac{\mathcal{J}}{M_S} + \frac{\mathcal{J}'}{M_R} \right) \gamma_0 \gg 1.$$ 

Under this assumption, the end-to-end SNR per bit can be approximated as

$$\gamma \approx \frac{A_1 \mathcal{J}' \mathcal{J}}{A_2 \mathcal{J} + A_3 \mathcal{J}'} \triangleq \gamma_{ap}. \quad (7.37)$$

Following the same procedure as in the preceding subsection, the cdf

$$P_{\gamma_{ap}}(x) = \Pr[\gamma_{ap} \leq x] U(x)$$

is obtained as

$$P_{\gamma_{ap}}(x) = \left\{ 1 - 2 \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi \Psi' \sqrt{\frac{C_2}{C_1}} \left( \frac{C_1 x}{\gamma_0} \right)^k \frac{1}{(k-1)!} \exp \left[ -\frac{x}{\gamma_0} \left( C_1 + C_2 \right) \right] \right. $$

$$\times \sum_{j_1=0}^{K_{j_3-j_2} - 1} \frac{(C_2 x/\gamma_0)^{j_1}}{j_1!} \sum_{j_2=0}^{j_1} \frac{(C_1)}{C_2} \frac{j_2}{2} \sum_{j_3=0}^{k-1} \frac{(C_2)}{C_1} \frac{j_3}{2}$$

$$\times \mathcal{K}_{j_3-j_2+1} \left( \frac{2x}{\gamma_0} \sqrt{C_1 C_2} \right) \left\} U(x). \quad (7.38)$$

From Equation (7.38) we can obtain

$$P_{\gamma_{ap}}(x) = 2 \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi \Psi' \sqrt{\frac{C_2}{C_1}} \left( \frac{C_1 x}{\gamma_0} \right)^k \frac{1}{(k-1)!} \exp \left[ -\frac{x}{\gamma_0} \left( C_1 + C_2 \right) \right]$$

$$\times \sum_{j_1=0}^{k'-1} \frac{(C_2 x/\gamma_0)^{j_1}}{j_1!} \sum_{j_2=0}^{j_1} \frac{(C_1)}{C_2} \frac{j_2}{2} \sum_{j_3=0}^{k-1} \frac{(C_2)}{C_1} \frac{j_3}{2} \left( \frac{C_1 + C_2}{\gamma_0} - \frac{j_1 + k}{x} \right) \mathcal{K}_{j_3-j_2+1} \left( \frac{2\sqrt{C_1 C_2} x}{\gamma_0} \right)$$

$$+ \frac{\sqrt{C_1 C_2}}{\gamma_0} \left[ \mathcal{K}_{j_3-j_2} \left( \frac{2\sqrt{C_1 C_2} x}{\gamma_0} \right) + \mathcal{K}_{j_3-j_2+2} \left( \frac{2\sqrt{C_1 C_2} x}{\gamma_0} \right) \right] U(x).$$
With the aid of [93, Equation (6.621.3)], the high-SNR approximation of the $n$th-order moment of $\gamma$ can be derived as

$$
E[\gamma^n] \approx \int_0^\infty x^n p_{\gamma|x}(x) \, dx
$$

$$
= 2\sqrt{\pi} \gamma_0^n \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \psi' \frac{C_k^l}{(k-1)!} \sum_{j_1=0}^{k-1} \sum_{j_2=0}^{l} \binom{j_1}{j_2} (7.39)
$$

$$
\times \sum_{j_3=0}^{k-1} \binom{k-1}{j_3} \frac{\Gamma(c_1 + c_2 + n) \Gamma(c_1 - c_2 + n) 2^{2c_2} C_2^2}{\Gamma(c_1 + n + 3/2)(\sqrt{C_1} + \sqrt{C_2})^{2(c_1+c_2+n)}} \mathcal{D}
$$

where

$$
c_1 = j_1 + k, \quad c_2 = j_3 - j_2 + 1, \quad C_0 = \left(\frac{\sqrt{C_1} - \sqrt{C_2}}{\sqrt{C_1} + \sqrt{C_2}}\right)^2
$$

$$
\mathcal{D} = \frac{(c_1 + n)^2 - c_2^2 (C_1 + C_2)}{(\sqrt{C_1} + \sqrt{C_2})^2} 2 F_1 \left( c_1 + c_2 + n + 1, c_2 + \frac{1}{2}; c_1 + n + \frac{3}{2}; C_0 \right)
$$

$$
+ \frac{1}{4} (c_1 - c_2 + n)(c_1 - c_2 + n + 1) 2 F_1 \left( c_1 + c_2 + n, c_2 - \frac{1}{2}; c_1 + n + \frac{3}{2}; C_0 \right)
$$

$$
+ \frac{4(c_1 + c_2 + n)(c_1 + c_2 + n + 1) C_1 C_2}{(\sqrt{C_1} + \sqrt{C_2})^4}
$$

$$
\times 2 F_1 \left( c_1 + c_2 + n + 2, c_2 + \frac{3}{2}; c_1 + n + \frac{3}{2}; C_0 \right)
$$

$$
- (j_1 + k)(c_1 + n + 1/2) 2 F_1 \left( c_1 + c_2 + n, c_2 + \frac{1}{2}; c_1 + n + \frac{1}{2}; C_0 \right).
$$

Using Equations (7.35) and (7.39), the approximate AoF is obtained. We will see later in Section 7.3.4 that this AoF matches closely with the AoF evaluated from $p_{\gamma}(x)$ by means of simulations.

**BER Analysis**

Another important performance measure which characterizes the end-to-end link quality is the BER. It is given by [222, Equation (5.1)]

$$
P_e = \int_0^\infty Q(\sqrt{x}) p_{\gamma|x}(x) \, dx. \quad (7.40)
$$
There is no closed-form solution to Equation (7.40) with \( p_y(x) \) given by Equation (7.67). By using the above high-SNR assumption, the BER can be approximated by

\[
P_e \approx \int_0^\infty Q(\sqrt{x}) P_{y_{ap}}(x) \, dx
\]

where the first equality is obtained by letting \( Y \) be a standard normal random variable and using [222, Equation (4.1)]. A direct calculation and using [93, Equation (6.621.3)] yields

\[
P_e \approx \frac{1}{2} \left( 1 - \sqrt{\frac{2}{\pi}} \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \sum_{k'=1}^{\mu'} \Psi \left( \frac{\sqrt{C_2}}{C_1} \right)^{j_3/2} \int_0^\infty \frac{2y^2}{\gamma_0} \psi_1 \left( \frac{1}{2} \right) \frac{\gamma_0}{\gamma} \right)
\]

7.3.2.2 DF Relaying

In the DF relaying scheme, the transmitted and received signals for the first hop are the same as Equations (7.22) and (7.23) respectively. Such received signals are decoupled and combined to produce the decoupled data bits as described in Equation (7.25). Subsequently, the relay makes hard decisions on the decoupled bits and encodes them in a similar manner to the way the source encodes the data bits. This yields the matrix \( \hat{B}_R \) of dimension \( M_R^{(1)} \times N_b \). As will be shown in Section 7.3.4, such a hard decision leads to better performance (at the cost of higher complexity) than the DCF relaying scheme. In the second hop, the transmitted signals at the relay and the received signals at the destination can be represented respectively as Equations (7.26) and (7.27) with \( \hat{Z}_R \) being replaced by \( \hat{B}_R \). Finally, the decision variables can be obtained as Equation (7.28). It is straightforward to show that the received SNRs per bit for the first and second hops are

\[
\gamma_1 = \frac{\bar{J} \gamma_0}{M_S} \quad \text{and} \quad \gamma_2 = \frac{J' \gamma_0}{M_R^{(1)}}
\]

respectively.
**SNR Outage Probability Analysis**

An SNR outage occurs if either the source–relay or relay–destination link is in outage. Therefore, we have

\[ P_{\text{out}} = \Pr[\min(\gamma_1, \gamma_2) \leq \gamma_{th}] \]

\[ = 1 - \Pr[\gamma_1 > \gamma_{th}] \Pr[\gamma_2 > \gamma_{th}] \]

\[ = 1 - \left[ \int_{(M_S/\gamma_0)\gamma_{th}}^{\infty} p_{\gamma_1}(x) \, dx \right] \left[ \int_{(M_R/\gamma_0)\gamma_{th}}^{\infty} p_{\gamma_2}(x) \, dx \right] \]

\[ = 1 - \left[ 1 - P_{\gamma_1} \left( \frac{M_S}{\gamma_0} \gamma_{th} \right) \right] \left[ 1 - P_{\gamma_2} \left( \frac{M_R}{\gamma_0} \gamma_{th} \right) \right] \]

\[ = 1 - \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi \left( \frac{C_1}{\gamma_0} \right)^k \frac{\Gamma(k, C_1 \gamma_{th}/\gamma_0)}{(k-1)!} \left[ \sum_{l'=0}^{L-1} \sum_{k'=1}^{\mu'} \Psi' \left( \frac{C_2}{\gamma_0} \right)^k \frac{\Gamma(k'+1, C_2 \gamma_{th}/\gamma_0)}{(k'-1)!} \right]. \]  \hspace{1cm} (7.43)

**BER Analysis**

The BER for this relaying scheme is given by

\[ P_e = P_{e1} (1 - P_{e2}) + P_{e2} (1 - P_{e1}) = P_{e1} + P_{e2} - 2P_{e1}P_{e2}, \] \hspace{1cm} (7.44)

where \( P_{e1} \) and \( P_{e2} \) are the BERs of the source–relay and relay–destination links respectively. According to Equations (7.42) and (7.58), the pdf of \( \gamma_1 \) is given by

\[ p_{\gamma_1}(x) = \frac{M_S}{\gamma_0} \frac{\gamma_0}{\gamma_1} \frac{\Gamma(k, C_1 \gamma_{th}/\gamma_0)}{(k-1)!} \] \hspace{1cm} (7.45)

Hence, we can derive \( P_{e1} \) as follows:

\[ P_{e1} = \int_0^{\infty} Q(\sqrt{x}) p_{\gamma_1}(x) \, dx \]

\[ = \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi \left( \frac{C_1}{\gamma_0} \right)^k \frac{\Gamma(k+1/2)}{(k-1)!} \frac{1}{2} \sqrt{\gamma_0} \] \hspace{1cm} \( 2F1 \left( 1, k + \frac{1}{2}; k + 1; \frac{2}{\gamma_0/C_1 + 2} \right) \). \hspace{1cm} (7.46)
where we have used [93, Equation (8.359.3)] and [93, Equation (6.455.1)] respectively to obtain the third and last equalities. Similarly, we have

\[
P_{e2} = \frac{1}{2} \sqrt{\frac{\gamma_0}{\pi}} \sum_{\ell' = 0}^{L-1} \sum_{k' = 1}^{\mu'} \psi_{\ell'} \Gamma(k' + 1/2) \left( \frac{2}{\gamma_0/C_2 + 2} \right)^{k'} \\
\quad \times \sqrt{\gamma_0 + 2C_2} \left( \frac{2}{\gamma_0/C_2 + 2} \right).
\]

(7.47)

### 7.3.3 Transmitter-CSI-Assisted Relay Systems

In contrast to the receiver-CSI-assisted relay systems, the source and relay have partial knowledge of the source–relay and relay–destination link channels respectively in the transmitter-CSI-assisted relay systems, i.e., the source knows the channel matrices \( \mathbf{H}_{\text{SR},l} \), \( l = 0, 1, \ldots, L - 1 \), and the relay knows the channel matrices \( \mathbf{H}_{\text{RD},l} \), \( l = 0, 1, \ldots, L - 1 \). To achieve both spatial and multipath diversities in the latter systems, the pre-Rake technique [111, 252], which is similar to the time reversal (TR) technique, is adopted at the transmitter side and matched filtering is performed at the receiver side. In the following, we describe the relaying schemes for these systems.

#### 7.3.3.1 AF Relaying

In this relaying scheme, the pre-Rake technique [111, 252] is used. In the first hop, the transmitted signal vector at the source can be modelled as

\[
\mathbf{x}_S(t) = \sqrt{\frac{E_f}{J}} \sum_{i=1}^{N_t} \sum_{l=0}^{L-1} \mathbf{H}_{\text{SR},L-1-l} \mathbf{1}_{M_R}^T \mathbf{b}^T(t, i, l), 
\]

(7.48)

where

\[
\mathbf{g}(t, i, l) = \begin{bmatrix}
w(t - N_b(i - 1)T_f - lT_w) \\
w(t - N_b(i - 1)T_f - T_f - lT_w) \\
\vdots \\
w(t - N_b(i - 1)T_f - (N_b - 1)T_f - lT_w)
\end{bmatrix}.
\]

It can be easily shown that the total transmit energy per bit from the source is the same as that in the aforementioned relaying schemes. Note from Equation (7.48) that all the antennas at the source transmit the same data bit \( b_k \) (\( k = 1, 2, \ldots, N_b \)) at the same time. Furthermore, the source convolves the data bits with the time-reversed version of the partial CIRs for the first-hop links. This results in a strong peak of the total received signal at each receive antenna of the relay, and then only a matched filter (matched to the UWB monopulse \( w(t) \)) is needed to receive this path (see Chapter 6 and [111]). To gain a better understanding, let us first consider the received signal at the relay. After passing
through the matched filter and sampling, the received signal can be described in a matrix form as

\[ Y_{R,L}(i) = \sqrt{\frac{E_f}{J}} \mathbf{H}_{SR,l} \odot \mathbf{H}_{SR,L-1,l}^T \mathbf{1}_{M_R^{(i)}} \mathbf{b}^T + \mathbf{N}_{R,L}(i), \quad l = 0, \ldots, L - 1, \quad i = 1, \ldots, N_f. \]

Note that one flash sampling for the received signal actually represents the sampling at time instants \( t = N_b(i - 1) T_f + l T_w, N_b(i - 1) T_f + l T_w + T_f, \ldots, N_b(i - 1) T_f + l T_w + (N_b - 1) T_f \), producing \( N_b \) signals which correspond to the \( N_b \) transmitted data bits \( b_1, b_2, \ldots, b_{N_b} \) respectively. Therefore, \( Y_{R,L}(i) \) is a matrix for each fixed \( i \) and \( l \), and the \( k \)th column of \( Y_{R,L}(i) \) is the signal vector related to the data bit \( b_k \) received at the different receive antennas. One can show that all diagonal elements of \( \mathbf{H}_{SR,l} \odot \mathbf{H}_{SR,L-1,l}^T \) achieve their peaks at \( l = L - 1 \). Therefore, in order to estimate \( \{b_k\}_{k=1}^{N_b} \), the relay only needs the signal

\[ Y_{R,L-1}(i) = \sqrt{\frac{E_f}{J}} \sum_{k=0}^{L-1} \mathbf{H}_{SR,k} \mathbf{H}_{SR,k}^T \mathbf{1}_{M_R^{(i)}} \mathbf{b}^T + \mathbf{N}_{R,L-1}(i), \quad i = 1, \ldots, N_f. \]  

(7.49)

The estimate of the data bit vector \( \mathbf{b} \) at the relay is given by

\[ \mathbf{z}_R = \sum_{n=1}^{M_R^{(i)}} \sum_{i=1}^{N_f} ([Y_{R,L-1}(i)]_n)^T. \]

(7.50)

Subsequently, all the elements of \( \mathbf{z}_R \) are normalized with respect to

\[ \mathbb{E}_{\mathbf{H}}[|z_{R,k}|^2] = N_f [E_f (J + \mathcal{I})^2 / J + (M_R^{(i)} N_0 / 2)] \quad \text{with} \]

\[ \mathcal{I} = \sum_{l=0}^{L-1} \sum_{n=1}^{M_R^{(i)}} \mathbf{M}_{R_l} \mathbf{H}_{SR,l} \mathbf{H}_{SR,l}^T \mathbf{I}_{nn'}, \]

which yields \( \{\tilde{z}_{R,k}\}_{k=1}^{N_b} \). This normalization makes the transmitted signal vector at the relay have the same average power as that at the source.

In the second hop, the transmitted signal vector at the relay can be expressed as

\[ \mathbf{x}_R(t) = \sqrt{\frac{E_f}{J}} \sum_{l=0}^{L-1} \sum_{i=1}^{N_f} \mathbf{H}_{RD,L-1-l}^T \mathbf{1}_{M_D} \tilde{z}_R^T \mathbf{g}(t - N_b N_f T_f, i, l), \]

(7.51)

where \( \tilde{z}_R = [\tilde{z}_{R,1} \tilde{z}_{R,2} \cdots \tilde{z}_{R,N_b}]^T \). The received signals are sent to a matched filter (matched to the UWB pulse \( w(t) \)) and then sampled. The destination requires only the following sampled outputs to decode the data bits:

\[ Y_{D,L-1}(i) = \sqrt{\frac{E_f}{J}} \sum_{k=0}^{L-1} \mathbf{H}_{RD,k} \mathbf{H}_{RD,k}^T \mathbf{1}_{M_D} \tilde{z}_R^T + \mathbf{N}_{D,L-1}(i), \quad i = 1, \ldots, N_f. \]  

(7.52)
Accordingly, the destination can form the vector of the decision variables as

$$
\mathbf{z}_D = \sum_{n=1}^{M_D} \sum_{i=1}^{N_t} (\mathbf{Y}_{D,L-1}(i)]_n)^T
$$

(7.53)

After some algebra, we obtain the overall end-to-end SNR per bit as follows:

$$
\gamma = \frac{\left[ E_b (J + I)(J' + I') \right]^2}{M_D E_b N_0 J + M_R^{(t)} E_b N_0 J' + M_R^{(t)} M_D J' N_0^2},
$$

(7.54)

where

$$
\mathcal{I}' = \sum_{l=0}^{L-1} \sum_{n=1}^{M_D} \sum_{n'=1}^{M_R^{(t)}} \sum_{j=1}^{M_D} \sum_{j',n' \neq n} \mathbf{H}_{RD,l} j_{n'} [\mathbf{H}_{RD,l}]_{nn'}. 
$$

Notice that \( \mathcal{I} \) (and/or \( \mathcal{I}' \)) vanishes for \( M_R^{(t)} = 1 \) (and/or \( M_D = 1 \)).

**SNR Outage Probability Analysis**

Similar to the analysis in the preceding subsection, the cdfs of \( \mathcal{I} \) and \( \mathcal{I}' \) are needed to derive the SNR outage probability of \( \gamma \) in Equation (7.54). Unfortunately, it is difficult, if not impossible, to find those cdfs. Therefore, the SNR outage probability is calculated by means of simulations, except for the case where \( M_R^{(t)} = M_D = 1 \). In this case, Equation (7.54) reduces to Equation (7.29) with \( M_S \) and \( M_R^{(t)} \) being replaced by \( M_R^{(t)} \) and \( M_D \) respectively. Hence, the SNR outage probability can be expressed as Equation (7.34) with \( C_1 \) and \( C_2 \) being replaced by \( C_3 = M_R^{(t)}/\Phi_{SR,l} \) and \( C_4 = M_D/\Phi_{RD,l'} \) respectively.

**AoF Analysis**

In the case of \( M_R^{(t)} = M_D = 1 \), we use the high-SNR approximation for \( \gamma \) in Equation (7.54) as described in the preceding subsection. Consequently, Equation (7.54) reduces to Equation (7.37) with \( A_2 \) and \( A_3 \) being replaced by \( A_5 = M_D E_b N_0 \) and \( A_6 = M_R^{(t)} E_b N_0 \) respectively. Following the same procedure as in Equation (7.39), the approximation of the \( n \)th-order moment of \( \gamma \) in this case is obtained as Equation (7.39) with \( C_1 \) and \( C_2 \) being replaced by \( C_3 \) and \( C_4 \) respectively. In other cases, the \( n \)th-order moment is computed via simulations. Finally, the corresponding AoF is calculated according to Equation (7.35).

**BER Analysis**

The BER is calculated by means of simulations except for the case where \( M_R^{(t)} = M_D = 1 \). In this case, the approximate BER under the high-SNR assumption can be obtained as Equation (7.41) with \( C_1 \) and \( C_2 \) being replaced by \( C_3 \) and \( C_4 \) respectively.

**7.3.3.2 DTF Relaying**

In the DTF relaying scheme, the transmitted and received signals for the first hop are the same as Equations (7.48) and (7.49) respectively. The relay performs the data detection...
by producing the estimates of the transmitted bits as Equation (7.50) and making hard decisions on them. The vector of the detected bits is given by \( \hat{b}_R = \text{sign}(z_R) \). Those hard decisions result in superior performance compared with the AF relaying scheme, as will be seen in Section 7.3.4. In the second hop, the transmitted signals at the relay and the received signals at the destination can be expressed as Equations (7.51) and (7.52) respectively, with \( \tilde{z}_R \) being replaced by \( \hat{b}_R \). Ultimately, the decision variables can be obtained as Equation (7.53). It is straightforward to show that the received SNRs per bit for the first and second hops are respectively,

\[
\gamma_1 = \frac{(J + I)\gamma_0}{M_R^{(t)}} \quad \text{and} \quad \gamma_2 = \frac{(J' + I')\gamma_0}{M_D}.
\]

(7.55)

**SNR Outage Probability Analysis**

Comparing Equation (7.55) with Equation (7.42), we observe that they are of similar forms. Hence, we can follow the same approach as in the preceding subsection to derive the SNR outage probability. In the case of \( M_R^{(t)} = M_D = 1 \), we obtain the SNR outage probability as Equation (7.43) with \( C_1 \) and \( C_2 \) being replaced by \( C_3 \) and \( C_4 \) respectively. In other cases, the SNR outage probability is obtained through simulations.

**BER Analysis**

Similarly, we follow the approach of the preceding subsection to calculate the BER for this scheme. When \( M_R^{(t)} = M_D = 1 \), the BER is obtained as Equation (7.44), where \( P_{e1} \) and \( P_{e2} \) are the same as Equations (7.46) and (7.47) respectively, with \( C_1 \) and \( C_2 \) being replaced by \( C_3 \) and \( C_4 \) respectively. Otherwise, the BER is obtained by means of simulations.

### 7.3.4 Numerical Results and Discussion

In this subsection, some numerical results are presented to show the effectiveness of the proposed schemes. The system setup is depicted in Figure 7.1, where we assume that the source and destination are separated by a distance of 8 m, and the relay is located halfway between them. The total multipath gains \( G_{SR} \), \( G_{RD} \) and \( G_{SD} \) are calculated according to the UWB path loss model described in [35, Equation (1)]. In generating the UWB energy-normalized channel coefficients \( \varphi_{SR,l} \), \( \varphi_{RD,l} \) and \( \varphi_{SD,l} \), \( l = 0, \ldots, L_t - 1 \), we assume that \( L_t = 50 \), \( m = 2 \), \( \Omega_{SR,0} = \Omega_{RD,0} = \Omega_{SD,0} = 0.054 \) and \( \varphi_{SR} = \varphi_{RD} = \varphi_{SD} = 0.95 \). For illustration purposes, we only consider the cases where \( M_S, M_R^{(t)}, M_R^{(t)} \cdot M_D \leq 2 \).

In all simulations, we obtain the SNRs (i.e., \( \gamma \)) of the decision variables as follows. First, the signal and noise components of the decision variables for the DCF relay system are exactly generated according to the processing illustrated in Equations (7.23)–(7.25), (7.27) and (7.28), while those for the AF relay system are exactly generated according to the processing illustrated in Equations (7.49)–(7.50), (7.52) and (7.53). The generation of the signal and noise components in the DF and DTF cases follows the above processing with the modifications described in Sections 7.3.2.2 and 7.3.3.2 respectively. Second, calculating the power ratio between the signal and noise components for the relevant decision variable yields its SNR. This procedure is repeated for several different channel
realizations. The histograms of such channel ensembles give a discrete approximation of the pdf of the SNR so that the SNR outage probability and AoF can be computed. To simplify the simulations, we assume that \( N_t = 1 \) and hence \( E_f = E_b/2 \).

In the legends of Figures 7.8–7.13, the numbers in parentheses represent \( M_S, M_R^{(t)}, M_R^{(t)} \) and \( M_D \) respectively for the relay systems, and \( M_S \) and \( M_D \) respectively for the nonrelay systems. The transmit energy per bit in the nonrelay systems is set equal to that in the relay systems, i.e., \( E_b \).

Figure 7.8 shows the SNR outage probabilities of the receiver-CSI-assisted relay systems (computed with Equations (7.34) and (7.43) for the DCF and DF cases respectively) and corresponding nonrelay systems, all with \( L = 20 \). The outage results for the nonrelay case are obtained by using

\[
P_{\text{out}} = 1 - \sum_{l=0}^{L-1} \sum_{k=1}^{\hat{\mu}} \Psi(k, \hat{\gamma}_0) \frac{\Gamma(k, \hat{\gamma}_0 \gamma_{\text{th}}/\gamma_0)}{(k-1)!},
\]

where \( \hat{\mu} = M_S M_D m, \hat{\gamma} = M_S \Phi_{SD,1} \) and \( \Psi \) stands for \( \Psi(l, k, \hat{\mu}, \{\Phi_{SD,q}\}_{q=0}^{L-1}) \). The SNR threshold \( \gamma_{\text{th}} \) is chosen to be 10 dB. It is shown that deploying multiple antennas improves the outage performance of those systems. Moreover, significant improvement in the SNR outage probabilities of the DCF and DF relay systems is observed when increasing the number of receive antennas, i.e., \( M_R^{(t)} = M_D = 2 \). This corresponds to the fact that in each hop, with the partial CSI at the receiver side, using multiple receive antennas can provide

![Figure 7.8](image_url)  

**Figure 7.8**  
Outage probabilities of the receiver-CSI-assisted nonrelay and relay systems, all with \( L = 20 \).
Figure 7.9  Outage probabilities of the transmitter-CSI-assisted nonrelay and relay systems, all with $L = 20$.

Figure 7.10  BERs of the receiver-CSI-assisted relay systems, all with $L = 20$. 
Figure 7.11  BERs of the transmitter-CSI-assisted relay systems, all with $L = 20$.  

Figure 7.12  The effect of the correlation coefficient $\rho$ on the performance of the receiver-CSI-assisted relay systems.
a higher spatial diversity gain compared with using multiple transmit antennas \[258\]. With the same values of \( M_S, M_R^{(t)}, M_D^{(t)}, M_D \) and \( L \), the DF relay system generally yields a performance gain over its DCF counterpart due to the additional signal processing, i.e., the hard decisions, at the relay.

Figure 7.9 shows the SNR outage probabilities of the transmitter-CSI-assisted (i.e., AF and DTF) relay systems and corresponding nonrelay systems with the same SNR threshold as before. The results, except those for the case where \( M_R^{(t)} = M_D = 1 \), are obtained by simulations and then labelled with ‘(simu.)’. When \( M_R^{(t)} = M_D = 1 \), the SNR outage probabilities of the AF and DTF relay systems are calculated based on Equations (7.34) and (7.43) with the parameter replacement described in Section 7.3.3.1. In the nonrelay case with \( M_D = 1 \), the SNR outage probability can be directly obtained from Equation (7.56). It is clear from Figure 7.9 that, regardless of \( E_b/N_0 \), increasing the number of transmit antennas remarkably ameliorates the outage performance, compared with the case of employing a single transmit and receive antenna per hop. This is because the spatial diversity provided by multiple transmit antennas is properly exploited in the transmitter-CSI-assisted systems. On the other hand, increasing the number of receive antennas is not beneficial because the data transmission structure as represented by Equations (7.48) and (7.51) is somewhat simple for the case when \( M_R^{(t)} > 1 \) or \( M_D > 1 \).\(^5\) Advanced STC

\(^5\)This may be explained, for example, by considering Equation (7.54) for the AF relay system. Recall that when \( M_R^{(t)} > 1 \) (or \( M_D > 1 \)), \( I \) (or \( I' \)) in Equation (7.54) exists and can take either positive or negative values due to the equiprobable positive or negative polarity of the UWB multipath components. The negative values of \( I \) and \( I' \) might decrease \( \gamma \) in Equation (7.54) and, hence, degrade the corresponding SNR outage performance.
schemes might improve the efficiency of using multiple receive antennas in this kind of relay system. As shown in Figure 7.9, the SNR outage probabilities of the DTF relay systems are less than those of their AF counterparts.

Comparing Figure 7.9 with Figure 7.8, one can see that the AF and DCF relay systems, both with \((1,1,1,1)\) and \(L = 20\), yield the same performance. This comes from the fact that the two systems (i.e., Equations (7.54) and (7.29)) are exactly the same when \(M_S = M_R^{(i)} = M_D = 1\). However, for \((M_S, M_R^{(i)}, M_R^{(i)}, M_D) = (2,1,2,1)\) and \(L = 20\), the AF relay system yields superior performance. Similar trends can be observed in the DTF and DF relay systems as well. Among the systems considered with \(L = 20\), the DF relay system with \((2,2,2,2)\) achieves the best performance.

In order to verify the analytical BER results developed in the preceding subsections, we plot the BER curves of the receiver-CSI-assisted systems and transmitter-CSI-assisted systems in Figures 7.10 and 7.11 respectively. The analytical high-SNR approximations are labelled with ‘(appr.)’. As seen from both figures, the simulated results closely match the approximate ones, and the tightness of the approximations improves as \(E_b/N_0\) increases. Obviously, the results in these two figures support the outage performance comparisons presented earlier.

Table 7.1 shows the approximate AoFs developed in Sections 7.3.2.1 and 7.3.3.1 and their simulated counterparts, when \(E_b/N_0 = 25\) dB and the product of the numbers of transmit antennas, receive antennas and Rake/pre-Rake fingers is fixed in each hop, specifically, \(M_SM_R^{(i)}M_D = 20\). We can observe that, for both DCF and AF relay systems, the approximate AoFs are in good agreement with the simulated ones. Recollect that Equations (7.29) and (7.54) are identical when \(M_S = M_R^{(i)} = M_D = 1\). This is the reason why the DCF relay system with \((1,1,1,1)\) and \(L = 20\) possesses the same approximate and simulated AoFs as its AF counterpart. The results given in Table 7.1 indicate that increasing the number of transmit or receive antennas reduces the severity of UWB fading experienced by those relay systems more efficiently than increasing the number of Rake/pre-Rake fingers does.

The spatial correlation between the antennas has not been taken into account so far. From a practical point of view, it is important to investigate how such correlation affects the performance of the proposed relay systems. In what follows, we examine this effect on their outage performance. We adopt the spatial correlation model as described in [118] and Chapter 2. In Appendix 7.A, detailed calculations of the pdfs and cdfs of \(J\) and \(J'\) in the spatial correlation case are provided. Based on these functions, the SNR outage probabilities of the DCF and DF relay systems are obtained by numerically computing the

<table>
<thead>
<tr>
<th>System</th>
<th>((M_S, M_R^{(i)}, M_R^{(i)}, M_D))</th>
<th>(L)</th>
<th>AoF (simu.)</th>
<th>AoF (appr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCF</td>
<td>((1,1,1,1))</td>
<td>20</td>
<td>0.0149</td>
<td>0.0139</td>
</tr>
<tr>
<td>DCF</td>
<td>((2,1,2,1))</td>
<td>10</td>
<td>0.0146</td>
<td>0.0131</td>
</tr>
<tr>
<td>DCF</td>
<td>((2,2,2,2))</td>
<td>5</td>
<td>0.0142</td>
<td>0.0129</td>
</tr>
<tr>
<td>AF</td>
<td>((1,1,1,1))</td>
<td>20</td>
<td>0.0149</td>
<td>0.0139</td>
</tr>
<tr>
<td>AF</td>
<td>((2,1,2,1))</td>
<td>10</td>
<td>0.0139</td>
<td>0.0131</td>
</tr>
</tbody>
</table>
relevant integral terms in Equations (7.31) and (7.43) respectively, and the SNR outage probabilities for the AF and DTF cases can be calculated similarly when \( M_R^{(r)} = M_D = 1 \). Figures 7.12 and 7.13 respectively illustrate the effect of the spatial correlation coefficient \( \rho \) on the outage performances of the receiver-CSI-assisted and transmitter-CSI-assisted relay systems. We set \( M_S M_R^{(r)} = M_R^{(t)} M_D L = 8 \) to reduce the simulation time. To make the outage curves readable in Figure 7.12, some results for \( \rho = 0.5 \) are excluded. From both figures, it can be seen that the SNR outage probabilities of such relay systems increase as the spatial correlation coefficient increases. In particular, the DF and DTF relay systems are slightly more vulnerable to the spatial correlation compared with the DCF and AF ones, respectively. Furthermore, it is apparent from Figure 7.12 that in both DCF and DF cases, the performance penalty of such correlation becomes more evident when using multiple transmit and receive antennas. Because the spatial correlation in UWB channels depends mainly on antenna spacing [152], the above observations suggest that careful design of this spacing is important for the UWB MIMO relay systems.

### 7.4 Opportunistic Relaying for UWB Systems

Recently, opportunistic relaying schemes, in which only the best relay is chosen to be active among all available relays, have been proposed by Bletsas et al. [25] and shown to be outage-optimal for a multi-relay scenario. In this section, the performance of the opportunistic relaying scheme and that of the relaying schemes discussed in Section 7.3 will be compared in terms of their SNR outage probabilities. Here, our discussion is confined to proactive opportunistic relaying [25], which means that the relay selection is performed prior to the source transmission.

In the case of opportunistic relaying, we set \( M_R^{(r)} = M_R^{(t)} = 1 \) for all available relays and \( M_S = M_D = 1 \), as used in [25]. For the convenience of presentation, let \( \alpha_{S q,l} \) and \( \alpha_{qD,l} \) be the \( l \)th path channel coefficient between the source and \( q \)th relay and that between the \( q \)th relay and destination respectively, let \( Q = \{1, 2, \ldots, |Q|\} \) be the set of \( |Q| \) available relays and let \( q^* \) be the index of the best relay. In order to conform with the CSI assumption made in [25] and provide a fair comparison with the relaying schemes discussed in the preceding section, we assume that the \( q \)th relay knows both \( \alpha_{S q,l} \), \( l = 0, 1, \ldots, L - 1 \), and \( \alpha_{qD,l} \), \( l = 0, 1, \ldots, L - 1 \). Under this assumption, two opportunistic relaying schemes, namely opportunistic AF and opportunistic DTF, can be considered for UWB data transmissions. In the opportunistic AF scheme, the data transmission structure from the source to the best relay is described by Equations (7.22)–(7.25), with \( M_S = M_R^{(r)} = M_R^{(t)} = M_D = 1 \), and the data transmission structure from the relay to the destination is described by Equations (7.51)–(7.53). The description of such data transmissions for the opportunistic DTF scheme can be done in the same way, except that \( \tilde{z}_{R,1} \) in Equations (7.51) and (7.52) is substituted by \( \tilde{b}_{R,1} \), since the best relay makes a hard decision on \( z_{R,1} \) in Equation (7.25) (note that \( N_b = 1 \) as \( M_S = M_R^{(t)} = 1 \)). Now, the best relay can be selected from the following policy:

\[
q^* = \begin{cases} 
\arg \max_{q \in Q} \gamma_q, & \text{opportunistic AF,} \\
\arg \max_{q \in Q} \left[ \min(\gamma_{q,1}, \gamma_{q,2}) \right], & \text{opportunistic DTF,}
\end{cases}
\]
where $\gamma_q$ is the overall end-to-end SNR per bit, $\gamma_{q,1}$ and $\gamma_{q,2}$ are the received SNRs per bit for the first and second hops respectively and

$$\gamma_q = \frac{J_q J_q' \gamma_0^2}{J_q \gamma_0 + J_q' \gamma_0 + 1},$$

$$\gamma_{q,1} = J_q \gamma_0, \quad J_q = \sum_{l=0}^{L-1} \alpha_{S,q,l}^2,$$

$$\gamma_{q,2} = J_q' \gamma_0, \quad J_q' = \sum_{l=0}^{L-1} \alpha_{D,q,l}^2.$$

It is not difficult to show that the SNR outage probabilities for both schemes can be written as

$$P_{\text{out},q^*} = \prod_{q=1}^{\mid Q \mid} P_{\text{out},q},$$

where $P_{\text{out},q}$ is given by Equations (7.34) and (7.43) respectively in the opportunistic AF and opportunistic DTF cases, both with $M_S = M_R^{(r)} = M_R^{(t)} = M_D = 1$.

Figure 7.14 Comparison of the opportunistic and MIMO relay systems.
In Figure 7.14, we plot the SNR outage probabilities of the opportunistic relay systems together with those of the MIMO relay systems discussed in Section 7.3. The SNR threshold $\gamma_{th}$ is chosen to be 10 dB. It is found that the opportunistic AF and opportunistic DTF systems with $(1,1,1,1)$ and $L = 20$ need at least five and four available relays respectively to achieve SNR outage performance comparable to the DCF and DF systems with $(2,2,2,2)$ and $L = 10$. More interestingly, in the outage probability range of interest, the opportunistic AF system with $|Q| = 5$ and the opportunistic DTF system with $|Q| = 4$ perform worse than the AF system with $(2,1,2,1)$ and $L = 12$ and the DTF system with $(2,1,2,1)$ and $L = 13$ respectively. These results imply that the deployment of more transmit and/or receive antennas can be more beneficial than inserting additional single-antenna relays.

7.5 Summary

In this chapter we have analysed and compared the BER performance of UWB relay systems for both coherent and noncoherent detection schemes. The selective-Rake scheme with the pilot-aided channel estimation is used for coherent detection, while the DTR scheme is used for noncoherent detection. For both the selective-Rake and DTR schemes, the data transmission rates of the single-antenna relay system and two-antenna relay system are half of the data rate of the nonrelay system. However, the performance improvement obtained from all the above relay systems comes at no additional energy requirement.

Numerical results have revealed how the channel estimation errors, the number of selective-Rake fingers and the number of antennas at the relay affect the BER performance of relevant relay systems. It has been found that increasing the number of antennas at the relay is generally better than increasing the number of selective-Rake fingers.

For the general two-hop UWB MIMO relay systems, we have investigated the DCF and DF relaying schemes with the partial CSI available at the receiver side and the AF and DTF relaying schemes with the partial CSI available at the transmitter side. The SNR outage probabilities of these systems have been derived and the closed-form approximations for the AoFs and BERs of the systems in the high SNR regime have been presented. Simulation results show that these approximations are quite accurate even in the whole SNR range of interest. The numerical results shed some light on the design of the UWB MIMO relay systems.

Meanwhile, there has been increasing interest in the multi-relay transmissions (also known as cooperative relaying or virtual antenna arrays) [11, 29, 59, 74, 83, 128, 149, 162, 193, 194, 257, 276]. However, most research studies have focused on the narrowband systems, and further investigations in the UWB case are still ongoing. In [149], we presented a two-hop UWB system employing multiple DTF relays. It is found that, to avoid error propagation, the SNR threshold should be properly designed at each relay so that the relay is switched on only when the received SNR exceeds the threshold. Although the tradeoff among the numbers of relays, antennas and Rake/pre-Rake fingers has been investigated in [149], the impact of the threshold on the system performance is not yet fully clear and systematic design guidelines for the threshold remain open.

It is interesting to extend the multi-relay idea to the case of ad hoc relaying, i.e., the relay nodes can be working in a multihop manner; and whether or not a specific relay
node participates in relaying the received signals to other nodes (either an intermediate relay node or the destination node) or chooses to be silent depends on the communication scenarios and the quality of the received signals. This kind of relaying scheme may cover the following scenario: the relay nodes are fixed somewhere, while both source and destination nodes are mobile devices, and at which hop a relay node (or relay nodes) will work depends on both the positions of the source and destination nodes as well as the scattering environments.

When the relay concept is introduced into UWB communication systems, there are many design freedoms to improve the system performance. The main available freedoms for designing more advanced UWB relay systems are:

F1 the number of antennas in two-hop relay systems;
F2 the number of relays in multihop relay systems;
F3 the modulation/coding design to alleviate the critical synchronization problem in UWB systems;
F4 the number of Rake/pre-Rake fingers;
F5 the delay $\tau_d$ between the relayed signal at the relay and the transmitted signal at the source to mitigate interference;
F6 the prespecified SNR threshold to avoid error propagation in multi-relay systems.

Up to now, freedoms F1, F3, F4 and F6 were partly examined in [9, 147–150], and freedom F5 was mentioned in [45, 46]. Freedom F2 remains unexplored, and there are many options for freedom F3. It is worthwhile conducting further investigations on the potential benefits by combining all the above design freedoms in UWB relay systems.

**Appendix 7.A Derivations of cdfs and pdfs of $\mathcal{J}$ and $\mathcal{J}'$**

Referring back to Section 7.3.1, let us define

$$
\mu = M_S M_R^{(r)} M_{D_m}, \quad \mu' = M_R^{(t)} M_{D_m},
$$

$$
\Phi_{SR,l} = G_{SR} \theta_{SR,l} = G_{SR} \frac{\Omega_{SR,l}}{m}, \quad \Phi_{RD,l} = G_{RD} \theta_{RD,l} = G_{RD} \frac{\Omega_{RD,l}}{m}
$$

for ease of exposition. We first consider the case that the channel coefficients $[H_{SR,l}]_{ij}$ are statistically independent and so are $[H_{RD,l}]_{ij}$. We refer to this case as the spatial uncorrelation case. Using Equations (7.20) and (7.21) and with the help of [12, Equation (7)], we obtain

$$
P_{\mathcal{J}}(x; \mu, \Phi_{SR,l}) = \frac{x^{\mu-1}}{\Phi_{SR,l}(\mu - 1)!} \exp \left( -\frac{x}{\Phi_{SR,l}} \right) U(x), \quad (7.57)
$$

$$
P_{\mathcal{J}}(x; \mu, \Phi_{SR,l}) = \left[ 1 - \frac{\Gamma(\mu, x/\Phi_{SR,l})}{(\mu - 1)!} \right] U(x)
$$

$$
= \left[ 1 - \exp \left( -\frac{x}{\Phi_{SR,l}} \right) \sum_{v=0}^{\mu-1} \frac{1}{v!} \left( \frac{x}{\Phi_{SR,l}} \right)^v \right] U(x).
$$
Applying Theorem 1 of [120], the pdf and cdf of $\mathcal{J}$ can be derived as

$$p_{\mathcal{J}}(x) = \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) P_{\mathcal{J}}(x; k, \Phi_{SR,l})$$

(7.58)

and

$$P_{\mathcal{J}}(x) = \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) P_{\mathcal{J}}(x; k, \Phi_{SR,l})$$

(7.59)

respectively, where

$$\Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) = \frac{(-1)^{(L-1)\mu} \Phi_{SR,l}^k}{[(\mu - 1)]^{L-1} \prod_{q=0}^{L-1} \Phi_{SR,q}^{\mu}} \sum_{i_1=k}^{\mu} \sum_{i_2=k}^{\mu} \cdots \sum_{i_{L-2}=k}^{\mu} \left\{ \frac{(2\mu - i_1 - 1)! (i_{L-2} + \mu - k - 1)!}{(\mu - i_1)! (i_{L-2} - k)!} \right\}$$

$$\times \left[ \frac{1}{\Phi_{SR,l}} - \frac{1}{\Phi_{SR,0+U(-\mu)}} \right]^{i_1-2\mu} \left[ \frac{1}{\Phi_{SR,l}} - \frac{1}{\Phi_{SR,r+U(\mu-l-1)}} \right]^{k-i_{L-2}-\mu} \prod_{p=1}^{L-3} \frac{(\mu + i_p - i_{p+1} - 1)!}{(i_p - i_{p+1})!} \left[ \frac{1}{\Phi_{SR,l}} - \frac{1}{\Phi_{SR,r+U(p-l)}} \right]^{i_{p+1}-i_p-\mu} \right).$$

Following [120, Equation (8)], we can calculate $\Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1})$ recursively as

$$\Psi(l, \mu - k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) = \frac{\mu}{k} \sum_{p,q=0, q \neq l}^{L-1} \frac{1}{\Phi_{SR,l}} \left( \frac{1}{\Phi_{SR,l}} - \frac{1}{\Phi_{SR,q}} \right)^{-p+1}$$

$$\times \Psi(l, \mu - k + p + 1, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1})$$

(7.60)

with

$$\Psi(l, k, \mu, \{\Phi_{SR,q}\}_{q=0}^{L-1}) = \frac{1}{\prod_{q=0, q \neq l}^{L-1} \Phi_{SR,q}^{\mu} \prod_{p=0, p \neq l}^{L-1} \left( \frac{1}{\Phi_{SR,p}} - \frac{1}{\Phi_{SR,l}} \right)^{-\mu}.}

Similarly, the pdf and cdf of $\mathcal{J}'$ are obtained as Equations (7.58) and (7.59) respectively, with $\mu$ being replaced by $\mu'$ and with $\Phi_{SR,q}$ (and $\Phi_{SR,l}$) being replaced by $\Phi_{RD,q}$ (and $\Phi_{RD,l}$).

Next, we extend the above derivation to the so-called spatial correlation case where the channel coefficients $[\mathbf{H}_{SR,l}]_{ij}$ are statistically correlated and so are $[\mathbf{H}_{RD,l}]_{ij}$. Following the approach outlined in [15], it is straightforward to show that the MGF of $\mathcal{J}_l$ is

$$\mathcal{M}_{\mathcal{J}_l}(s) = \left[ \det \left( \mathbf{I} - s \Phi_{SR,l} \mathbf{\Gamma}_{SR} \right) \right]^{-m},$$

(7.61)
where $\Upsilon_{SR}$ is the channel correlation matrix. For more details about UWB spatial correlation modelling, readers are referred to [118]. For analytical simplicity, it is assumed that the correlation properties at the transmitter side are independent of those at the receiver side [106]. Under this assumption, the correlation matrix is modelled as $\Upsilon_{SR} = \Upsilon_S \otimes \Upsilon_R^{(r)}$, where $\Upsilon_S$ and $\Upsilon_R^{(r)}$ are the $M_S \times M_S$ transmit and $M_R^{(r)} \times M_R^{(r)}$ receive covariance matrices respectively. Moreover, we assume an exponential correlation model, which means that $[\Upsilon_S]_{pq} = \rho^{|p-q|}$ and $[\Upsilon_R^{(r)}]_{pq} = \rho^{|p-q|}$, where $\rho$ is the correlation coefficient between two neighbouring antennas, $p,q = 1,2,\ldots,M_S$ and $p',q' = 1,2,\ldots,M_R^{(r)}$. Let $\{\lambda_c\}_{c=1}^{N_\lambda}$ be the distinct eigenvalues of $\Upsilon_{SR}$ where $\lambda_c$ has algebraic multiplicity $\kappa_c$. Then the MGF can be expressed as [15]

$$M_{J_l}(s) = \prod_{c=1}^{N_\lambda} \frac{1}{(1 - s \Phi_{SR,l}\lambda_c)^{\kappa_c m}}. \quad (7.62)$$

Using a partial fraction expansion of the product in Equation (7.62), we get

$$M_{J_l}(s) = \sum_{c=1}^{N_\lambda} \sum_{j=1}^{\kappa_c m} \frac{\varepsilon_{c,j}}{(1 - s \Phi_{SR,l}\lambda_c)^j}.$$

It is easy to verify that the coefficients $\varepsilon_{c,j}$ do not depend on $\Phi_{SR,l}$. They depend only on parameter $\rho$. By taking the inverse Laplace transform of $M_{J_l}(-s)$, the pdf of $J_l$ is obtained as follows:

$$p_{J_l}(x) = \sum_{c=1}^{N_\lambda} \sum_{j=1}^{\kappa_c m} \frac{\varepsilon_{c,j} x^{j-1} \exp \left( -\frac{x}{\Phi_{SR,l}\lambda_c} \right)}{(\Phi_{SR,l}\lambda_c)^j (j-1)!} U(x)$$

$$= \sum_{c=1}^{N_\lambda} \sum_{j=1}^{\kappa_c m} \varepsilon_{c,j} p_{\Upsilon}(x; j, \Phi_{SR,l}\lambda_c), \quad (7.63)$$

where $\Upsilon$ is the Gamma distributed random variable. Thus, we have

$$P_{J_l}(x) = \sum_{c=1}^{N_\lambda} \sum_{j=1}^{\kappa_c m} \left\{ 1 - \frac{\Gamma[j,x/(\Phi_{SR,l}\lambda_c)]}{(j-1)!} \right\} U(x) = \sum_{c=1}^{N_\lambda} \sum_{j=1}^{\kappa_c m} \varepsilon_{c,j} P_{\Upsilon}(x; j, \Phi_{SR,l}\lambda_c). \quad (7.64)$$

Unlike in the spatial uncorrelation case, the pdf and cdf of $J$ for the spatial correlation case cannot be simply expressed as the forms of Equations (7.63) and (7.64) respectively, except for the case where $L = 1$. However, the pdf can be numerically computed as the inverse Laplace transform of $\prod_{l=L}^{L-1} M_{J_l}(-s)$, and then the cdf can be readily obtained. Likewise, the MGF of $J'_l$ is written as Equation (7.61), with $M_{SR}$, $M_{R}^{(r)}$, $m$, $\Phi_{SR,l}$ and $\Upsilon_{SR}$ being replaced by $M_{R}^{(t)}$, $M_D$, $\Phi_{RD,l}$ and $\Upsilon_{RD} = \Upsilon_R^{(t)} \otimes \Upsilon_D$ respectively. Hence, the pdf and cdf of $J'_l$ can be derived in the same way as those of $J_l$, and the pdf and cdf of $J'$ can be numerically evaluated.
Appendix 7.B Derivation of Equation (7.33)

Using Equation (7.57) and [93, Equations (8.352.2) and (1.111)], the integral \( Q_3 \) in Equation (7.32) can be rewritten as

\[
Q_3 = \frac{\exp\left[-\frac{(A_2 \gamma_{th})}{(A_1 \Phi_{RD,l'})}\right]}{\Phi_{SR,l}(k - 1)!} \sum_{j_1=0}^{k-1} \sum_{j_2=0}^{j_1} \frac{1}{j_1! j_2!} \left(-\frac{A_2 \gamma_{th}}{A_1 \Phi_{RD,l'}}\right)^{j_1} \left(\frac{A_4}{A_2} + \frac{A_3}{A_1} \gamma_{th}\right)^{j_2} 
\]

\[
\times \int_{(A_3/A_1)\gamma_{th}}^{\infty} x^{k-1} \exp\left[-\frac{x}{\Phi_{SR,l}} - \frac{\gamma_{th}(A_1 A_4 + A_2 A_3 \gamma_{th})}{A_1 \Phi_{RD,l'}(A_1 x - A_3 \gamma_{th})}\right] \left(x - \frac{A_3}{A_1} \gamma_{th}\right)^{-j_2} dx 
\]

(7.65)

After tedious algebraic manipulations and with the help of [93, Equation (3.471.9)], the integral on the right-hand side of Equation (7.65) can be expressed as

\[
\int_{(A_3/A_1)\gamma_{th}}^{\infty} x^{k-1} \exp\left[-\frac{x}{\Phi_{SR,l}} - \frac{\gamma_{th}(A_1 A_4 + A_2 A_3 \gamma_{th})}{A_1 \Phi_{RD,l'}(A_1 x - A_3 \gamma_{th})}\right] \left(x - \frac{A_3}{A_1} \gamma_{th}\right)^{-j_2} dx 
\]

\[
= 2 \exp\left(-\frac{A_3 \gamma_{th}}{A_1 \Phi_{SR,l'}}\right) \sum_{j_3=0}^{k-1} \left(k - 1\right) \left(\frac{A_3}{A_1} \gamma_{th}\right)^{k-j_3-1} 
\]

\[
\times \left[\frac{A_2 \Phi_{SR,l}/\gamma_{th}}{A_1 \Phi_{RD,l'}} \left(\frac{A_4}{A_2} + \frac{A_3}{A_1} \gamma_{th}\right)\right]^{(j_3-j_2+1)/2} 
\]

\[
\times K_{j_3-j_2+1} \left[2 \sqrt{\frac{A_2 \gamma_{th}}{A_1 \Phi_{SR,l'} \Phi_{RD,l'}}} \left(\frac{A_4}{A_2} + \frac{A_3}{A_1} \gamma_{th}\right)\right]. 
\]

(7.66)

Plugging Equation (7.66) into Equation (7.65) yields Equation (7.33).

Appendix 7.C The pdf of the End-to-End SNR per Bit for the DCF Relay System

Taking the derivative of Equation (7.34) with respect to \( \gamma_{th} \), we obtain the pdf for \( \gamma \) of Equation (7.29) as

\[
p_{\gamma}(x) = 2 \sum_{l=0}^{L-1} \sum_{k=1}^{\mu} \Psi_{L-1} \sum_{l'=0}^{\mu'} \Psi'_{L-1} \sqrt{\frac{C_2}{C_1}} \left(1 + \frac{1}{x}\right) \left(\frac{C_1 x}{\gamma_0}\right)^k \exp\left[-\frac{x}{\gamma_0} (C_1 + C_2)\right] 
\]

\[
\times \sum_{j_1=0}^{k-1} \frac{(C_2 x / \gamma_0)^{j_1}}{j_1!} \sum_{j_2=0}^{j_1} \left(\frac{j_1}{j_2}\right) \left[\frac{C_1}{C_2} \left(1 + \frac{1}{x}\right)\right]^{j_2/2} \sum_{j_3=0}^{k-1} \left(\frac{k - 1}{j_3}\right) \left[\frac{C_2}{C_1} \left(1 + \frac{1}{x}\right)\right]^{j_3/2} 
\]

\[
\times \left( \left[ \frac{1}{\gamma_0} (C_1 + C_2) - \frac{j_1 + k}{x} + \frac{1 + j_2 + j_3}{2x(1 + x)} \right] K_{j_3 - j_2 + 1} \left[ \frac{2}{\gamma_0} \sqrt{C_1C_2x(1 + x)} \right] \\
+ \frac{1}{2\gamma_0} \left( 2 + \frac{1}{x} \right) \sqrt{\frac{C_1C_2x}{1 + x}} \left\{ K_{j_3 - j_2} \left[ \frac{2}{\gamma_0} \sqrt{C_1C_2x(1 + x)} \right] \\
+ K_{j_3 - j_2 + 2} \left[ \frac{2}{\gamma_0} \sqrt{C_1C_2x(1 + x)} \right] \right\} \right) U(x). \tag{7.67}
\]
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